Lehrstuhl für Informatik I
Effiziente Algorithmen und
wissensbasierte Systeme
Universität Würzburg

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Homework Assignment #8
Computational Geometry (Winter Term 2016/17)

Exercise 1

Let P be a set of n points in the plane. Give an $O(n \log n)$ time algorithm to find for each point p in P another point $a(p)$ in P that is closest to p. **[3 points]**

Exercise 2

Consider the beach line in Fortune's sweep for computing the Voronoi diagram of a finite set P of points in the plane.

a) Do the breakpoints of the beach line, i.e., the intersection points of two adjacent arcs on the beach line, always move downwards when the sweep line moves downwards? Prove this or give a counterexample. **[4 points]**

b) Give an example where the parabola defined by some site p in P contributes more than one arc to the beach line. Can you give an example where one site contributes a linear number of arcs? **[4 points]**

Please turn over.

Exercise 3

Suppose you are an anti-nuclear activist who wants to live as far away as possible from all known nuclear power plants. However, you still want to live within some preferred region, say Europe. This situation can be formalized as follows. The set of nuclear power plants is represented by a set S of n points in the plane. For the sake of simplicity, we (initially) assume that your preferred region is represented by a rectangle R (R does not necessarily contain all the points of S). The task is to find a point $p \in R$ (your place of residence) that maximizes the distance $\min_{s \in S} d(p, s)$ to the closest nuclear power plant.

a) Show that every optimal place of residence is a corner of the Voronoi Diagram $Vor(S)$, a corner of R, or an intersection point between the boundary of R and an edge of $Vor(S)$. **[6 points]**

b) Suppose the Voronoi Diagram $Vor(S)$ is already known. Give an algorithm that finds an optimal place of residence in $O(n)$ time. **[3 points]**

c) Now suppose that your preferred region is a convex polygon P (P does not necessarily contain all the points of S) with m corners. Again, the Voronoi Diagram $Vor(S)$ is already known. Give an algorithm that finds an optimal place of residence in $O(m + n)$ time. **[7 extrapoints]**

This assignment is due at the beginning of the next lecture, that is, on December 21 at 10:15. Solutions will be discussed in the tutorial on Friday, December 23, 14:15–15:45 in room SE I.