Lehrstuhl für Informatik I
Effiziente Algorithmen und
wissensbasierte Systeme
Universität Würzburg

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Dr. Joachim Spoerhase
M.Eng. Dongliang Peng

Homework Assignment #4
Computational Geometry (Winter Term 2016/17)

Exercise 1

We consider the following problem in the plane: Given a polygon P and a convex polygon Q, we want to scale P such that, after displacement, P completely fits into Q and is as large as possible. That is to say, we are looking for a vector $\vec{v} \in \mathbb{R}^2$ that allows for the largest possible scaling factor $\lambda > 0$ such that $\lambda \cdot P + \vec{v} \subseteq Q$.

Give a linear program that solves this problem. **[5 points]**

Exercise 2

Algorithm 1: RandMax

Input: Finite set $A \subset \mathbb{R}$
Output: Maximum $\max_{a \in A} a$ of A

```
1  if |A| = 1 then
2      return the only element a ∈ A
3  else
4      a = randomly chosen element from A
5      b = RandMax(A \ {a})
6      if b ≥ a then
7          return b
8      else
9          unnecessarily test every element of A \ {a} to ensure that a is maximum
10         return a
```

Consider Algorithm 1, which calculates the maximum of a set of numbers. First, give a tight bound on the worst-case running time of the algorithm. Then, consider the random choice of the element a and show that the expected running time is strictly less than the worst-case running time of the algorithm. **[6 points]**

Exercise 3

Consider n trains that are moving on parallel tracks. Train z_i ($i = 1, \ldots, n$) is moving with constant speed v_i and is at position p_i at time 0. At time 0, only time starts; all trains already run at their respective speeds. Design an algorithm that lists, in $O(n \log n)$ time, for a given number $t_{stop} > 0$, any train that has been ahead of the others at least once in the time interval $[0, t_{stop}]$.

Hint: Convert the problem into a geometric problem; then solve it with algorithms known from the lecture. **[9 points]**

This assignment is due at the beginning of the next lecture, that is, on November 30 at 10:15. Solutions will be discussed in the tutorial on Friday, December 2, 14:15–15:45 in room SE I.