

Homework Assignment #12

Computational Geometry (Winter Term 2016/17)

Exercise 1

Let S be a set of n points in the plane.

- Suppose all points from S are collinear. Let r be a parameter with $1 \leq r \leq n$. Draw a fine simplicial partition for S of size r . What is the crossing number of your partition? **[4 points]**
- Now suppose that the points in S lie on a $\sqrt{n} \times \sqrt{n}$ grid. (Assume for simplicity that n is a square number.) Let r be a parameter with $1 \leq r \leq n$. Draw a fine simplicial partition for S of size r and crossing number $O(\sqrt{r})$. **[4 points]**

Exercise 2

In this exercise we want to solve the recurrences from the lecture. You can assume that the partition size r is at least 3.

- Let $M(n)$ be the maximum number of nodes in a partition tree for a set of n points in the plane. Prove that the recurrence for $M(n)$ given in the lecture (and in Lemma 16.2 in the book) solves to $M(n) \in O(n)$. **[3 points]**
- Let $Q(n)$ be the maximum query time in a partition tree for a set of n points in the plane. Prove that the recurrence for $Q(n)$ given in the lecture (and in Lemma 16.3 in the book) solves to $Q(n) \in O(n^{1/2+\epsilon})$. **[3 extrapoints]**
- Suppose that we have a partition tree, except that the simplicial partitions used in the construction are not necessarily fine. What does this mean for the amount of storage used by the partition tree? And for its query time? **[3 points]**

Exercise 3

Let T be a set of n triangles in the plane. An *inverse range counting* query asks to count the number of triangles from T containing a query point p .

- a) Let O and U be the set of upper and lower edges from T . Further, let P be the ray starting in the query point p and moving vertically upwards. Describe how to determine the desired number of triangles from the number of intersections between P and the triangle edges in O and U , respectively. **[2 points]**
- b) Now design a data structure for inverse range counting queries by augmenting the multi-level partition trees from the lecture. **[4 points]**
- c) Analyze the memory consumption and the query time of your data structure. **[3 extrapoints]**

This assignment is due at the beginning of the next lecture, that is, on February 1 at 10:15. Solutions will be discussed in the tutorial on Friday, February 3, 14:15–15:45 in room SE I.