Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems

Institute for Informatics

Computational Geometry

Winter term 2016/17

Triangulating Polygons

or

Guarding Art Galleries

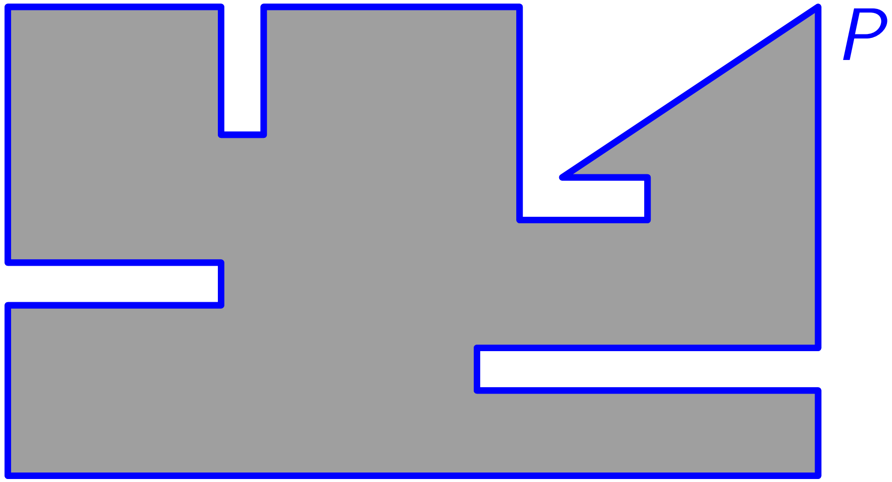
Lecture #3

Joachim Spoerhase

Chair for Informatics I

Guarding an Art Gallery

Given a *simple* polygon P (i.e., no holes, no self-intersection)...



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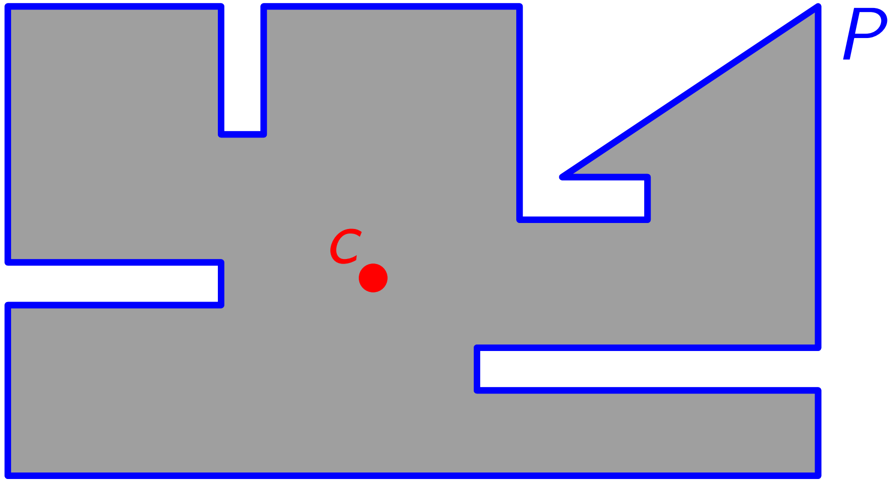

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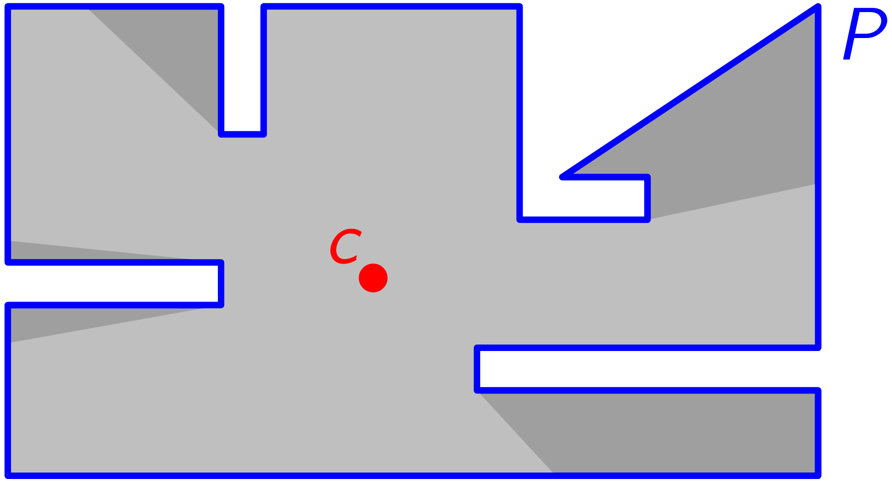

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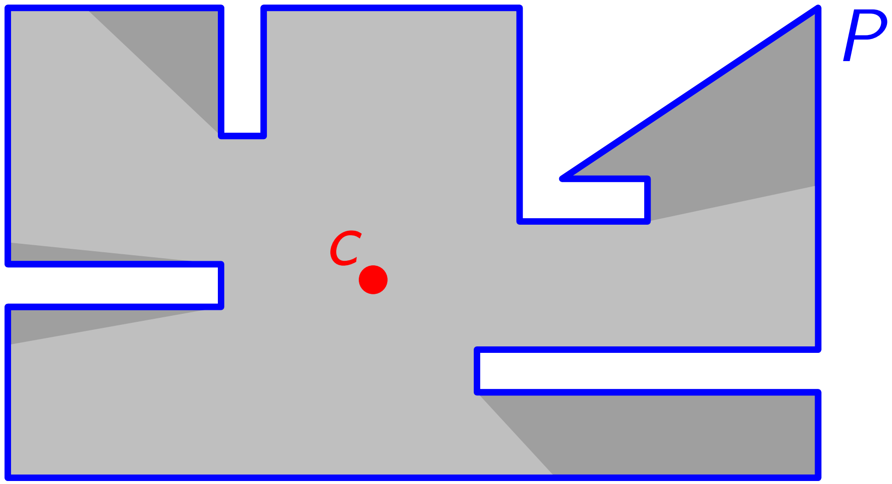

Observation. Camera c "sees" a star-shaped region

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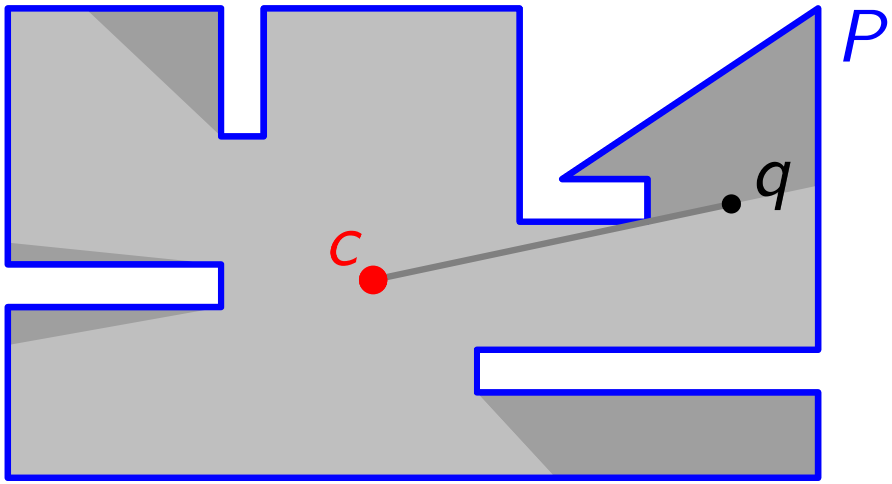

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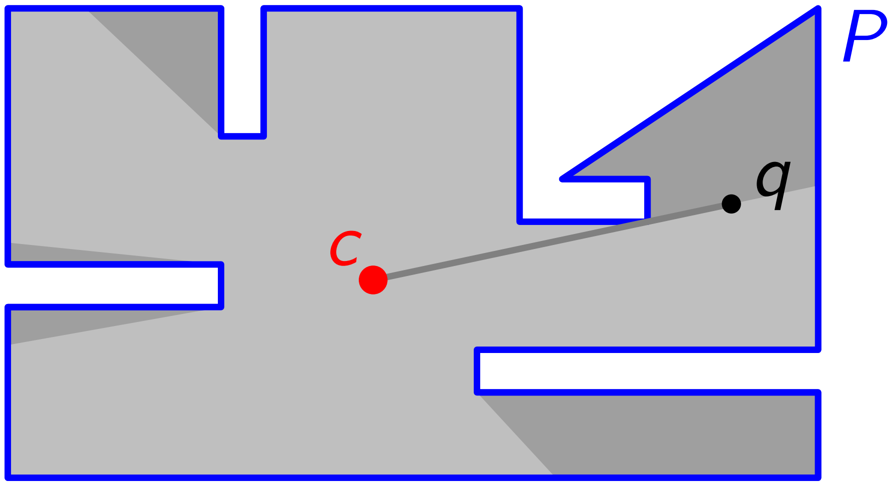

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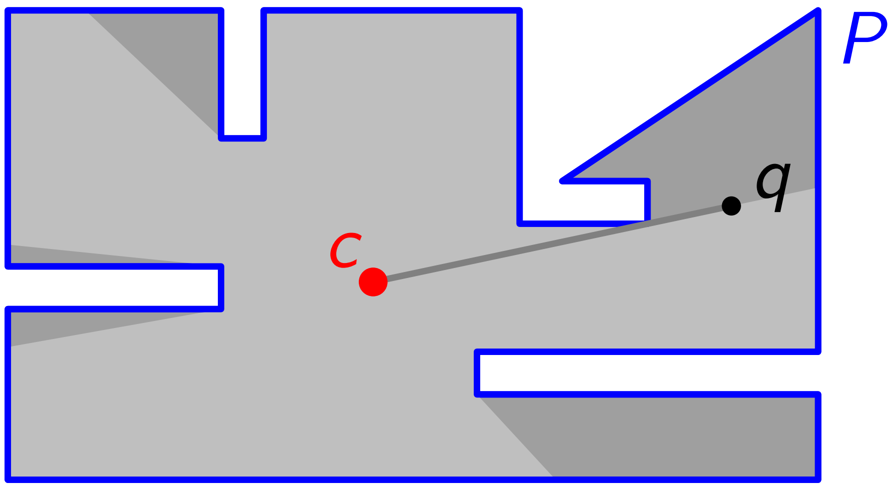

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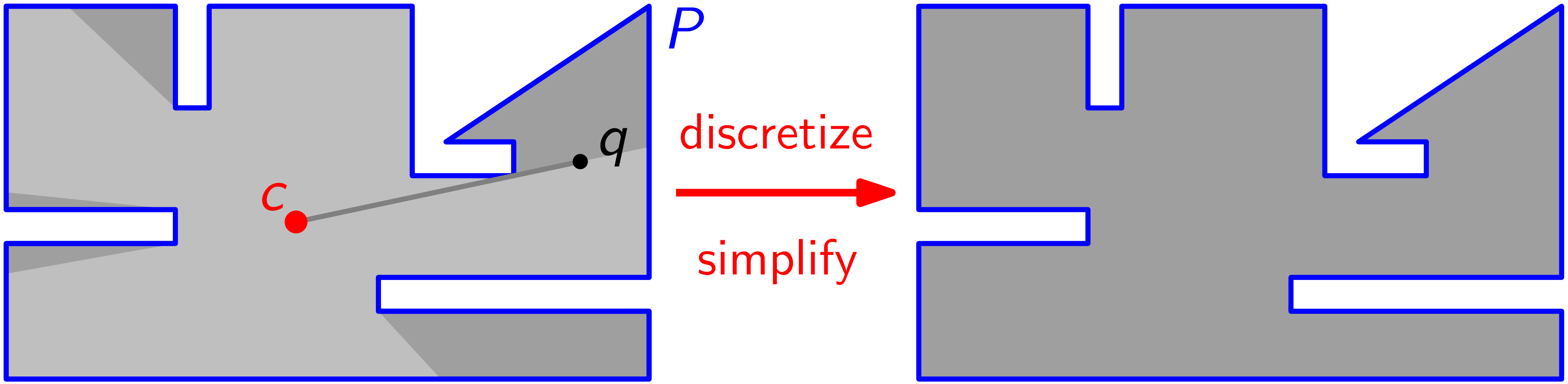

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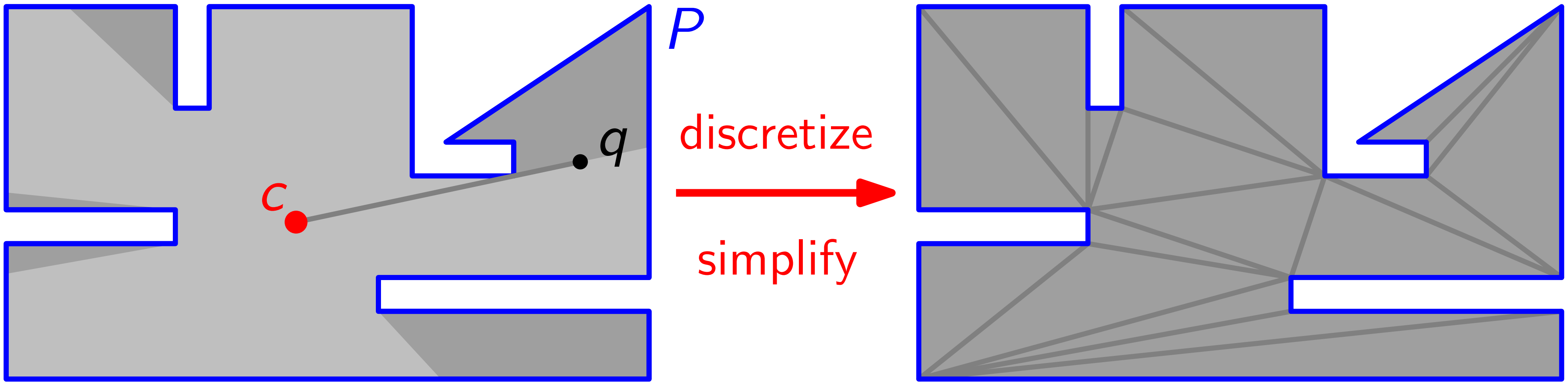

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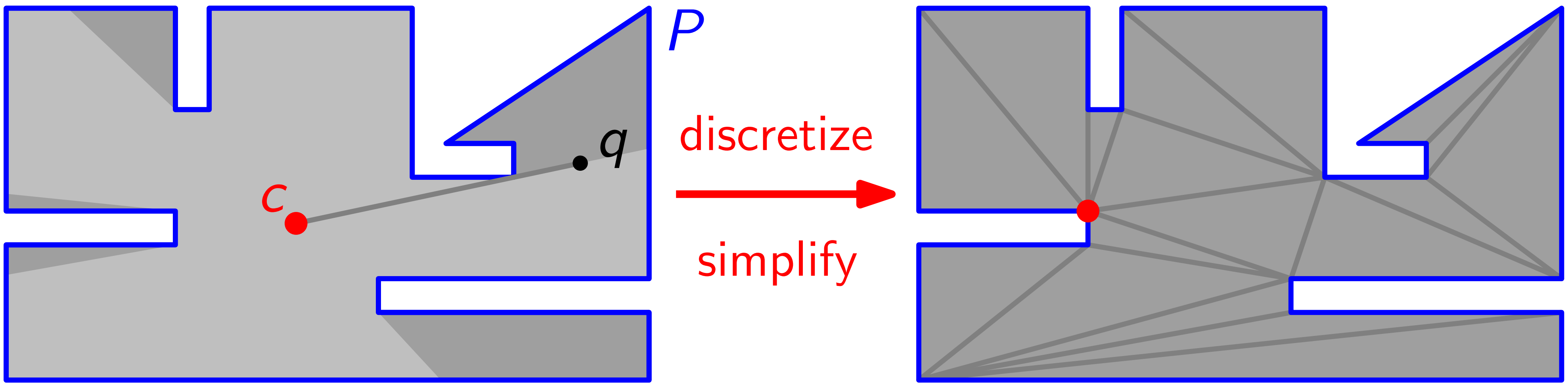

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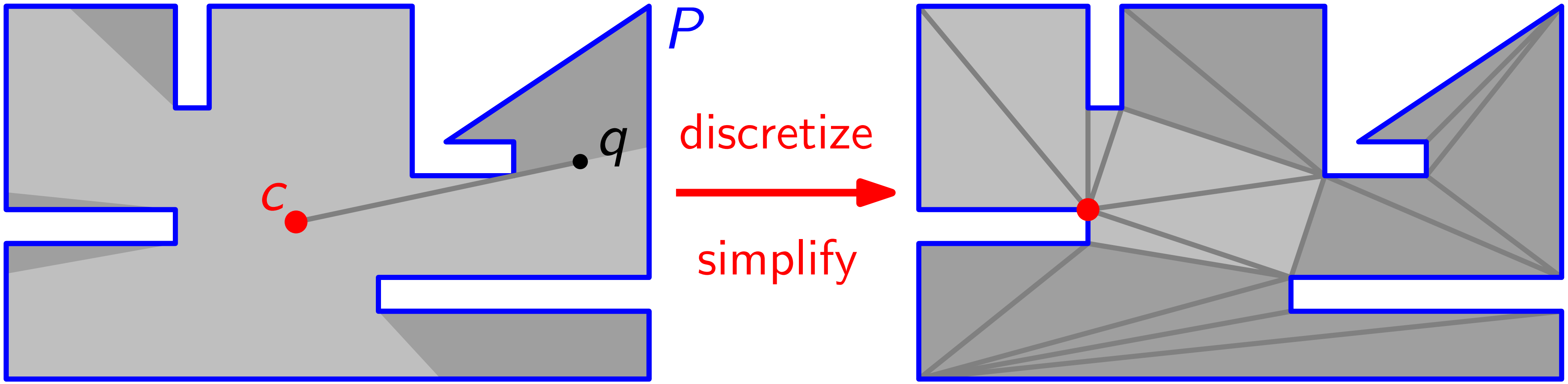

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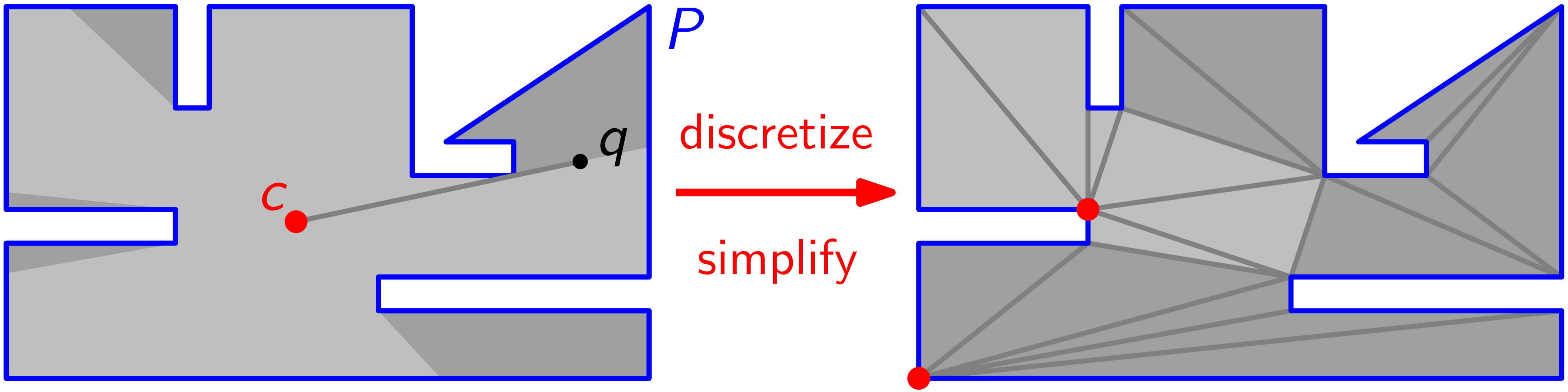

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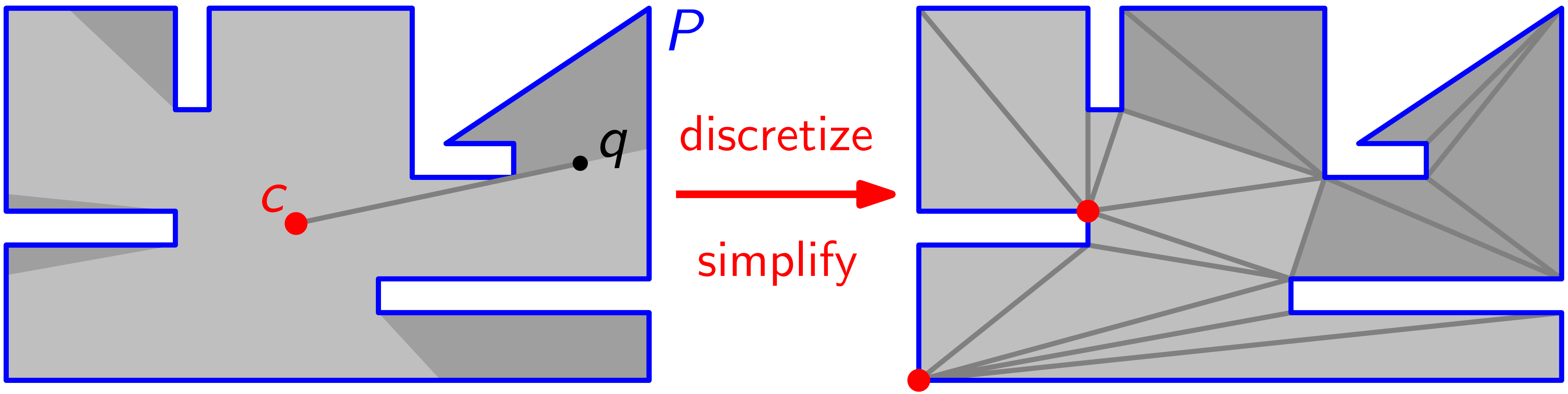

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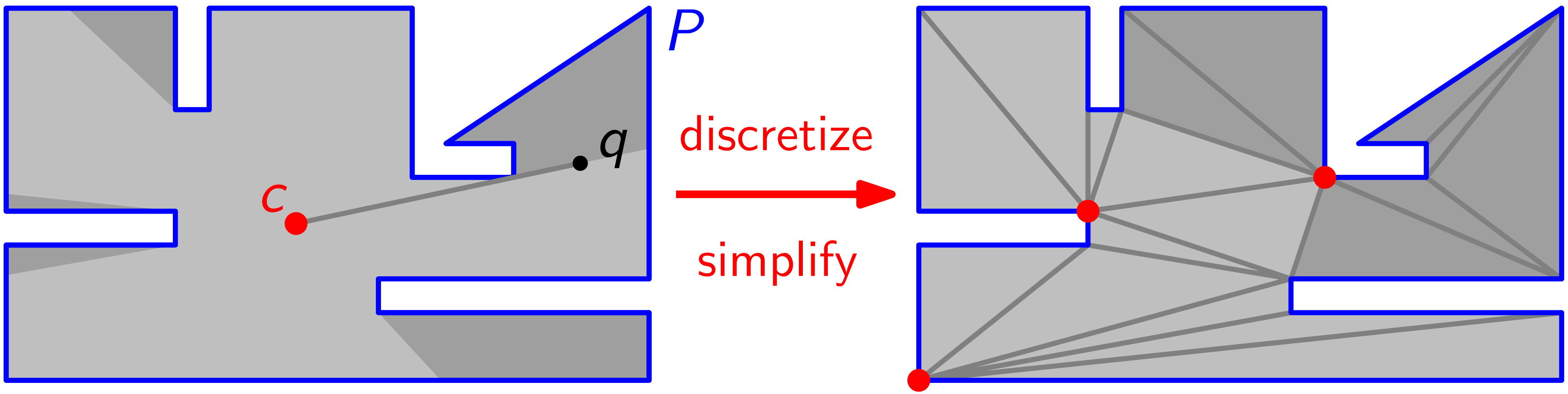

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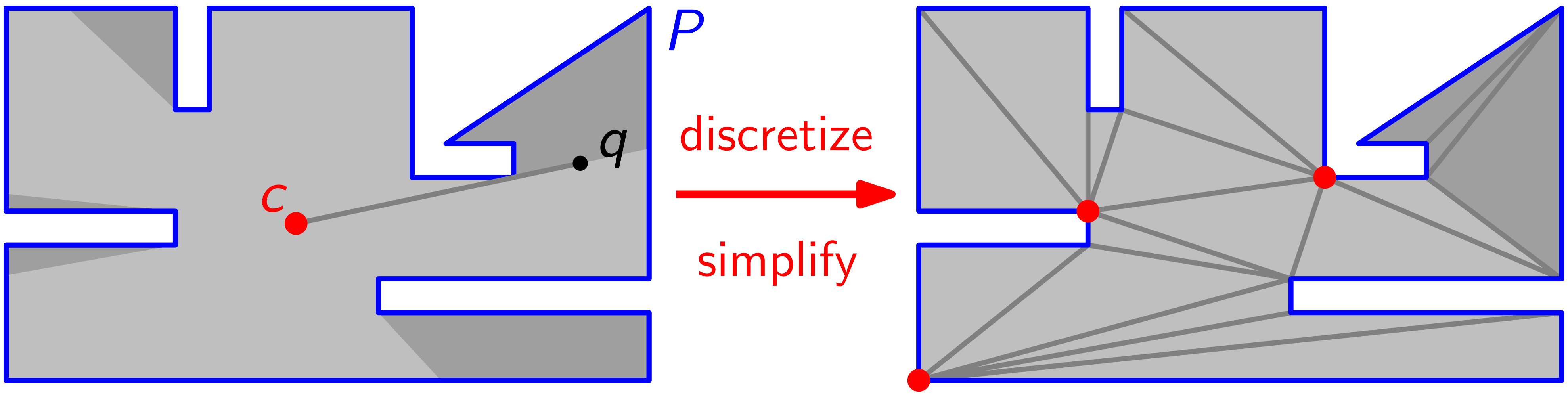

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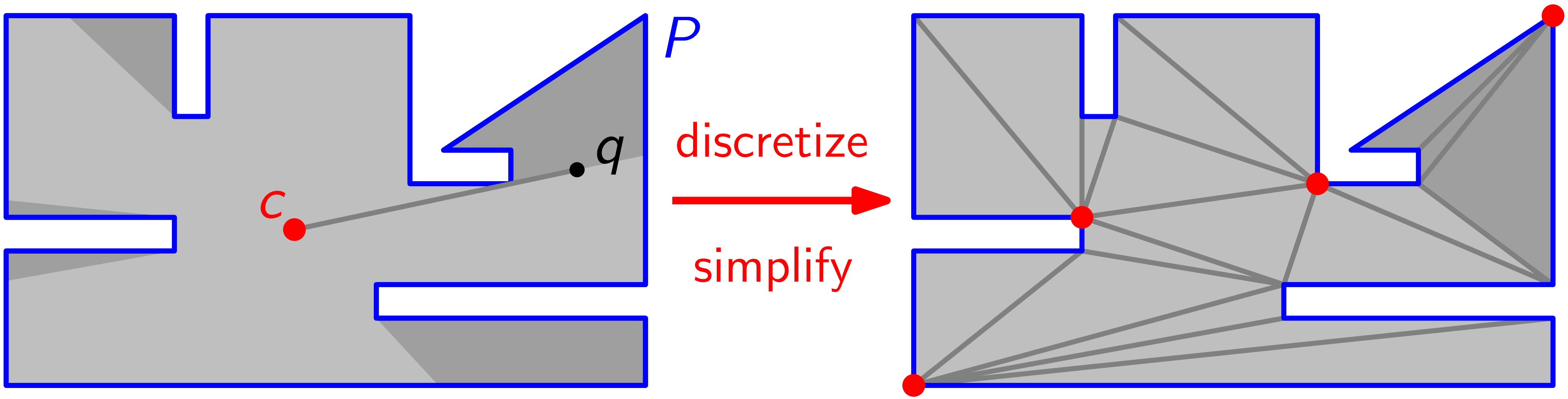

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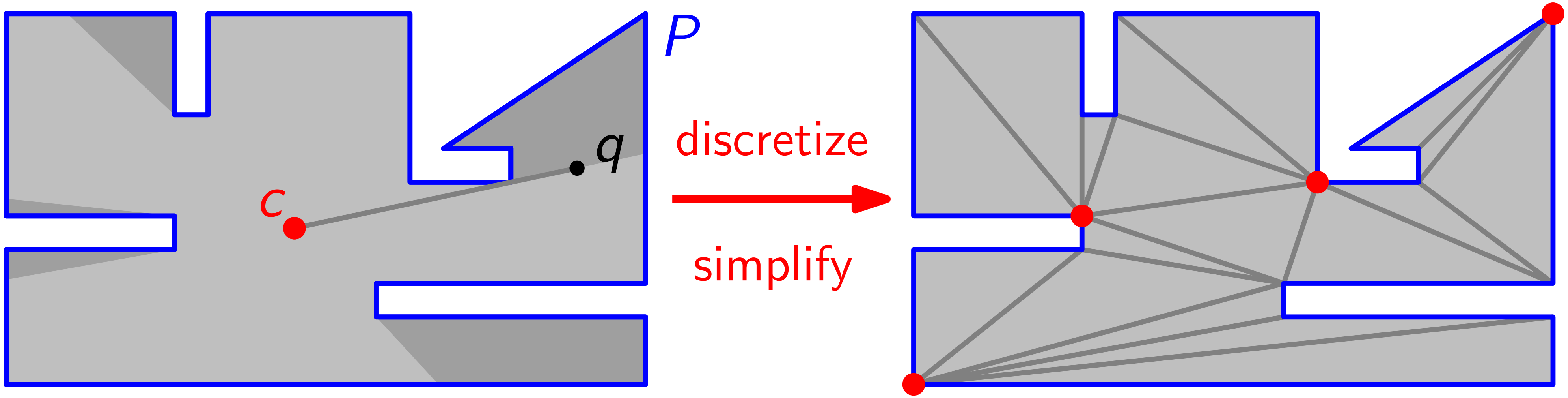

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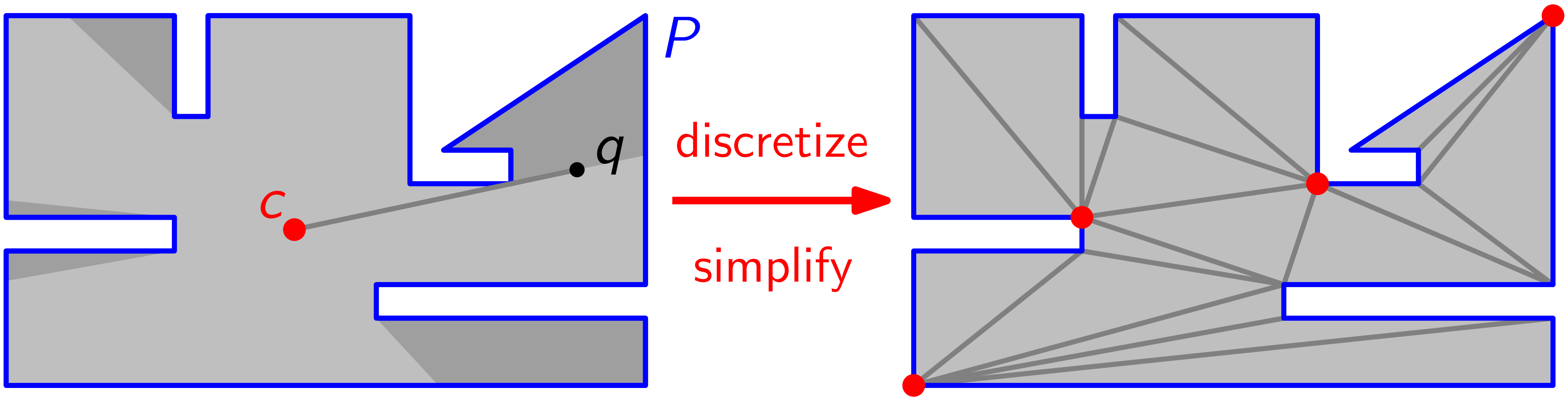

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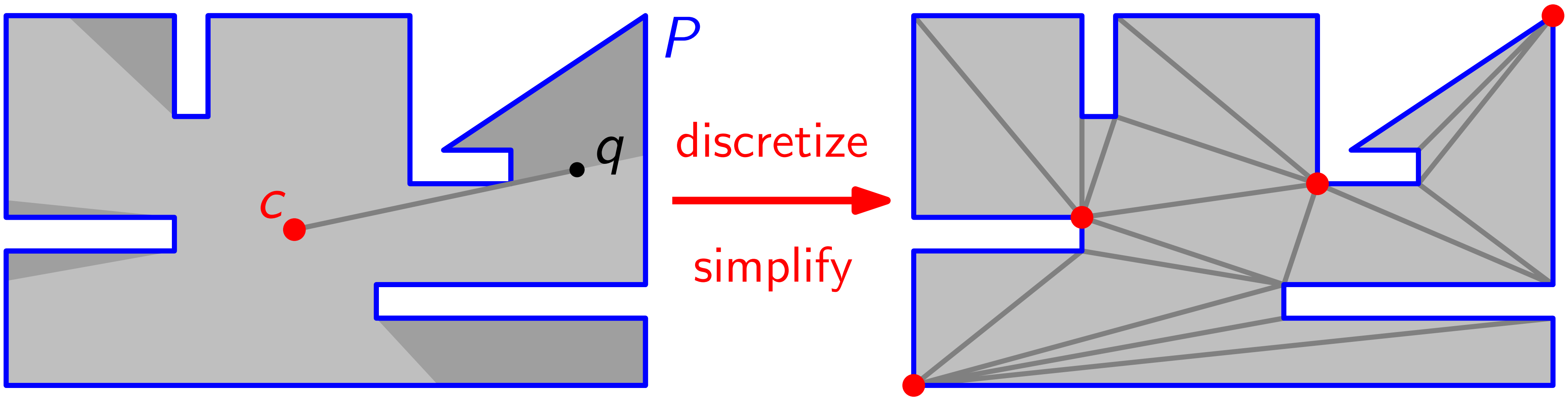

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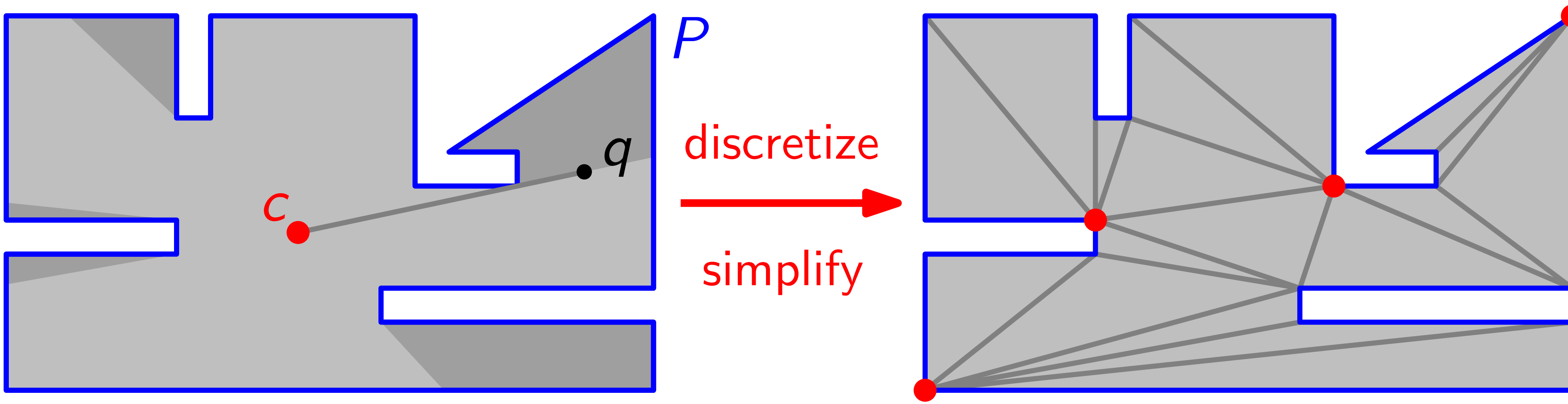

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Theorem. How can we prove these?

1. Every simple polygon can be triangulated.
2. Any triangulation of a simple polygon with n vertices consists of $n - 2$ triangles.

The Art Gallery Theorem [Chvátal '75]

Theorem. For surveilling a simple polygon with n vertices, $\lfloor n/3 \rfloor$ cameras are sometimes necessary and always sufficient.

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Exercise. Find, for arbitrarily large n, a polygon with n vertices, where $\approx n/3$ cameras are necessary.

[2 minutes]

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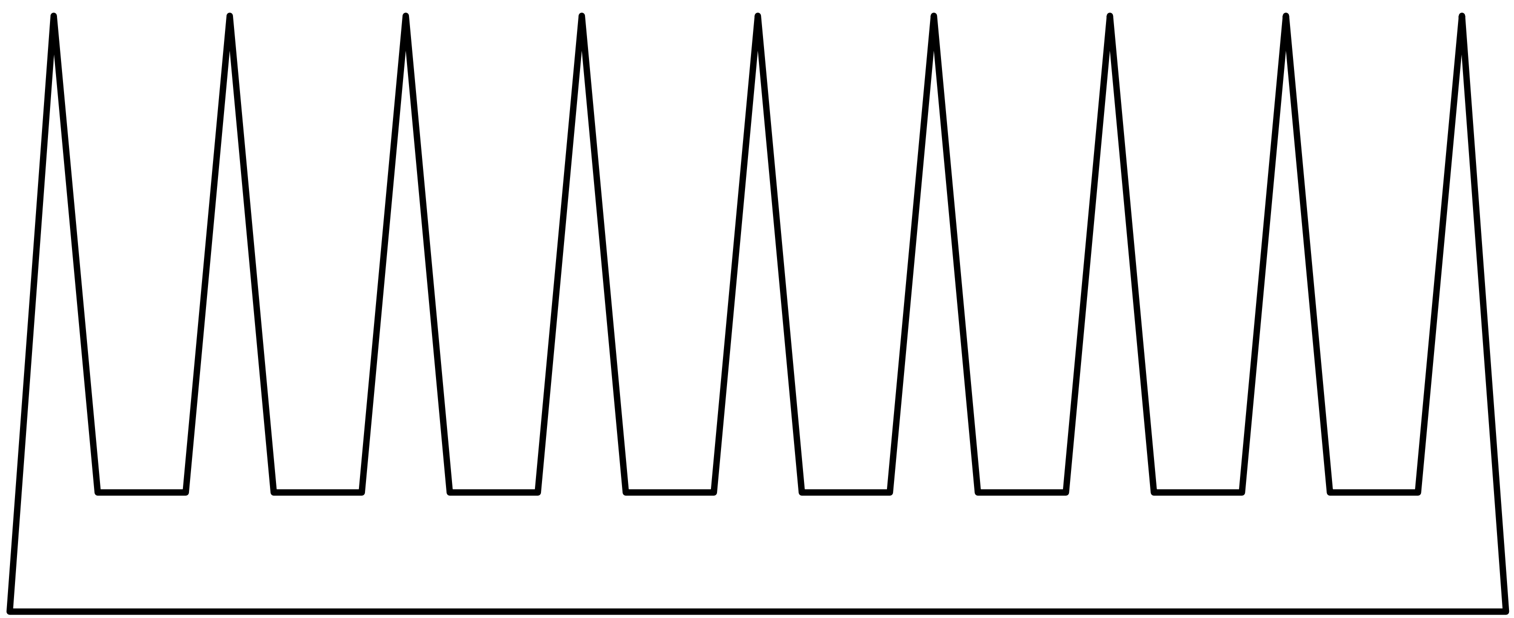

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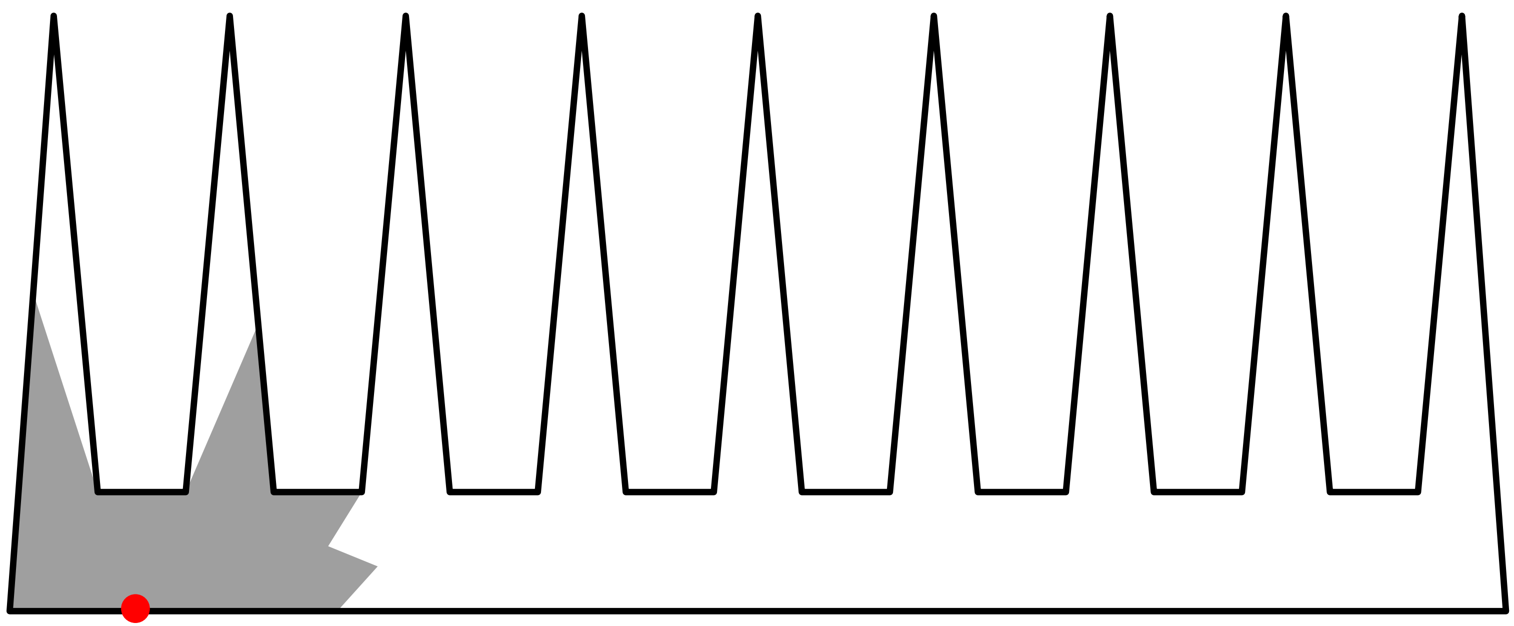

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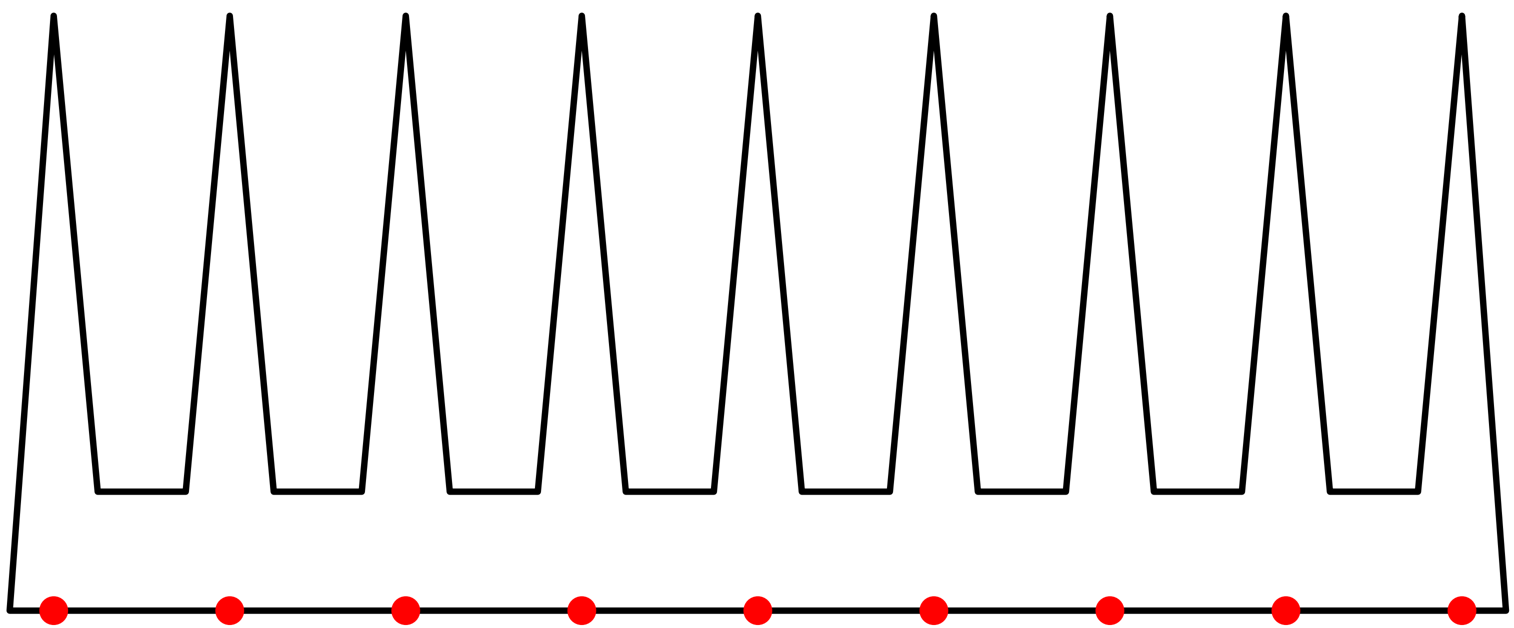

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rectilinear

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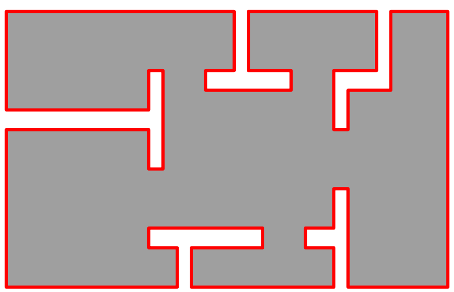
n/4



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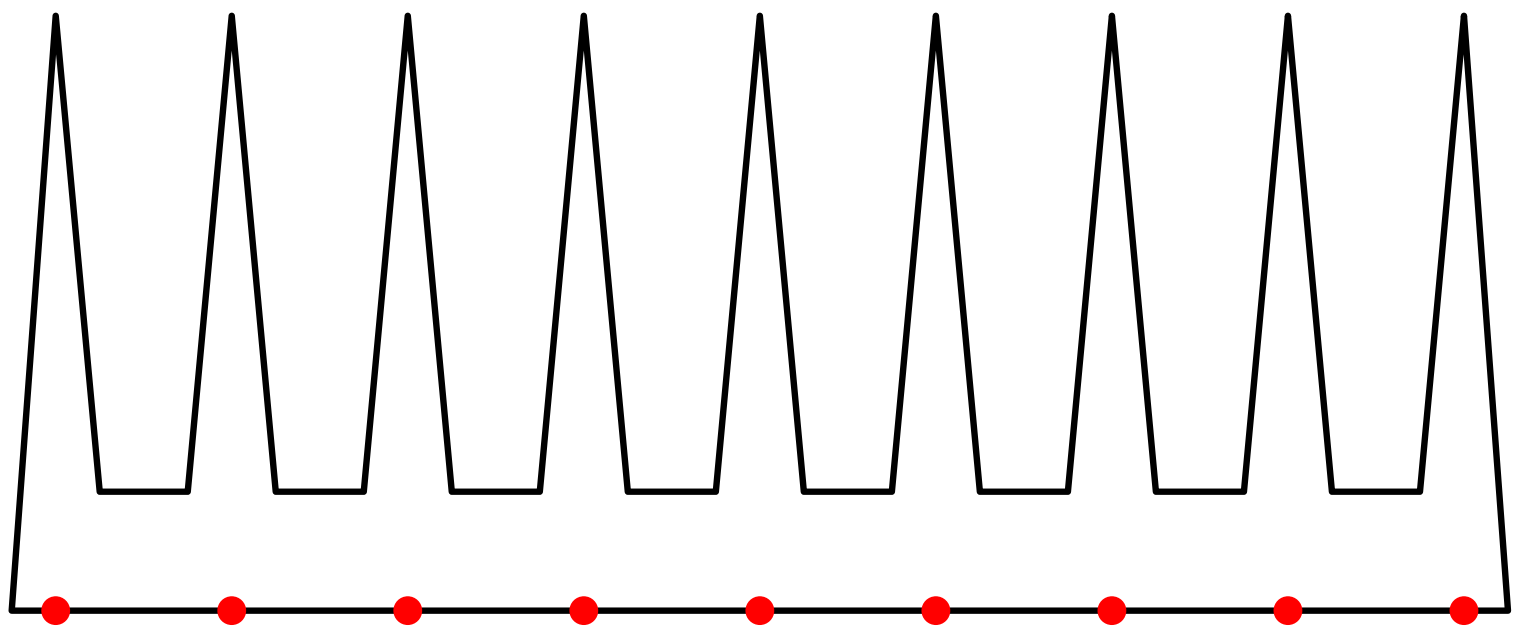

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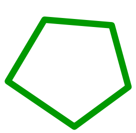

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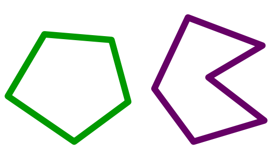

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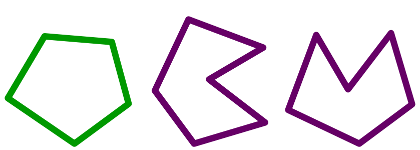

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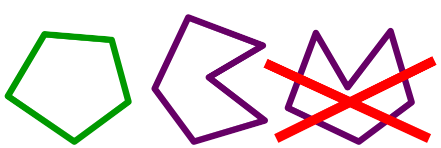

Partitioning a Polygon into Monotone Pieces

Idea: Classify vertices of given simple polygon P

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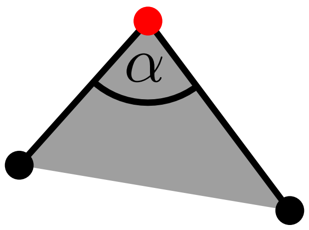

if $\alpha < 180°$

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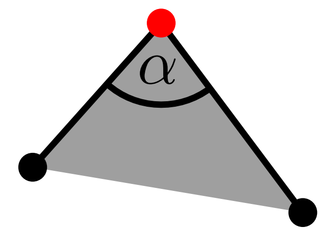
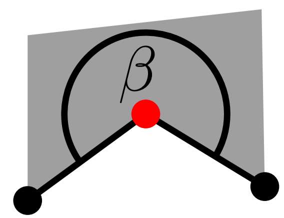


if $\alpha < 180°$

- *split* vertex



if $\beta > 180°$

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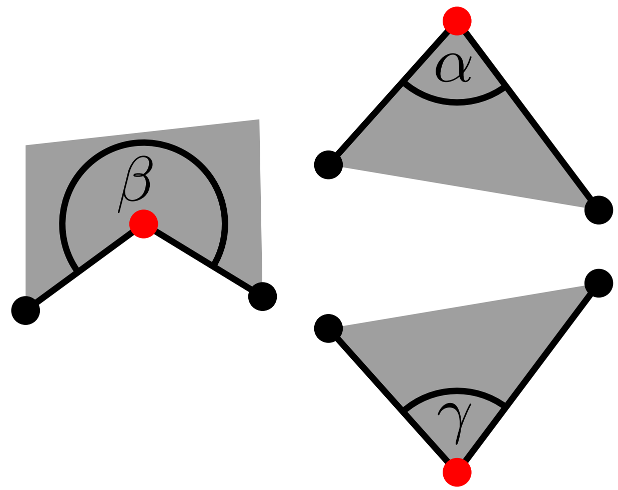
- *start* vertex — if $\alpha < 180°$
- *split* vertex — if $\beta > 180°$
- *end* vertex — if $\gamma < 180°$



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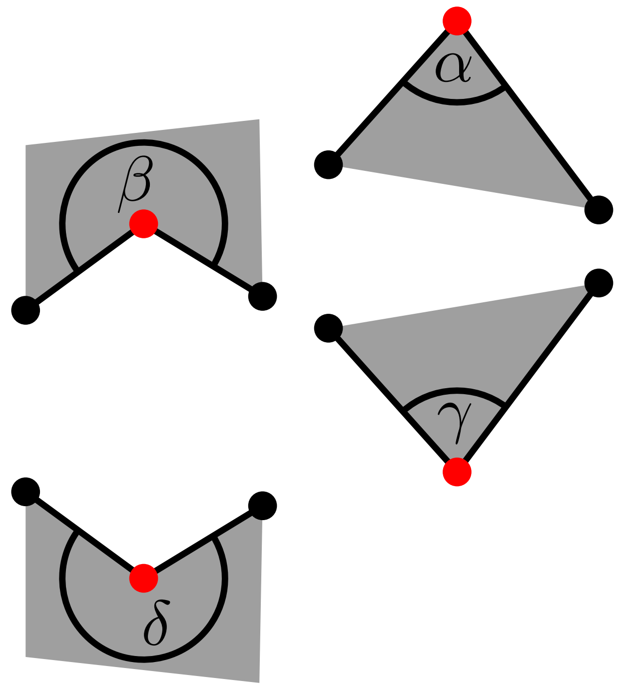
– *turn* vertices:
  vertical component of walking direction changes
  - *start* vertex — if $\alpha < 180°$
  - *split* vertex — if $\beta > 180°$
  - *end* vertex — if $\gamma < 180°$
  - *merge* vertex — if $\delta > 180°$



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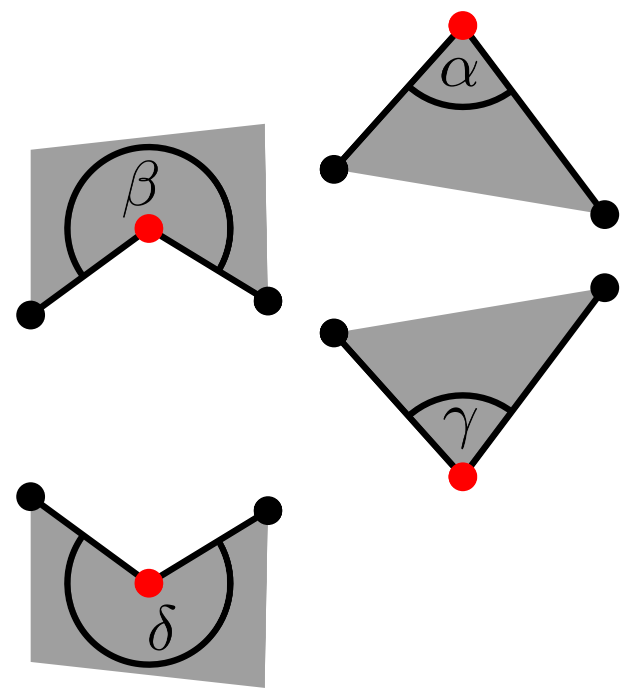
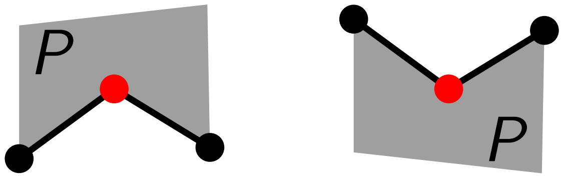


– *regular* vertices

Lemma: Let P be a simple polygon. Then P is y-monotone $\Leftrightarrow$ P has neither split vertices nor merge vertices.

Towards an Algorithm



Idea: Add diagonals to "destroy" split and merge vertices.

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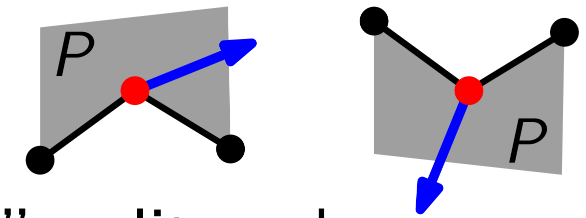

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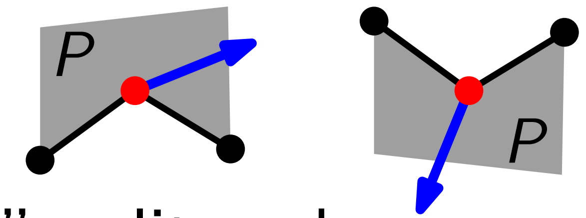

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Problem: Diagonals must not cross

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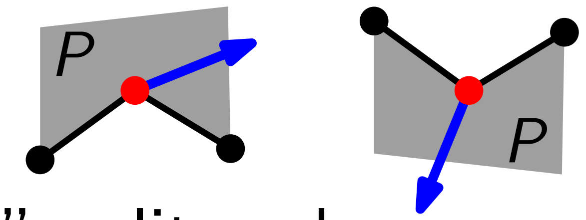
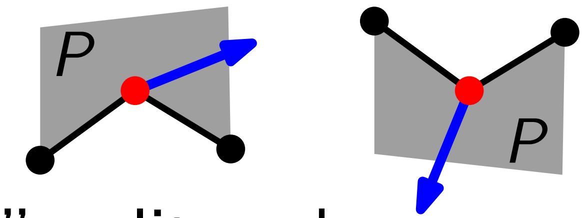


Idea: Add diagonals to “destroy” split and merge vertices.

Problem: Diagonals must not cross
– each other
– edges of P

Towards an Algorithm



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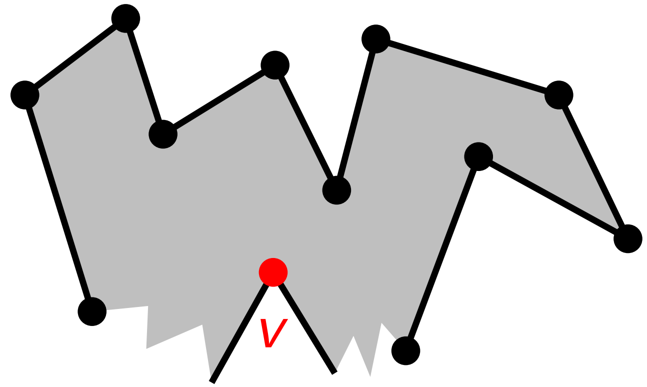
1) Treating split vertices



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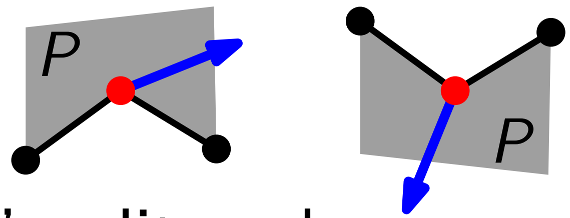

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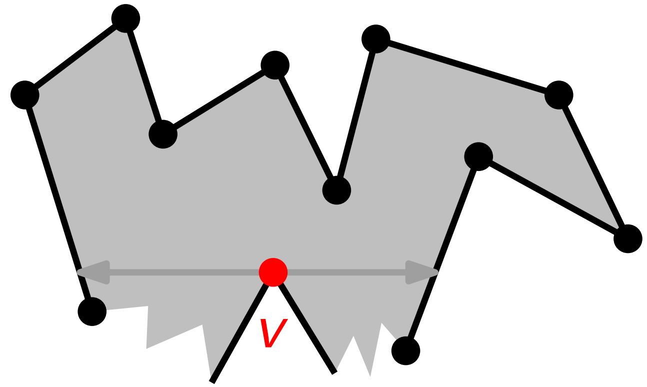

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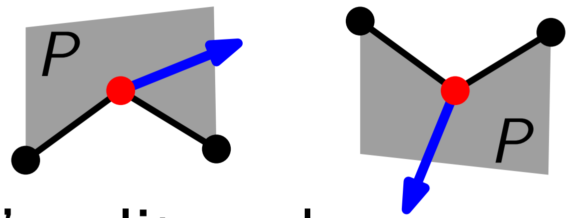

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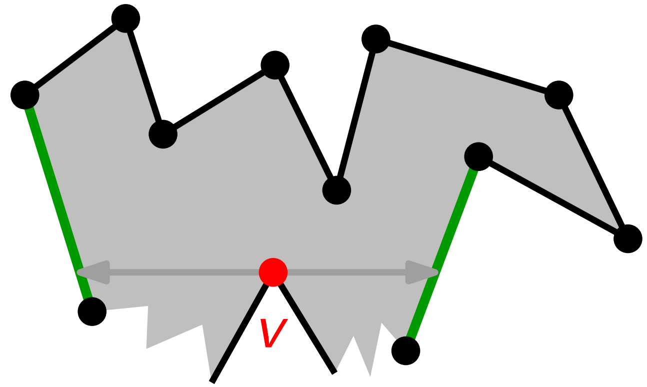

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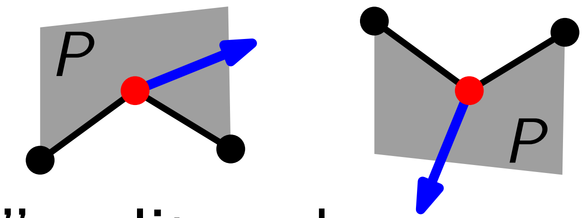

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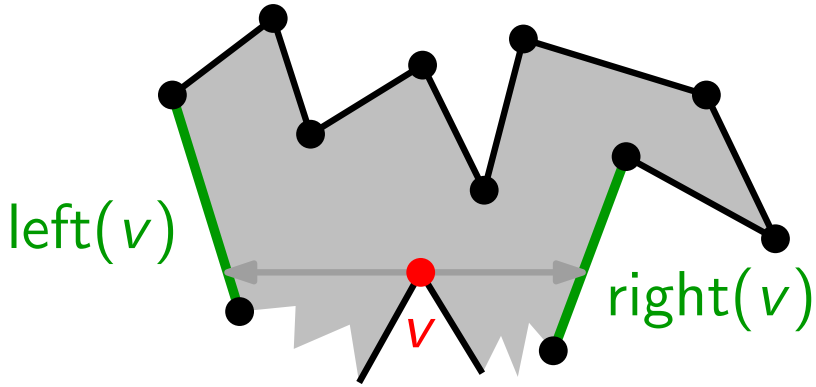

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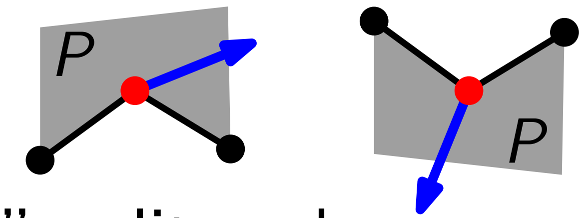

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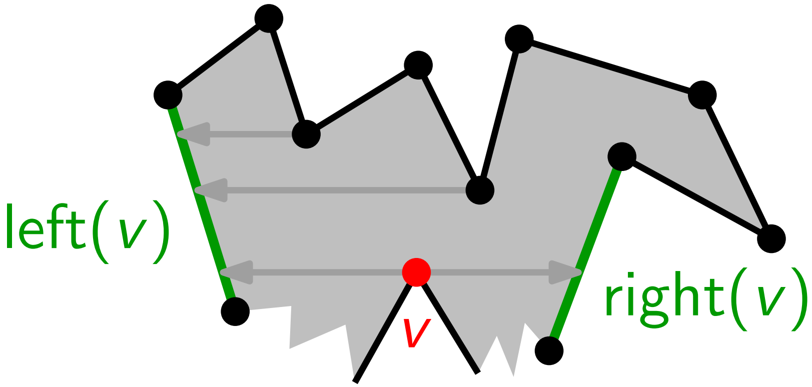

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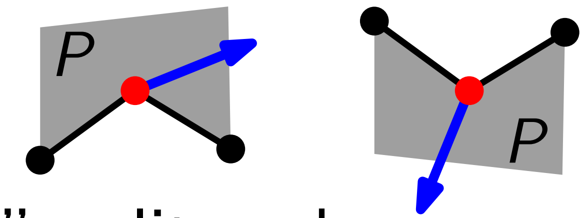

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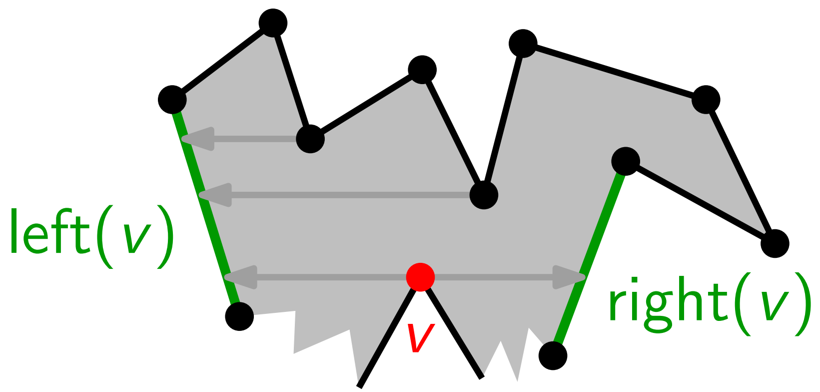

Connect v to vertex $w^\star$ having minimum y-coordinate among all vertices w above v and with $\text{left}(w) = \text{left}(v)$.

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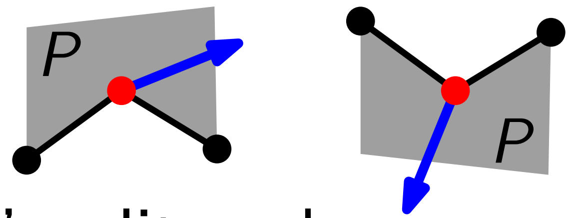

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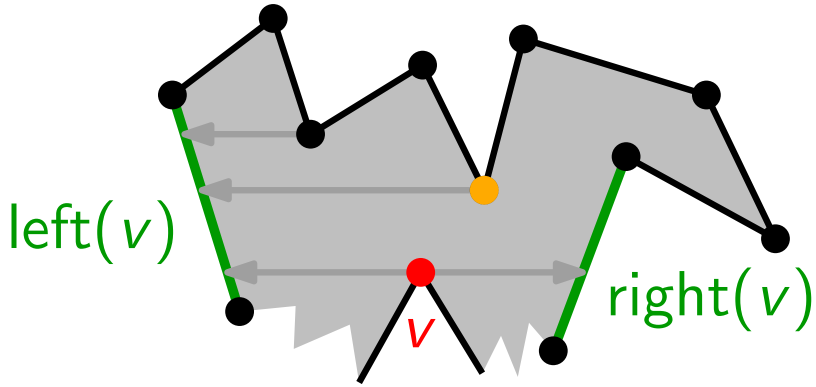

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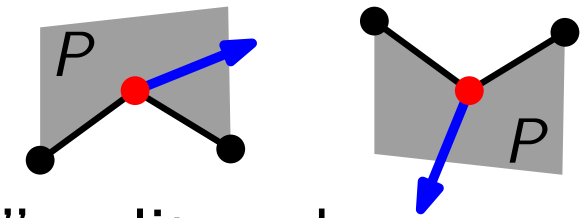

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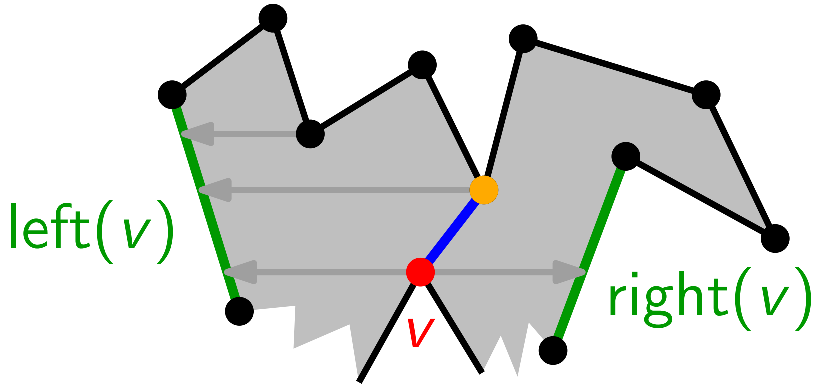

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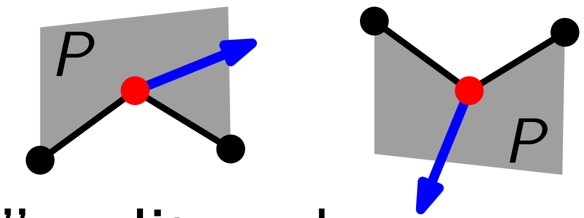

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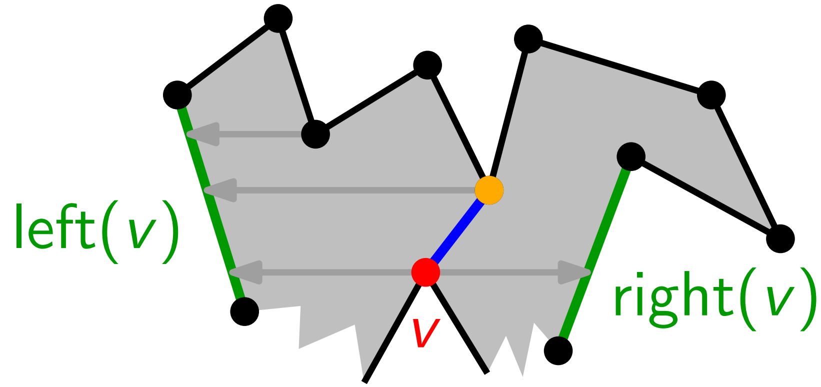

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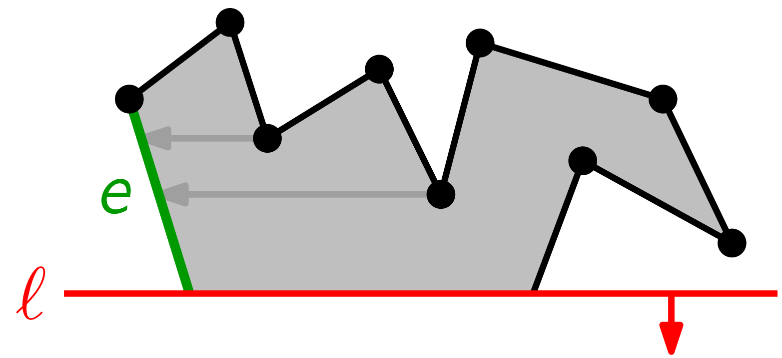

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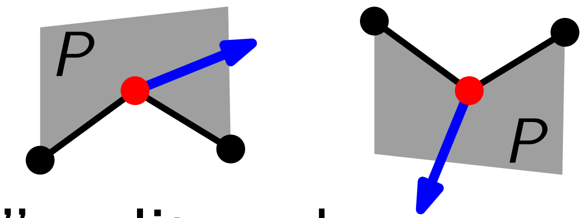

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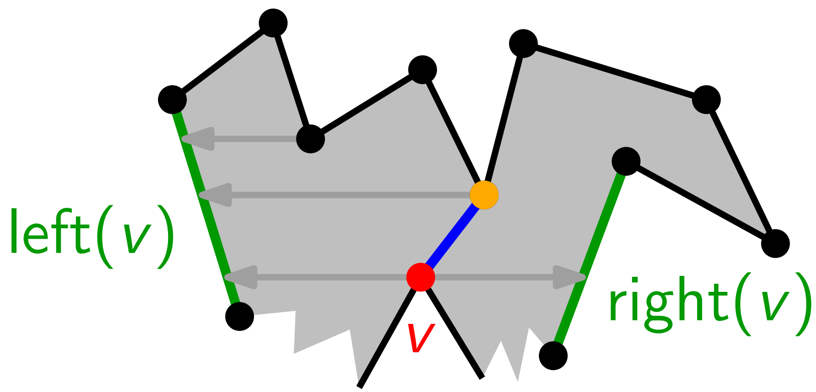

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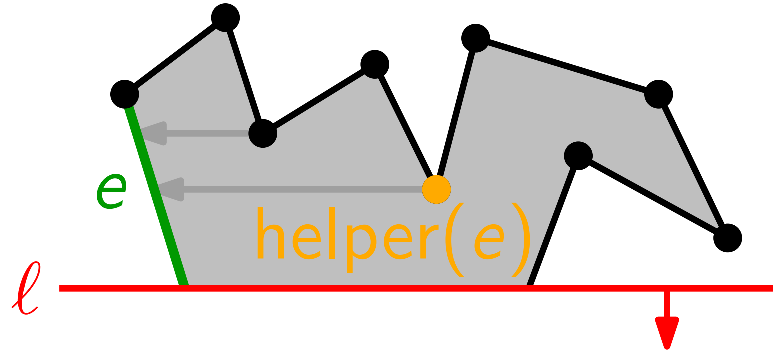

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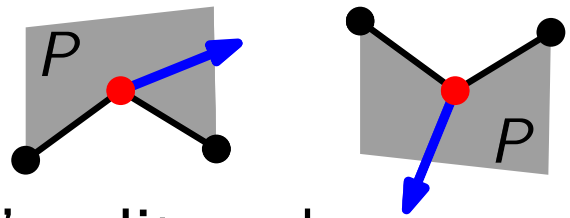

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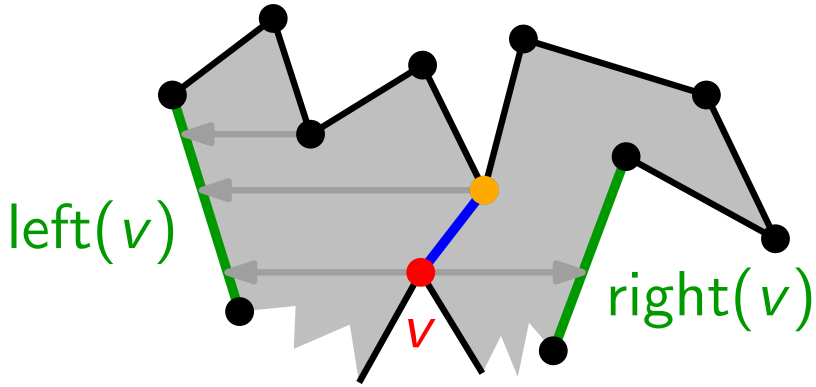

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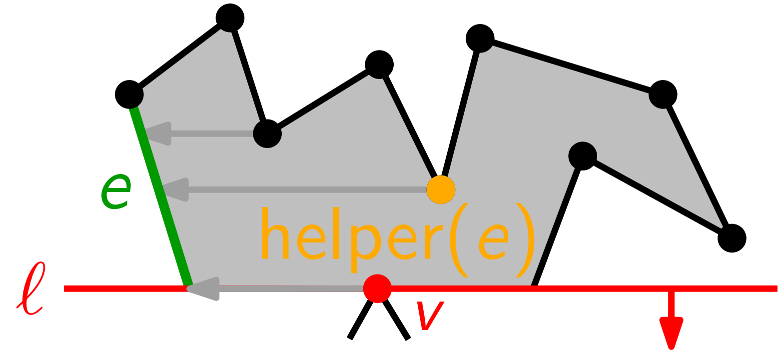

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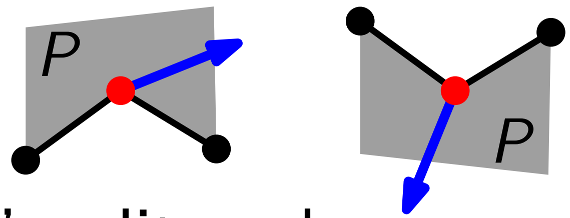

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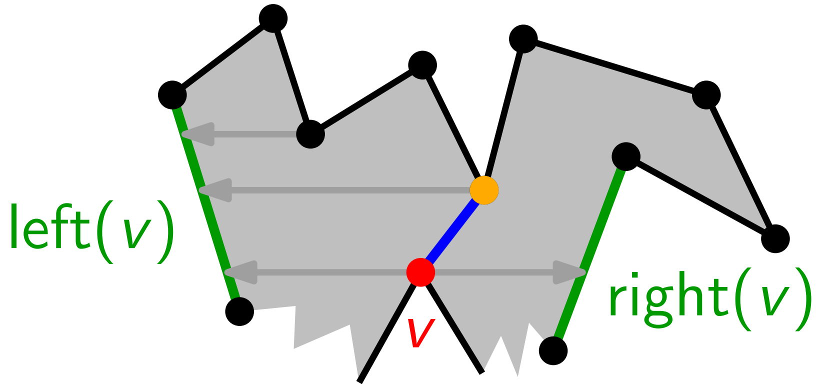

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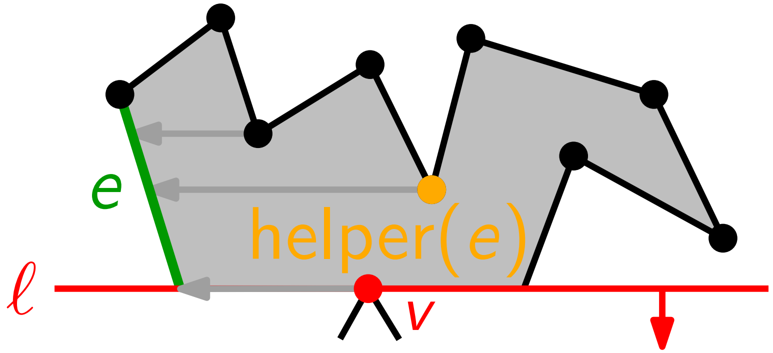

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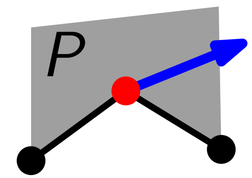

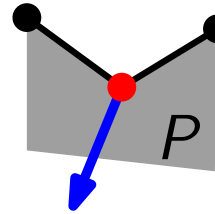

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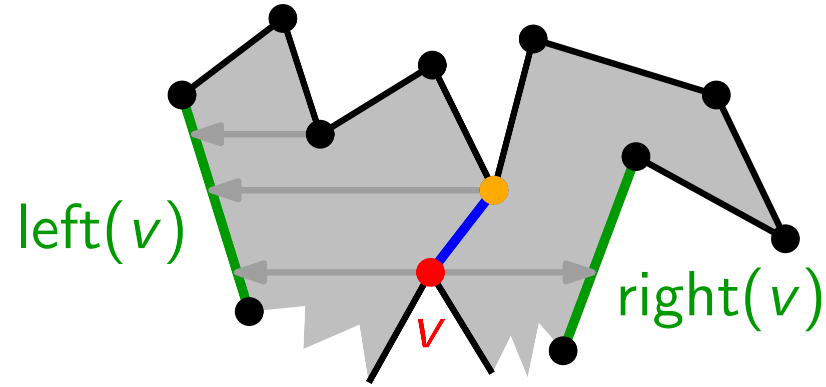

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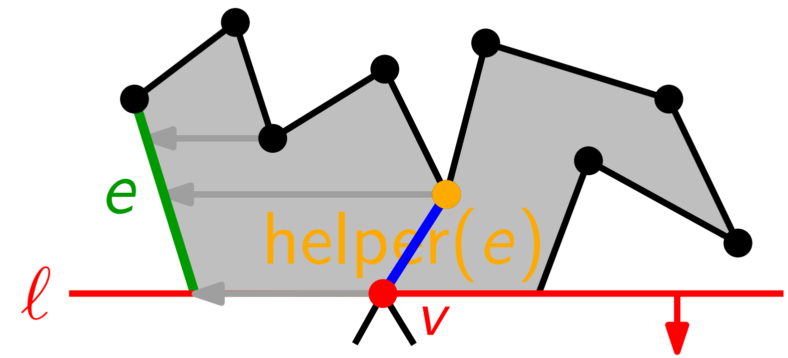

An Algorithm

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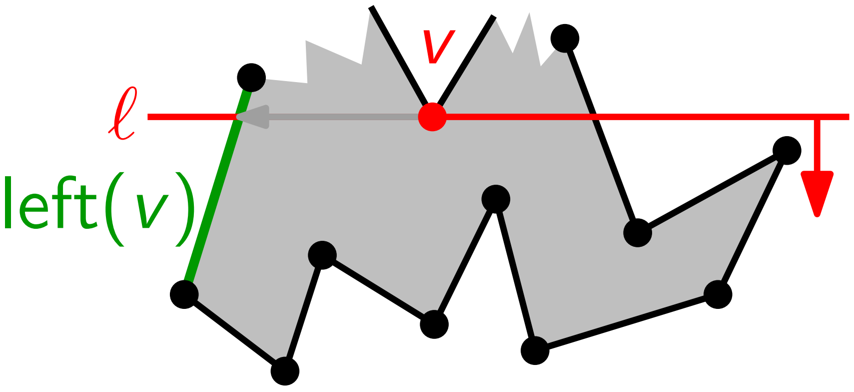

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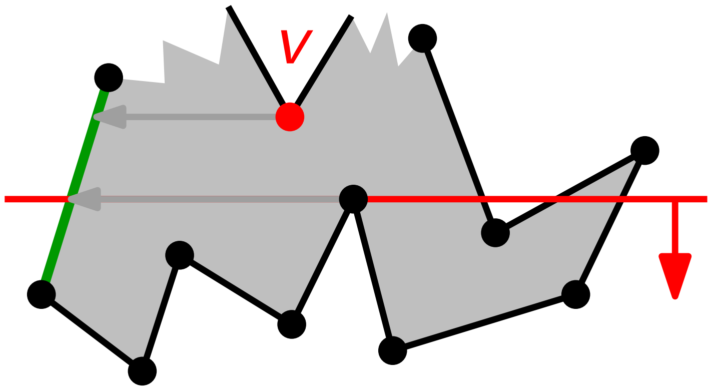

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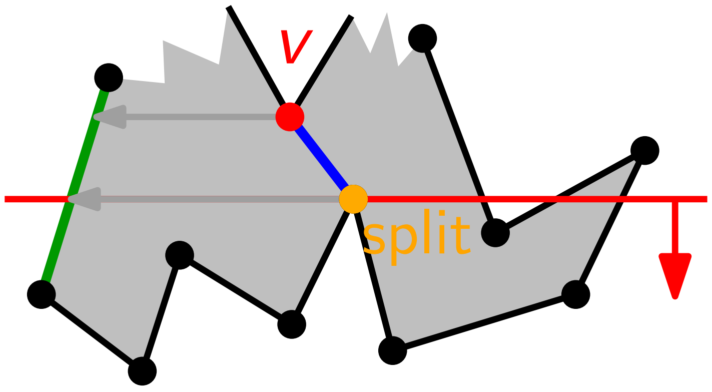

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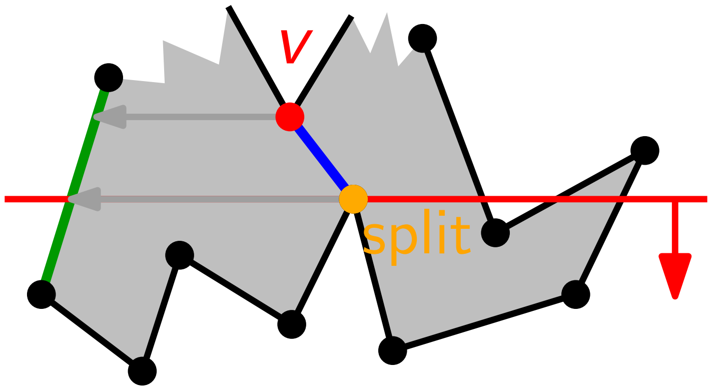

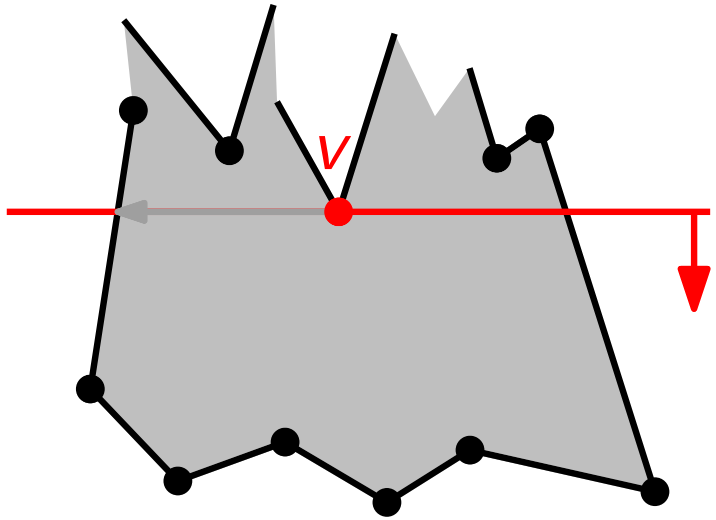

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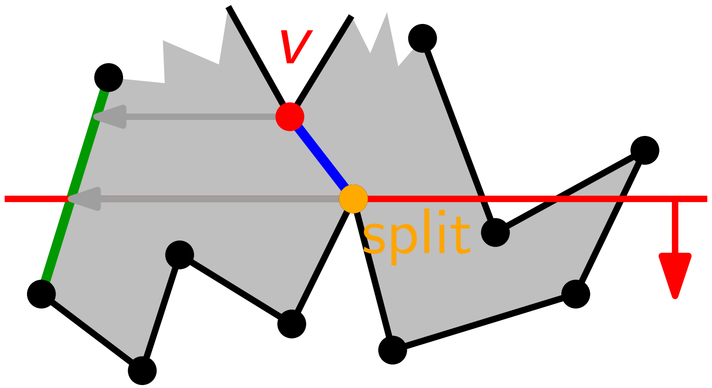

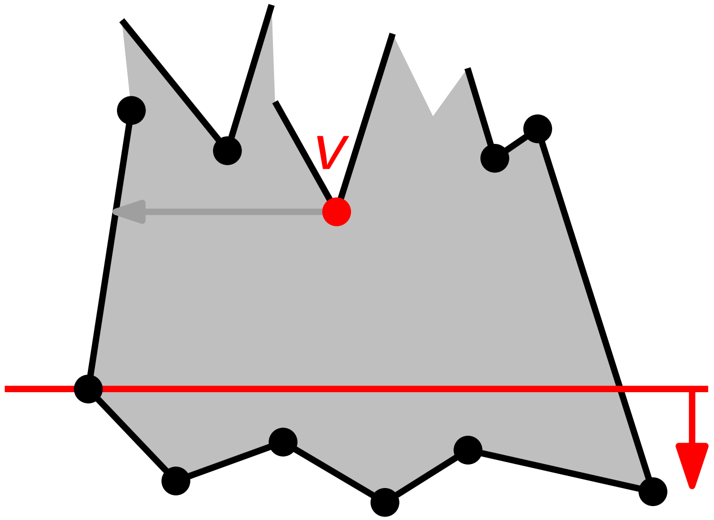

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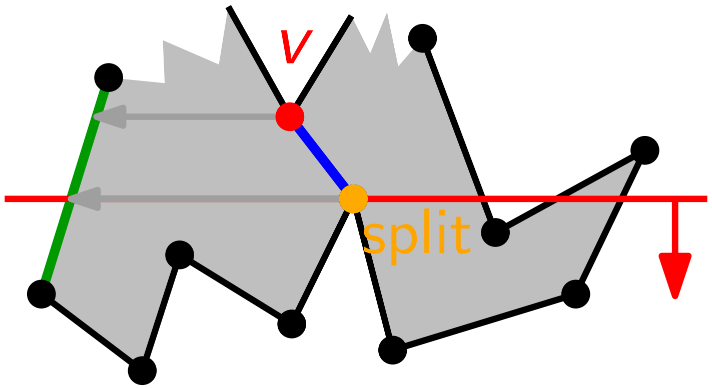

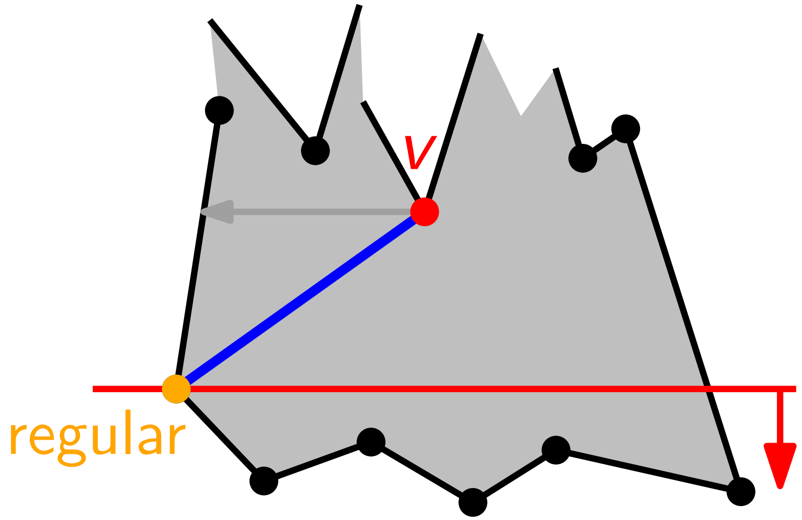

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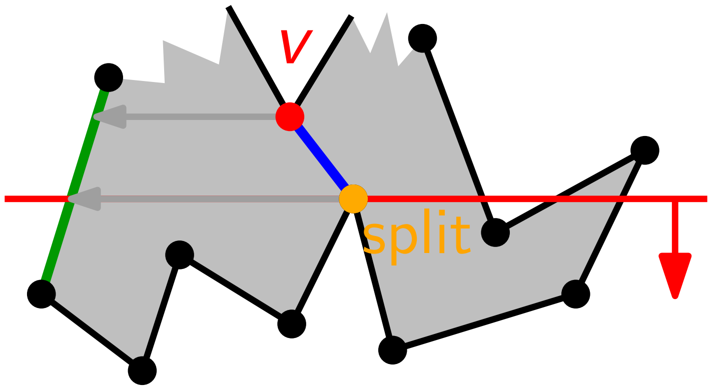

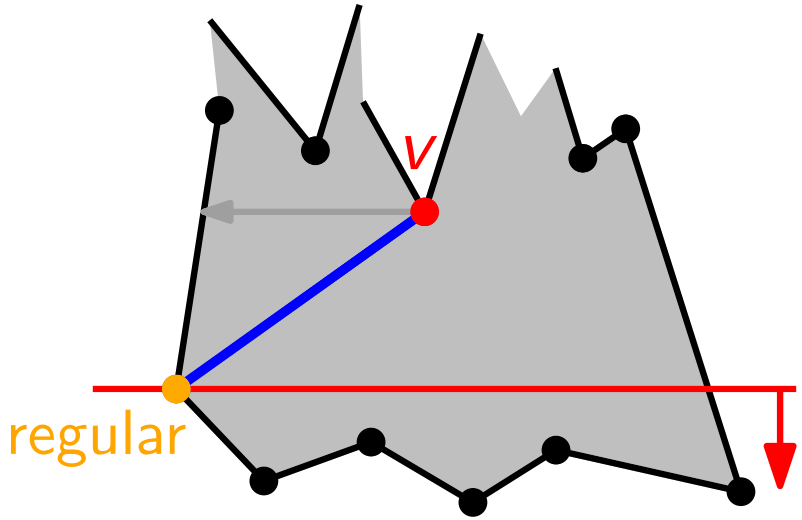

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$\mathcal{D} \leftarrow$ DCEL($V(P), E(P)$)
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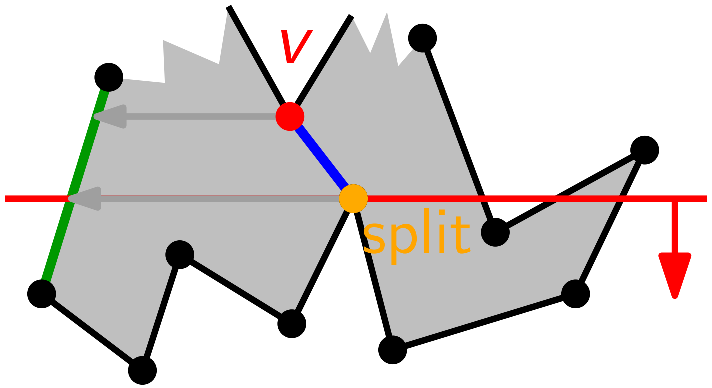

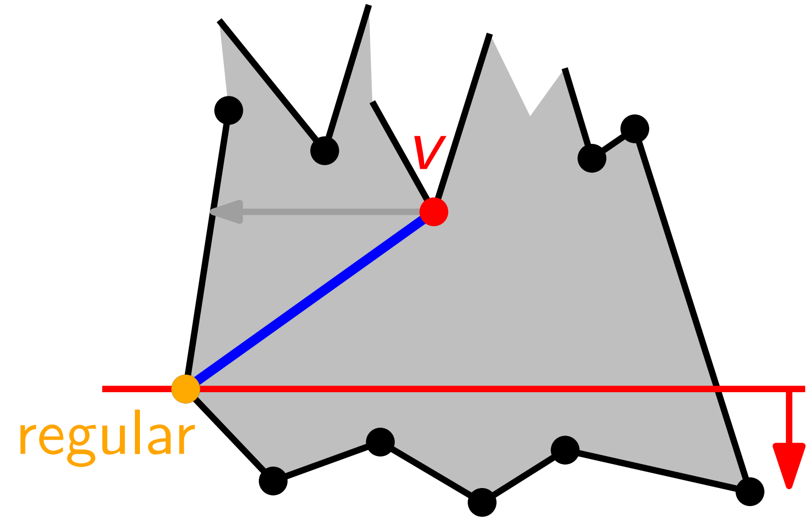

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{ *doubly-connected edge list:*
data structure for planar subdivisions

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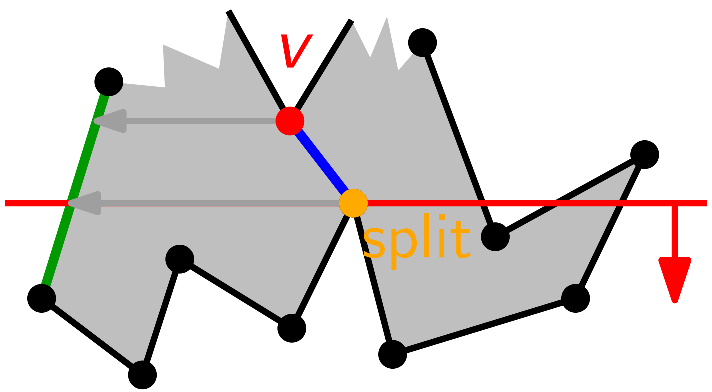

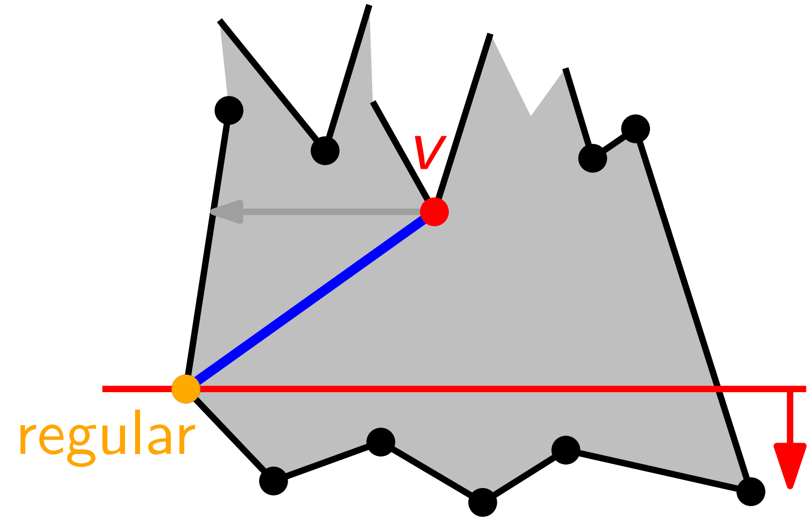

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An Algorithm

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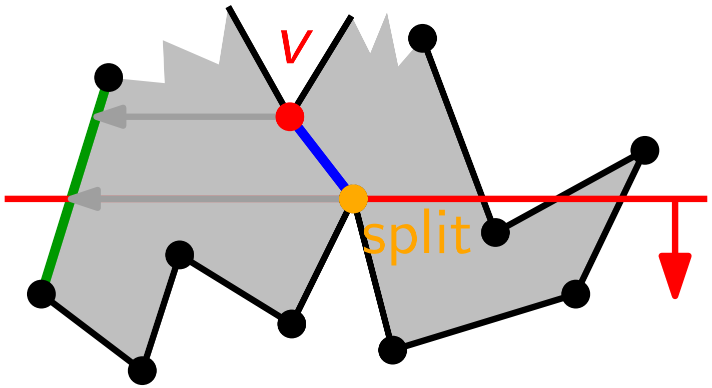

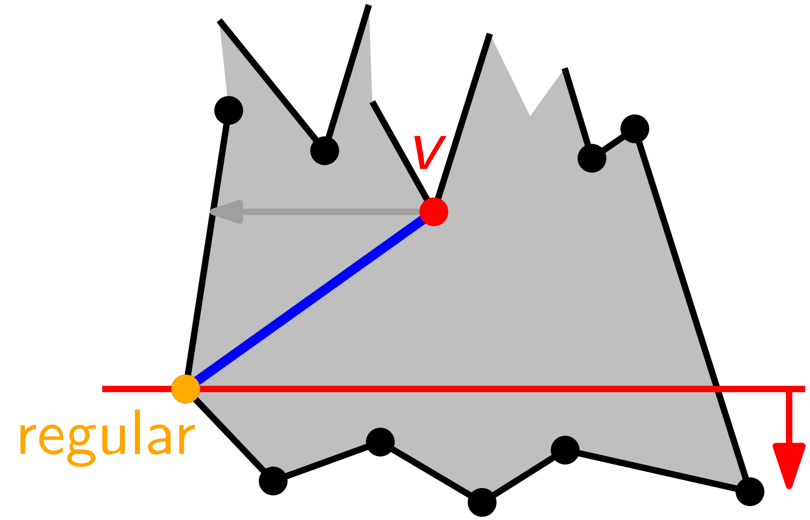

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  $v \leftarrow \mathcal{Q}$.extractMax()
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return DCEL $\mathcal{D}$

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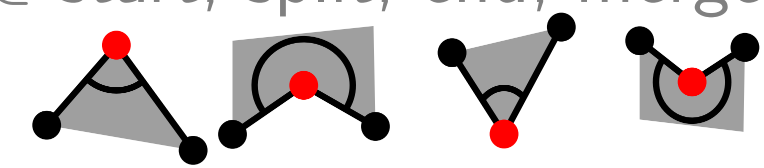
$\in$ start, split, end, merge, regular



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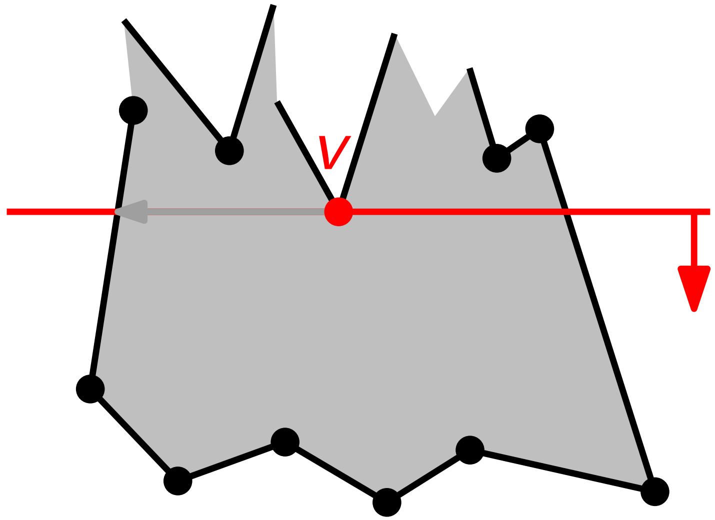

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handleMergeVertex(vertex v**)**
$e \leftarrow$ edge following v cw
if helper(e) merge vtx **then**
  $\mathcal{D}$.insert(diag(v, helper(e)))
$\mathcal{T}$.delete(e)
$e' \leftarrow \mathcal{T}$.edgeLeftOf($v$)
if helper(e') merge vtx **then**
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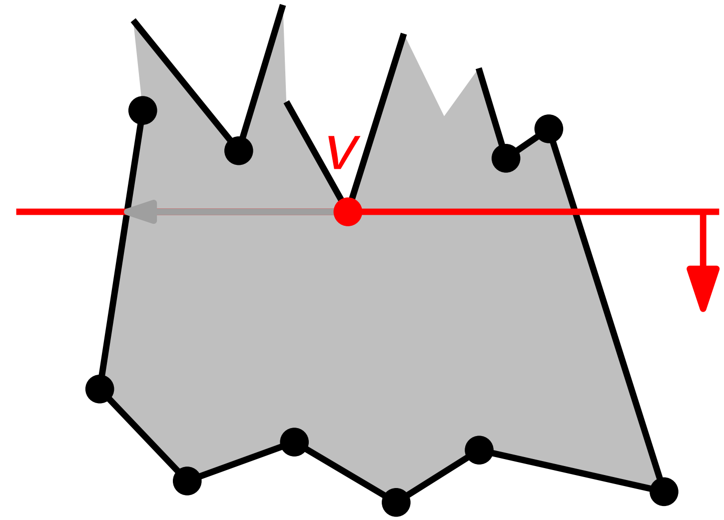
An Algorithm

2) Treating merge vertices



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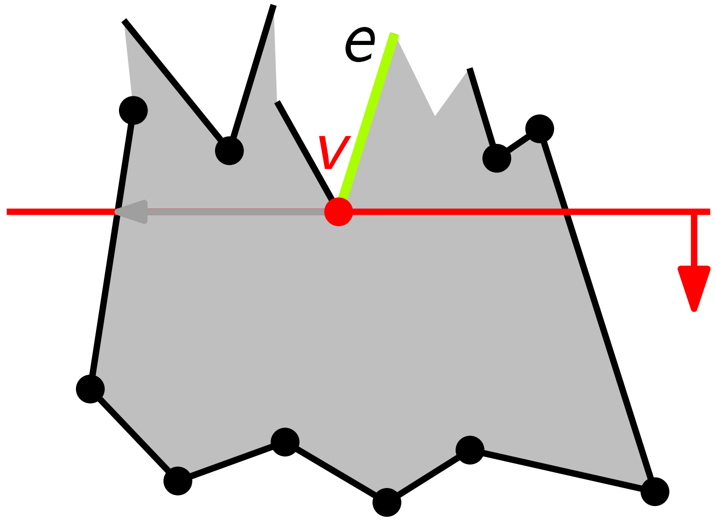

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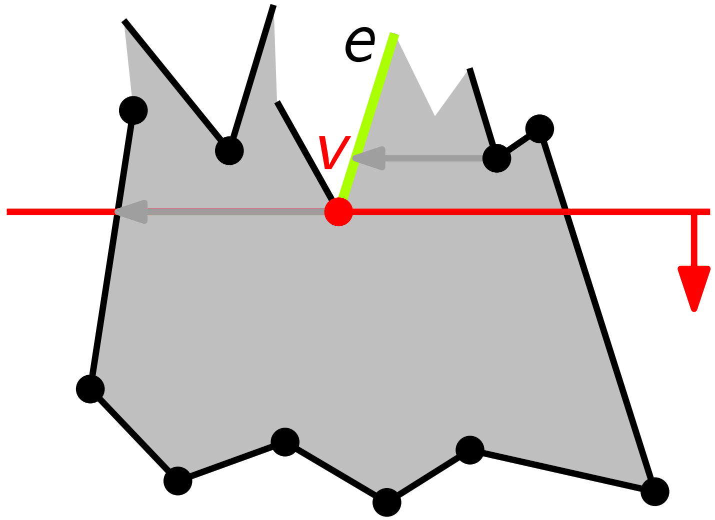

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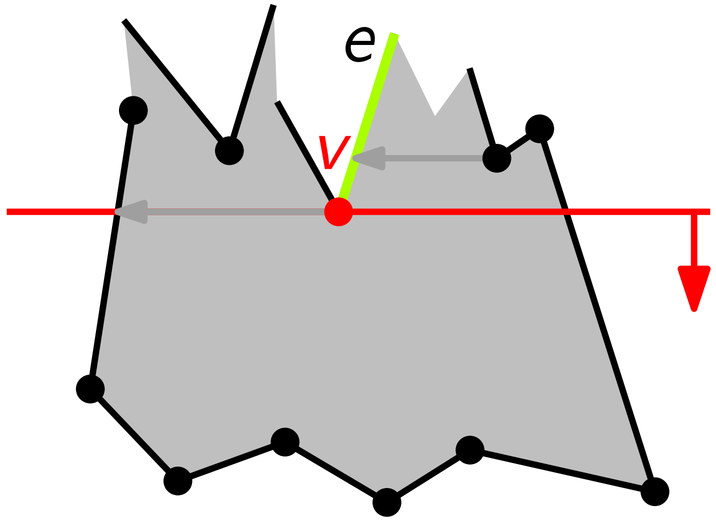

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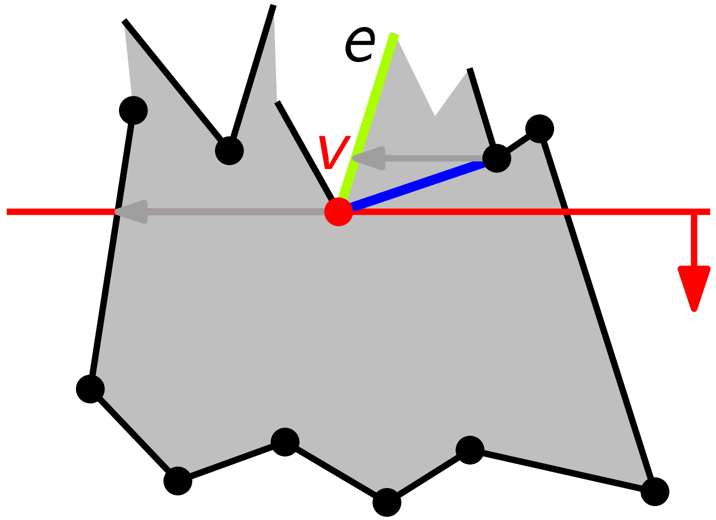

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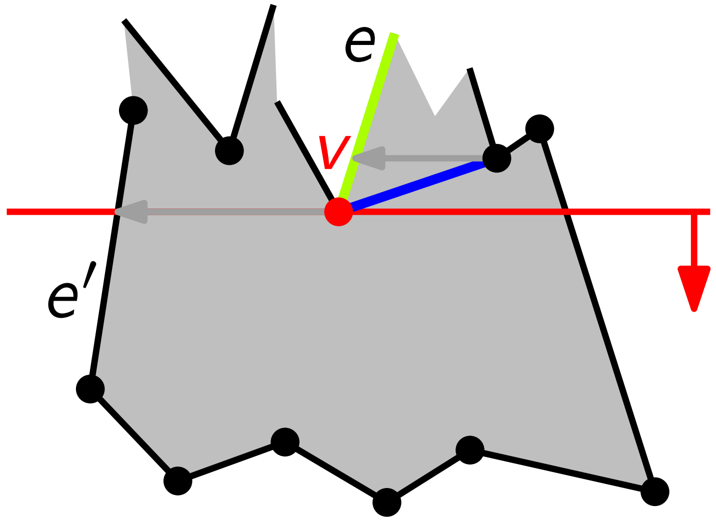

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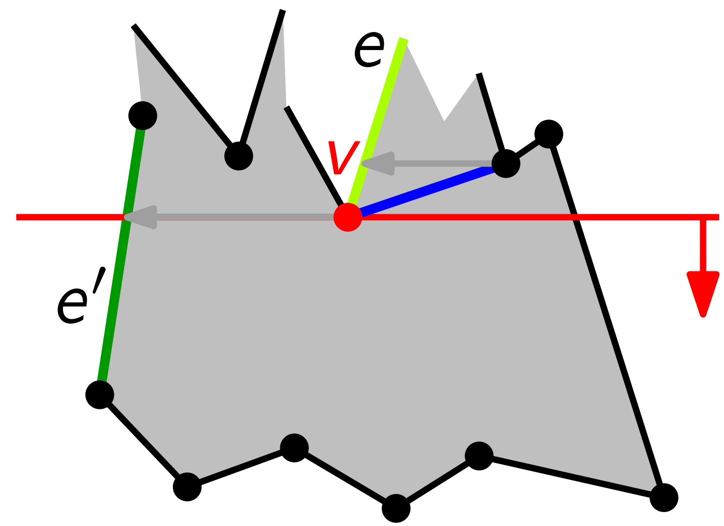

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An Algorithm

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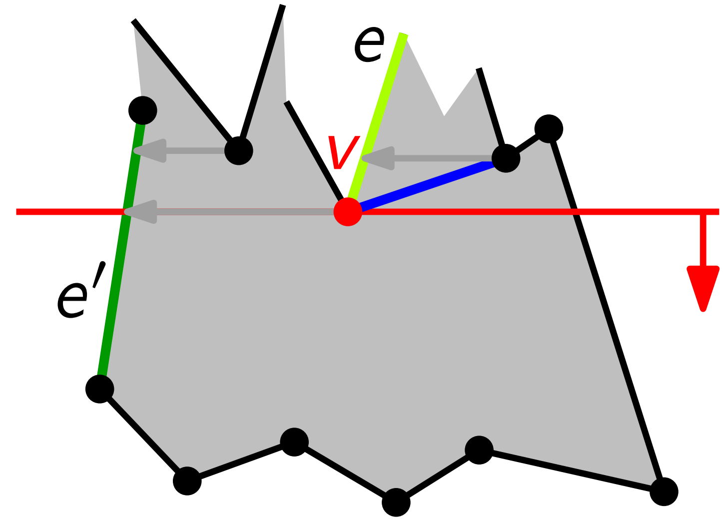

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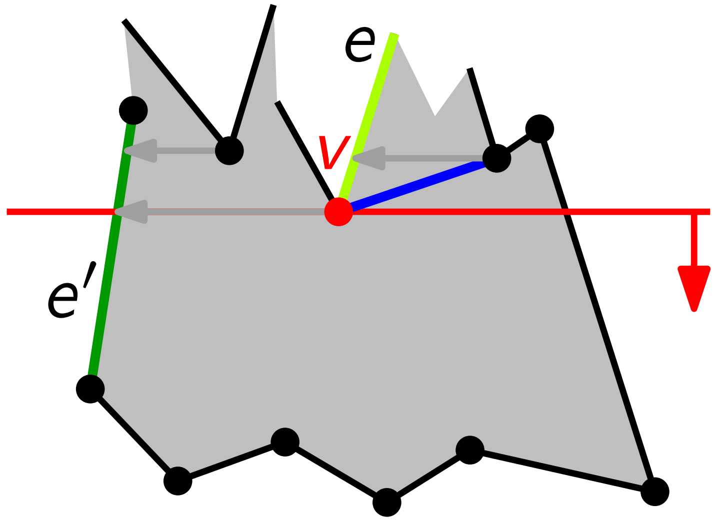

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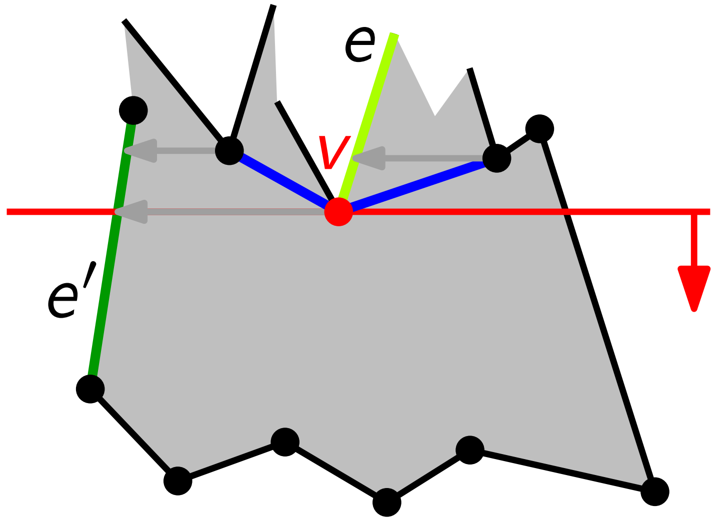

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Analysis

Lemma. makeMonotone() adds a set of non-intersecting diagonals to P such that P is partitioned into y-monotone subpolygons.

Analysis

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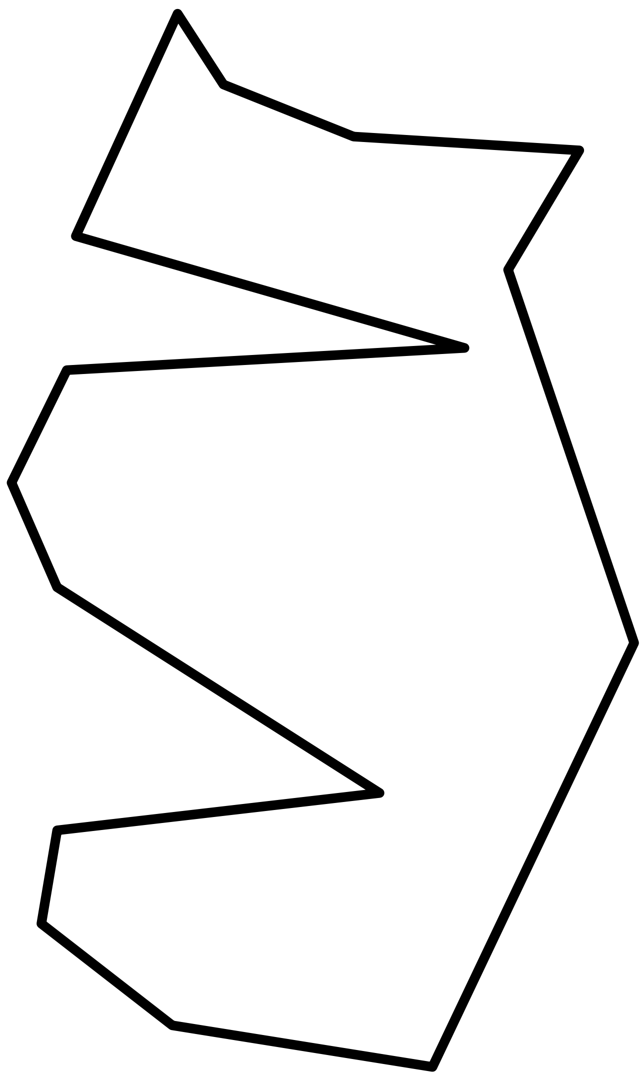
Lemma. A simple polygon with n vertices can be subdivided into y-monotone polygons in $O(n \log n)$ time.

Triangulating a y-Monotone Polygon P

Approach: greedy, going from top to bottom



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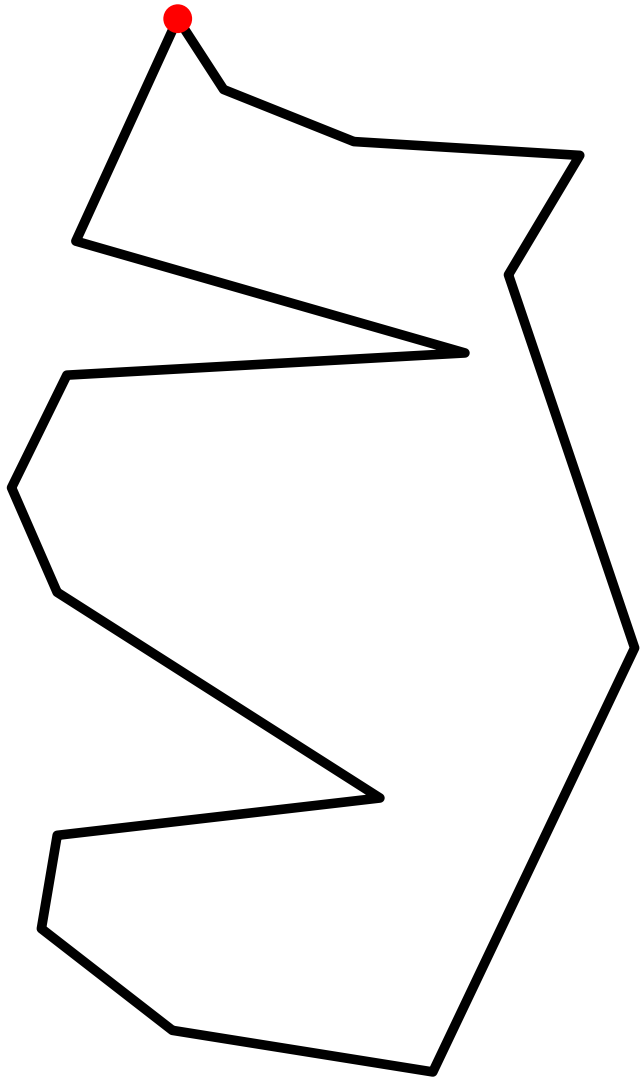

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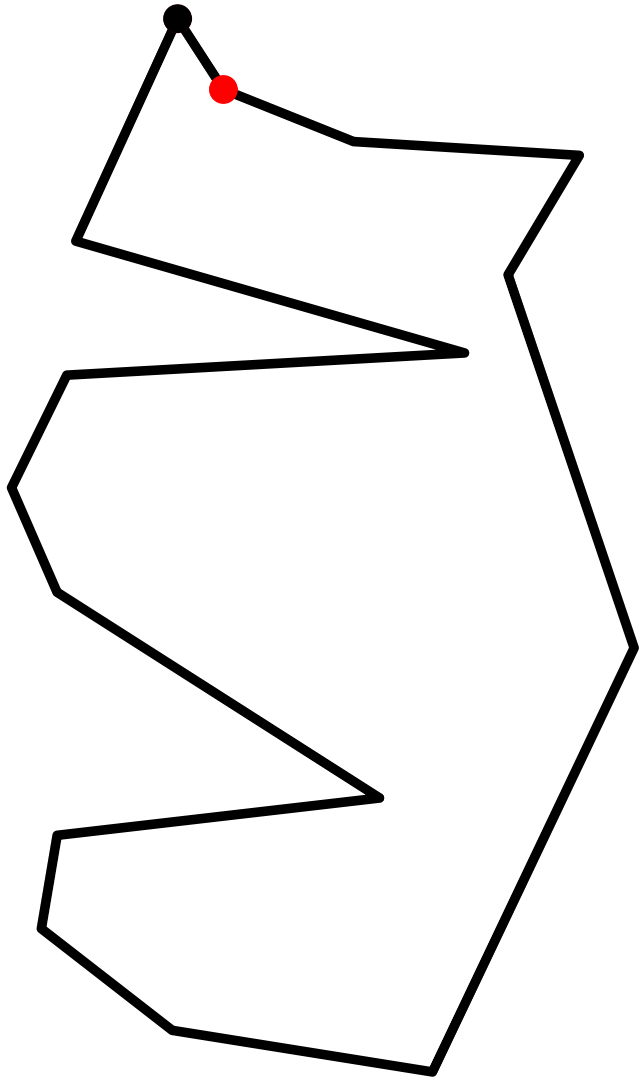

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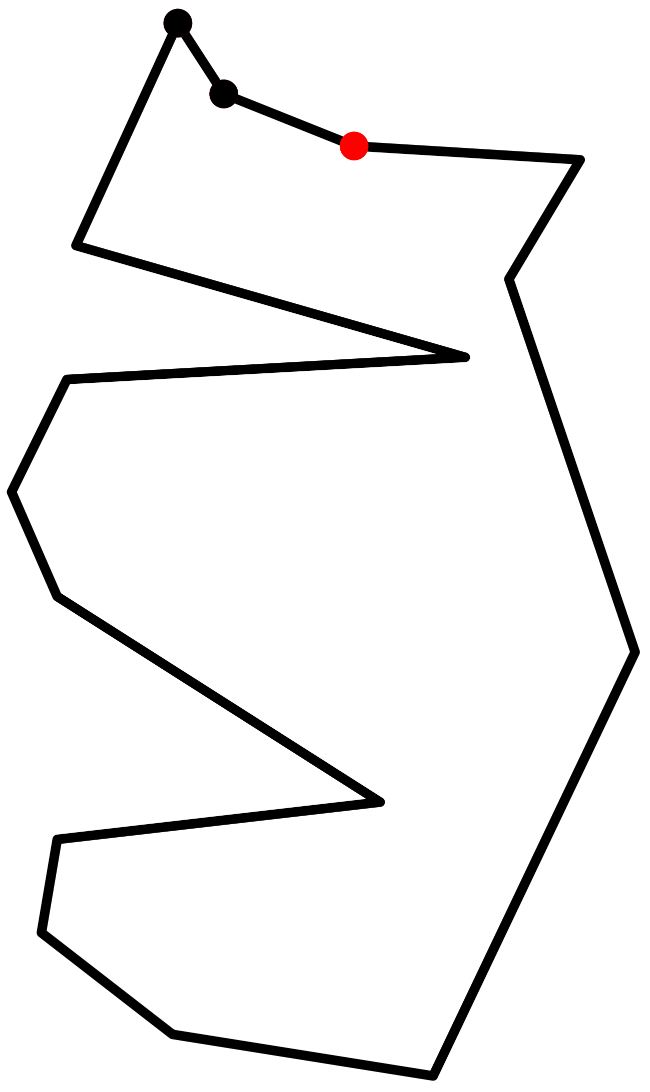

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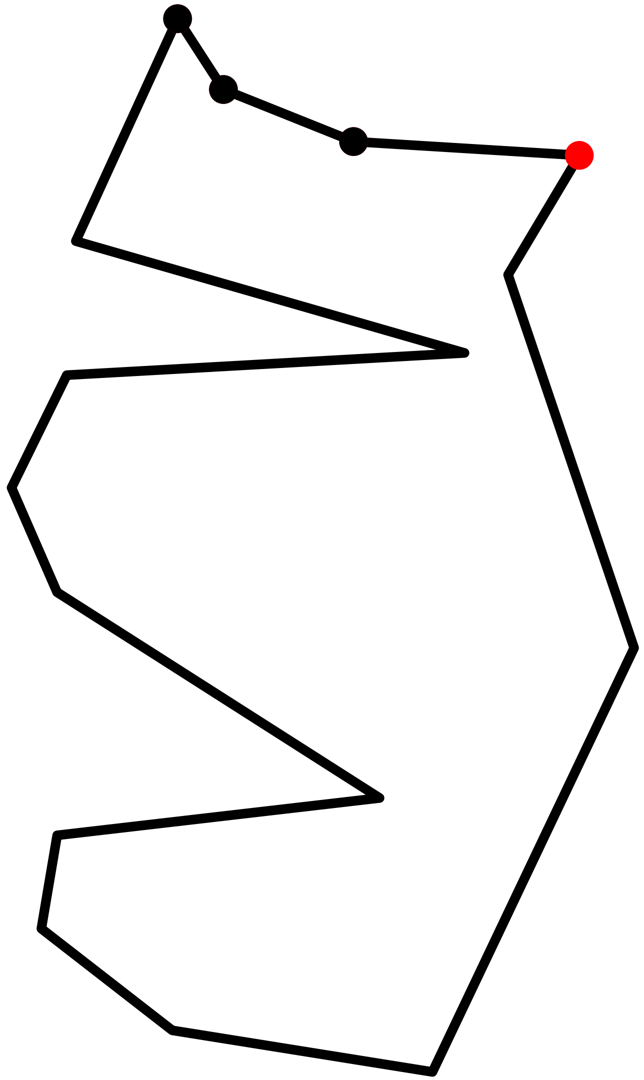

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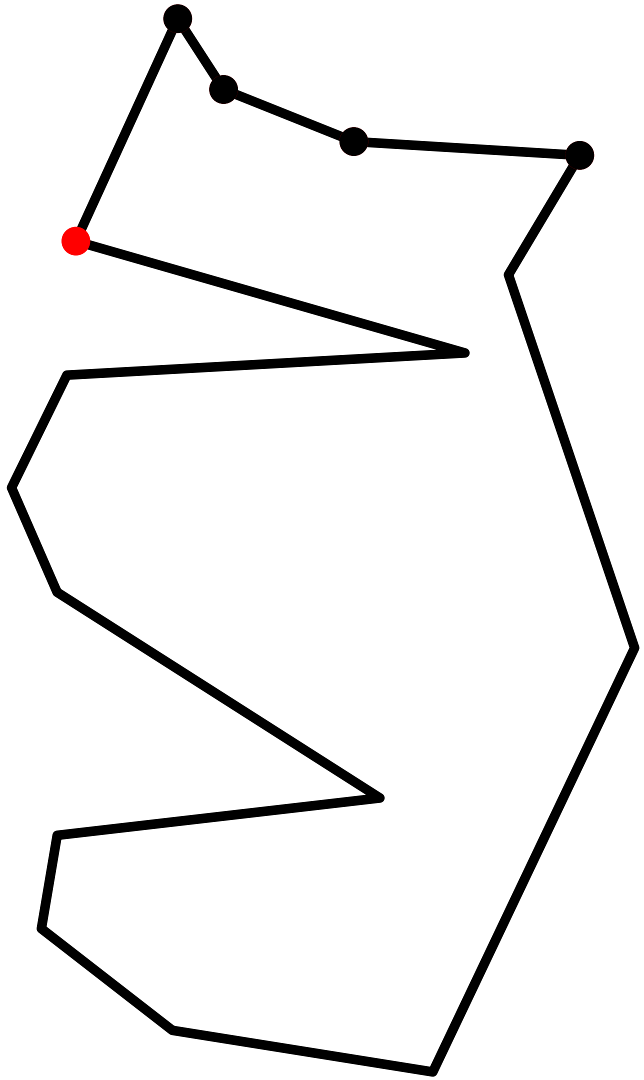

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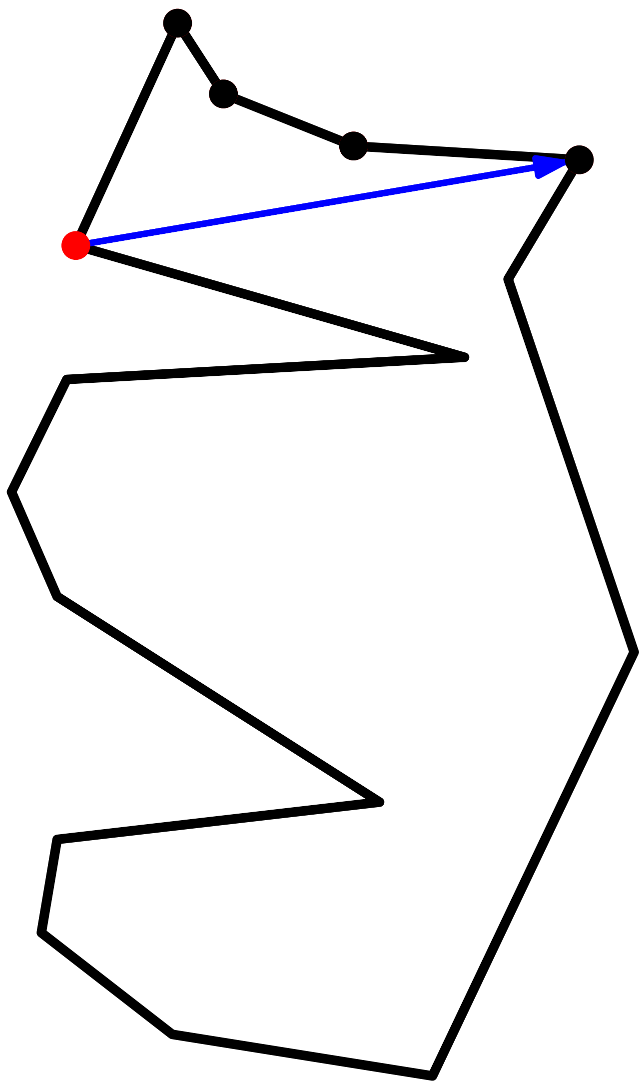

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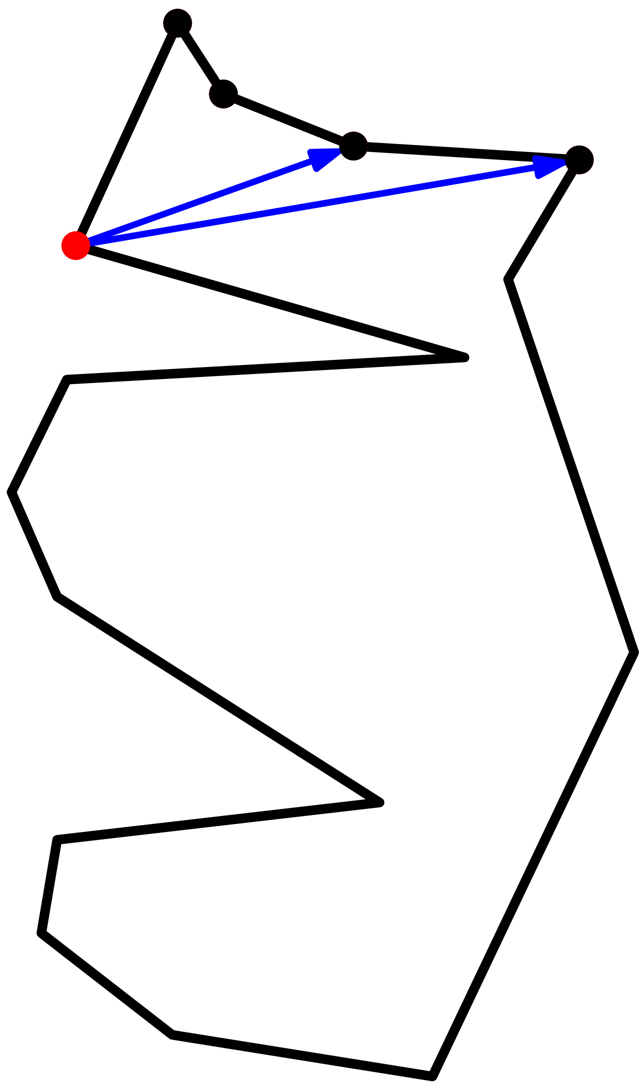

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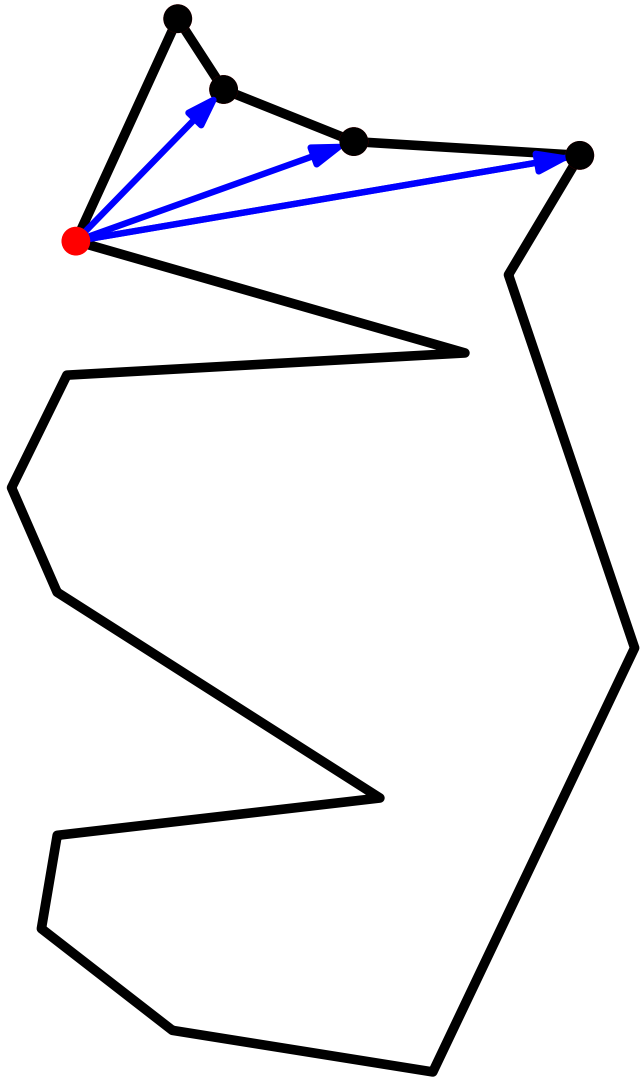

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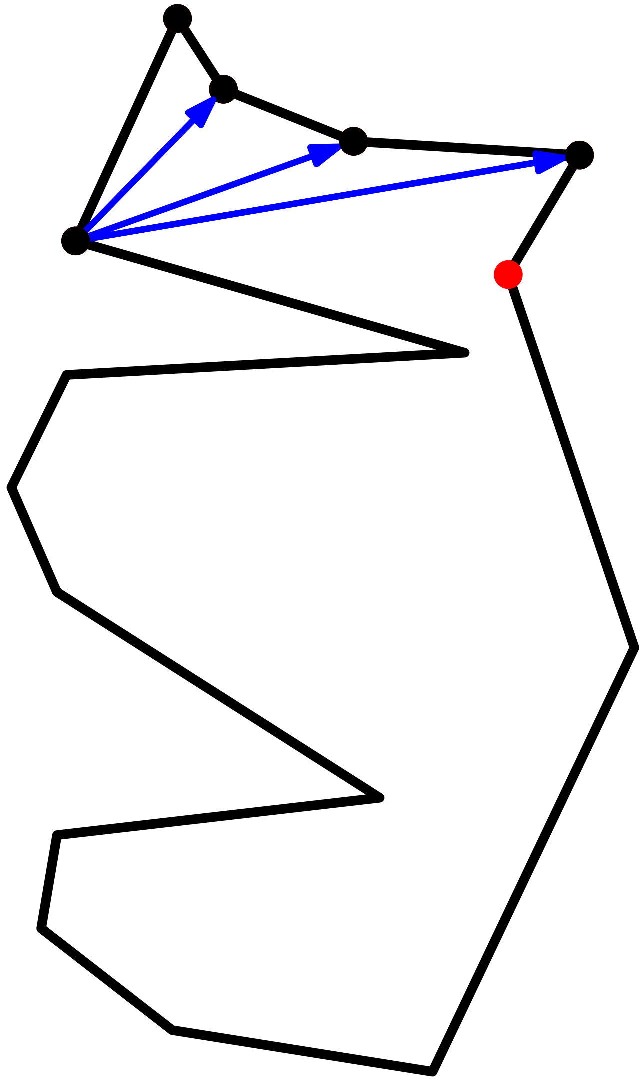

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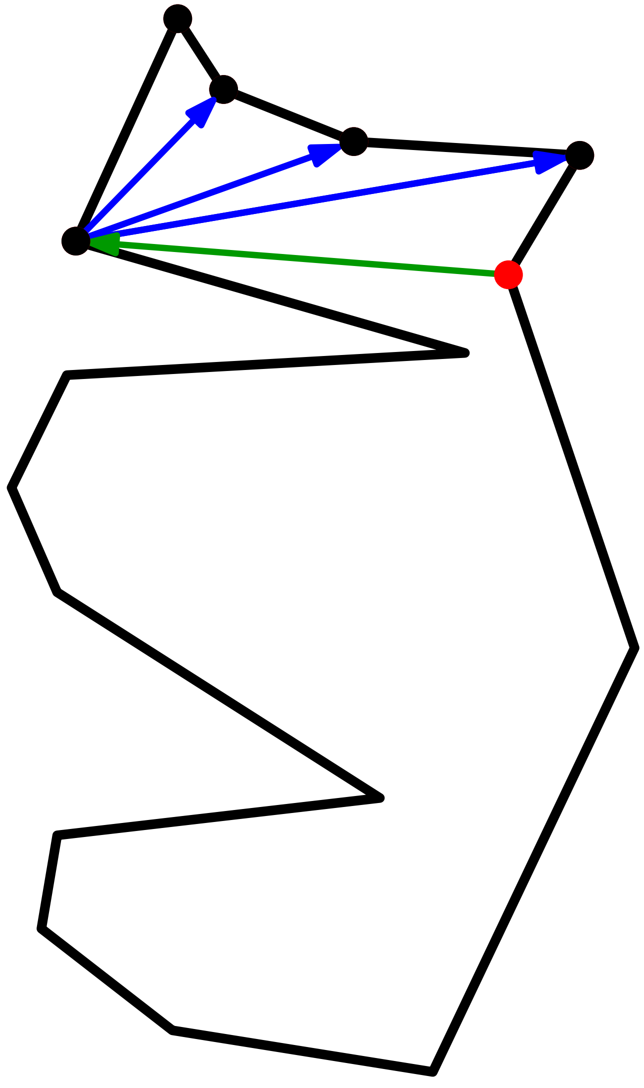

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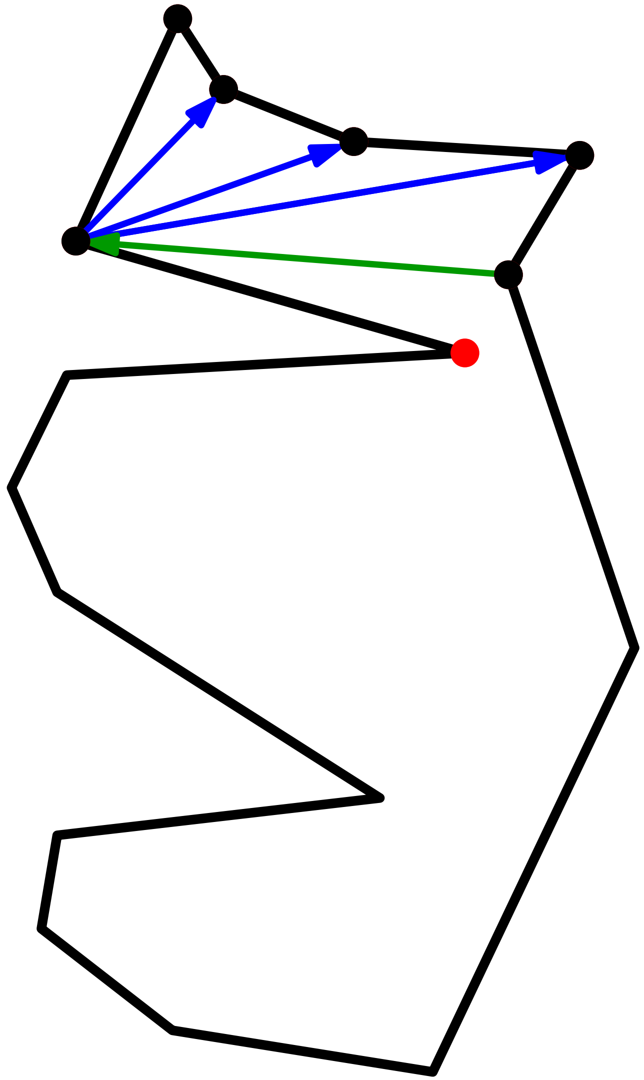

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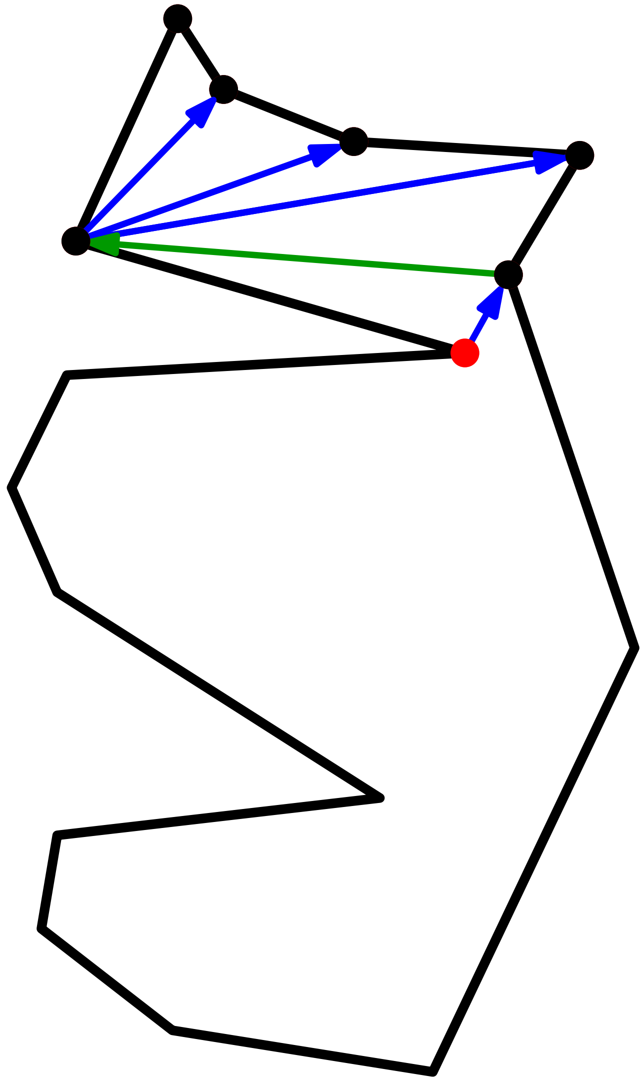

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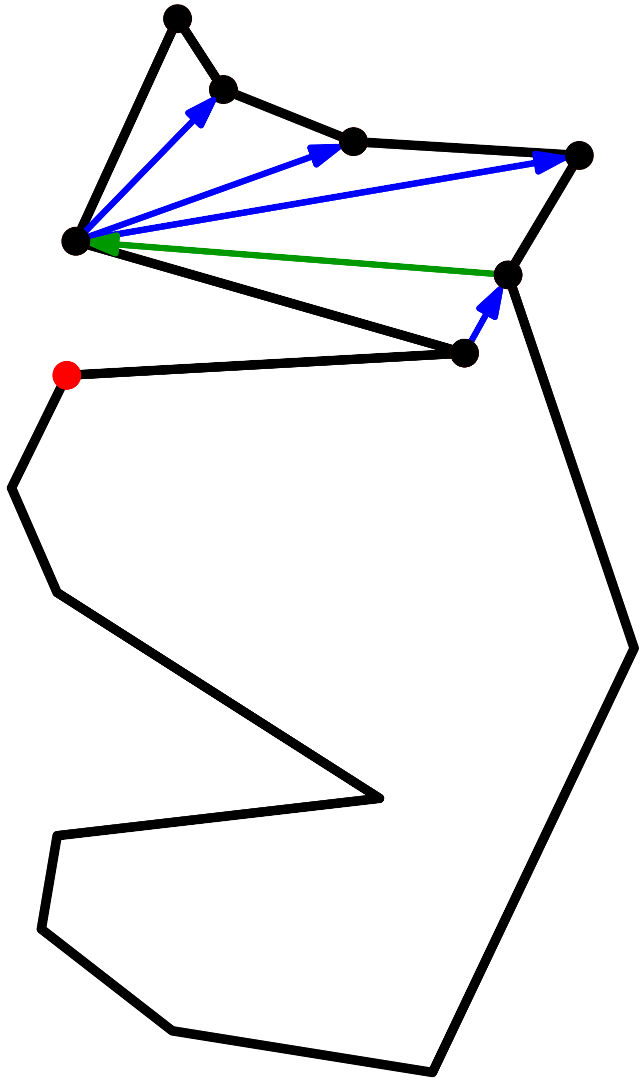

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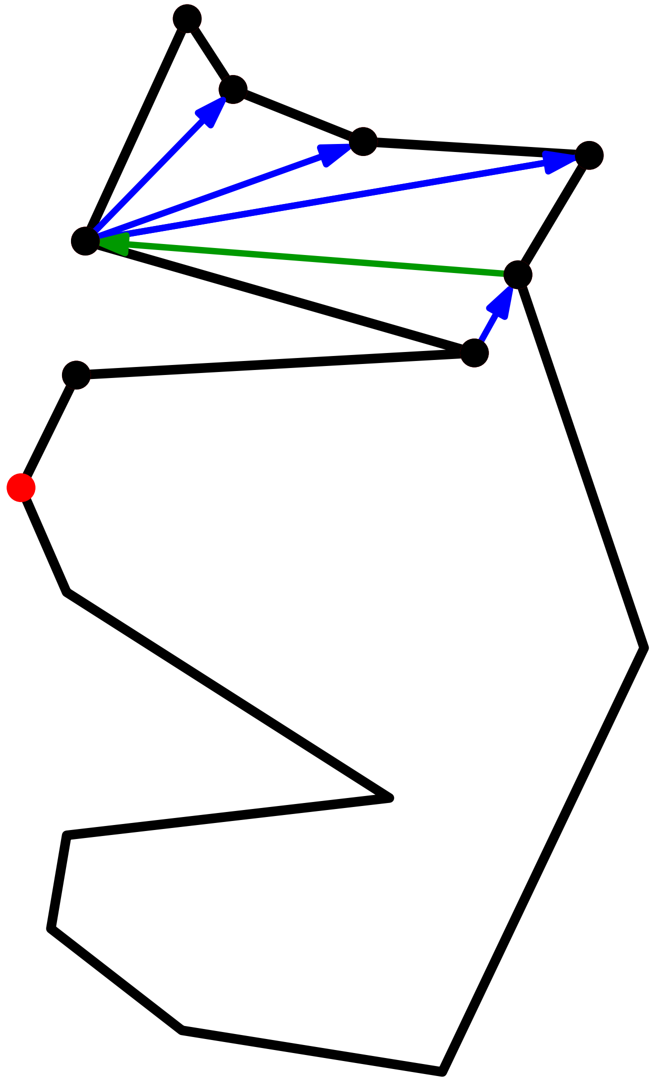

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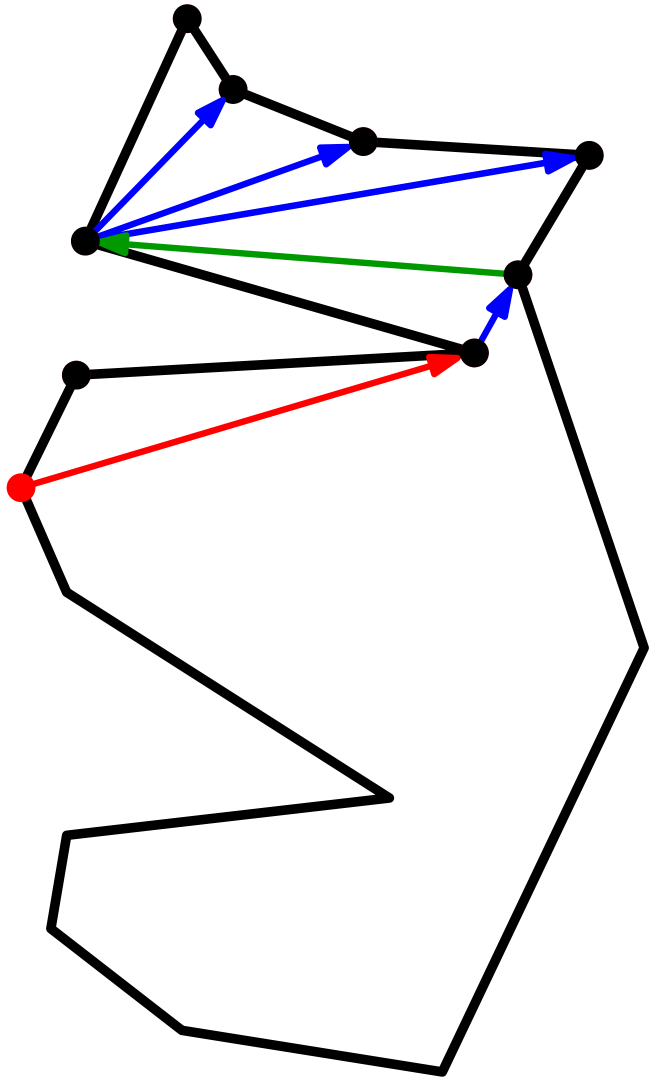

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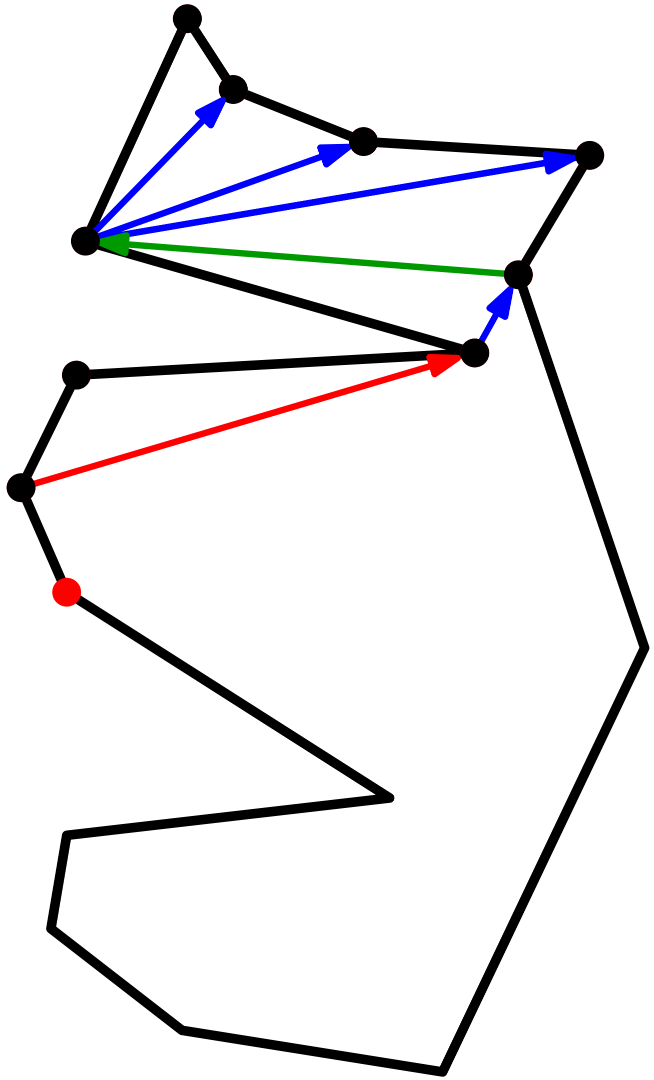

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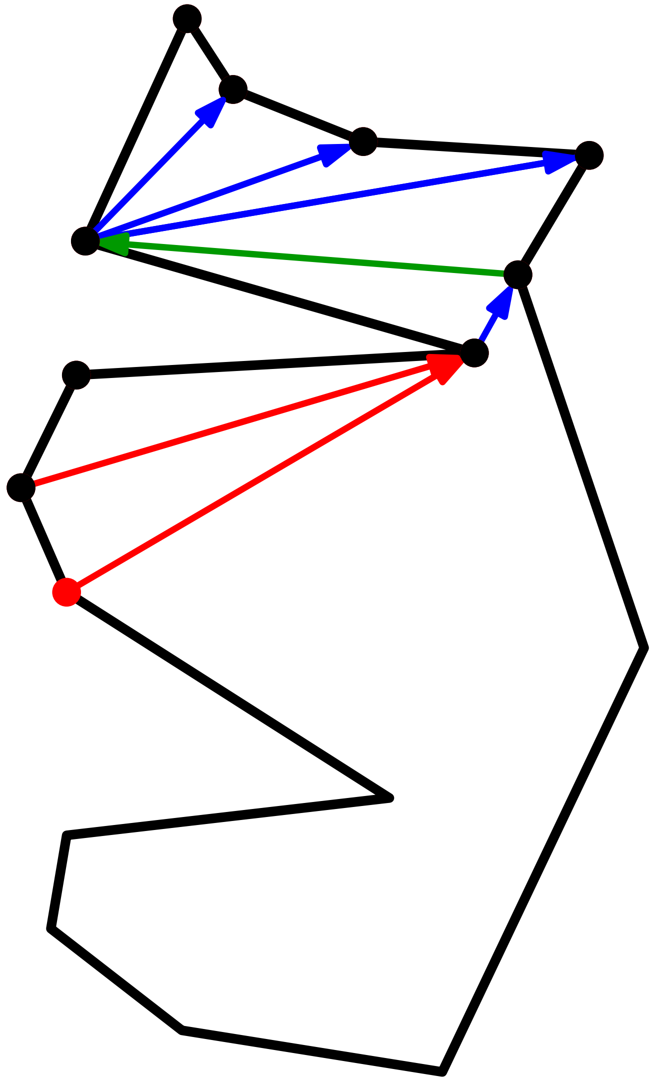

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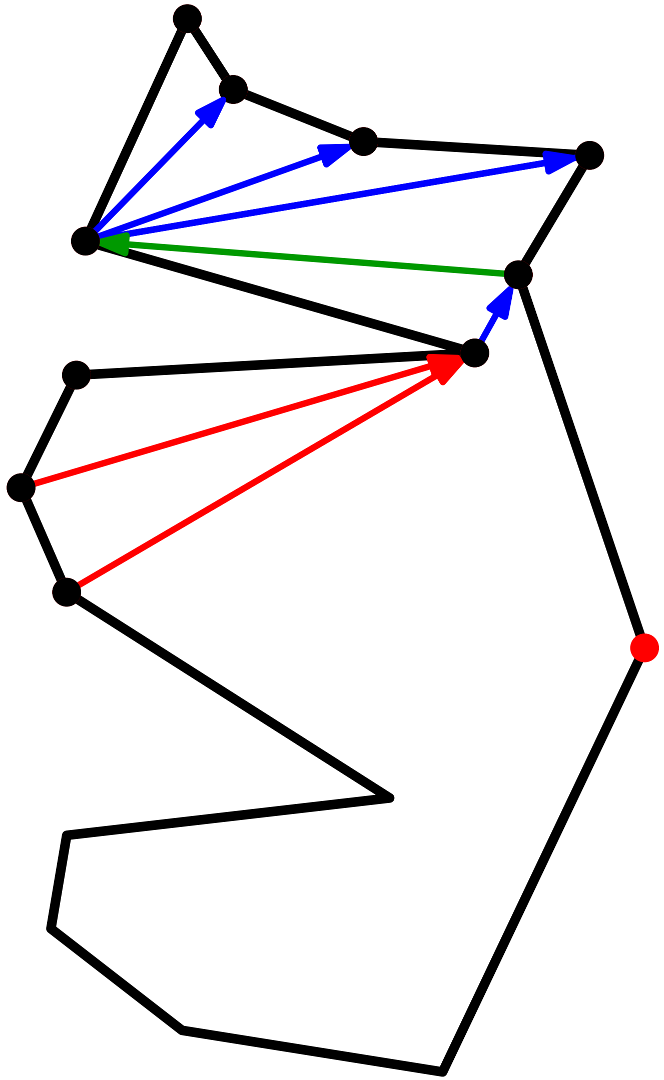

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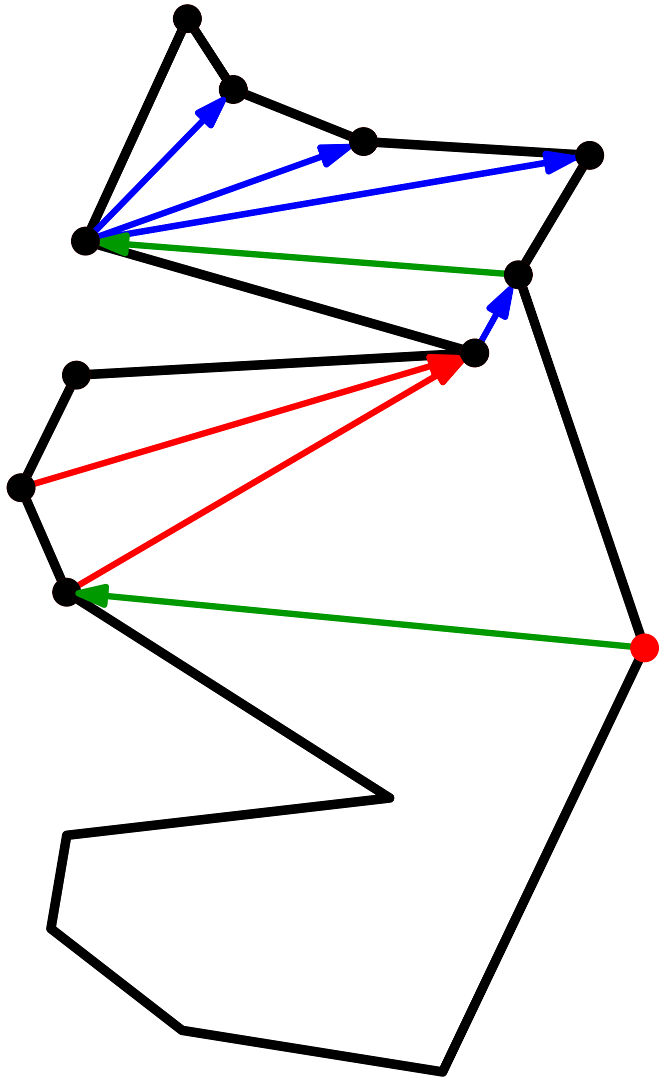

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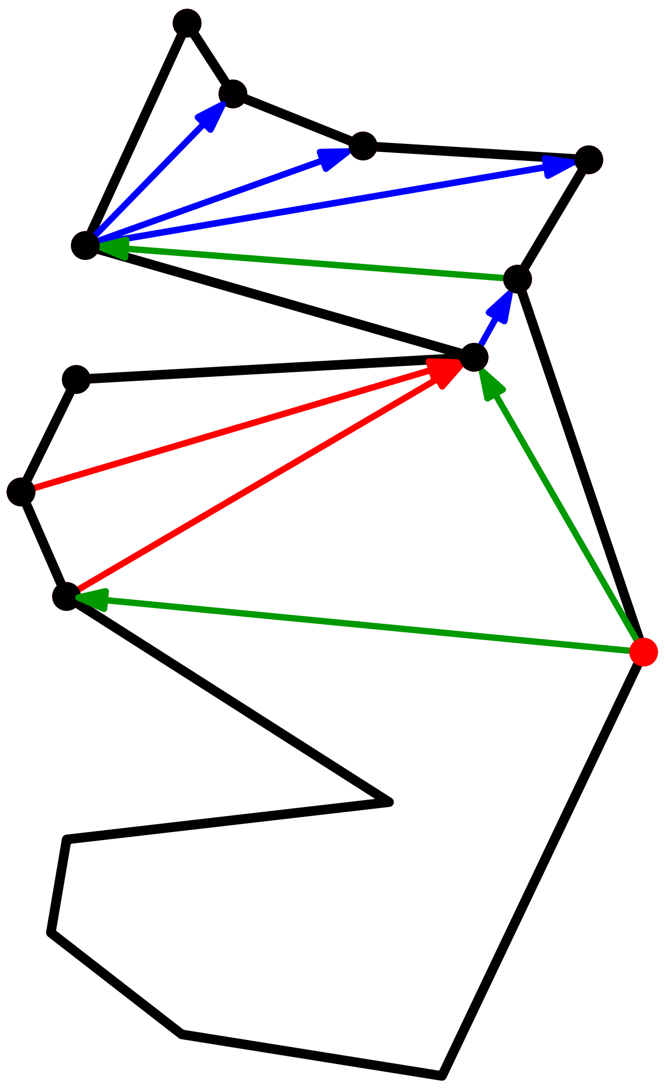

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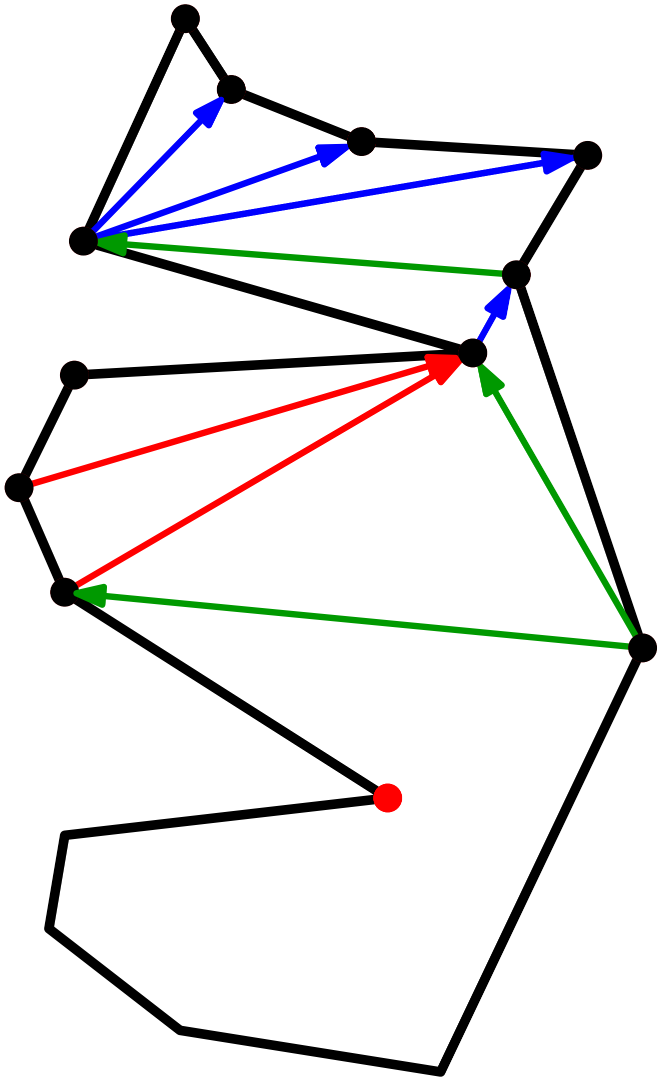

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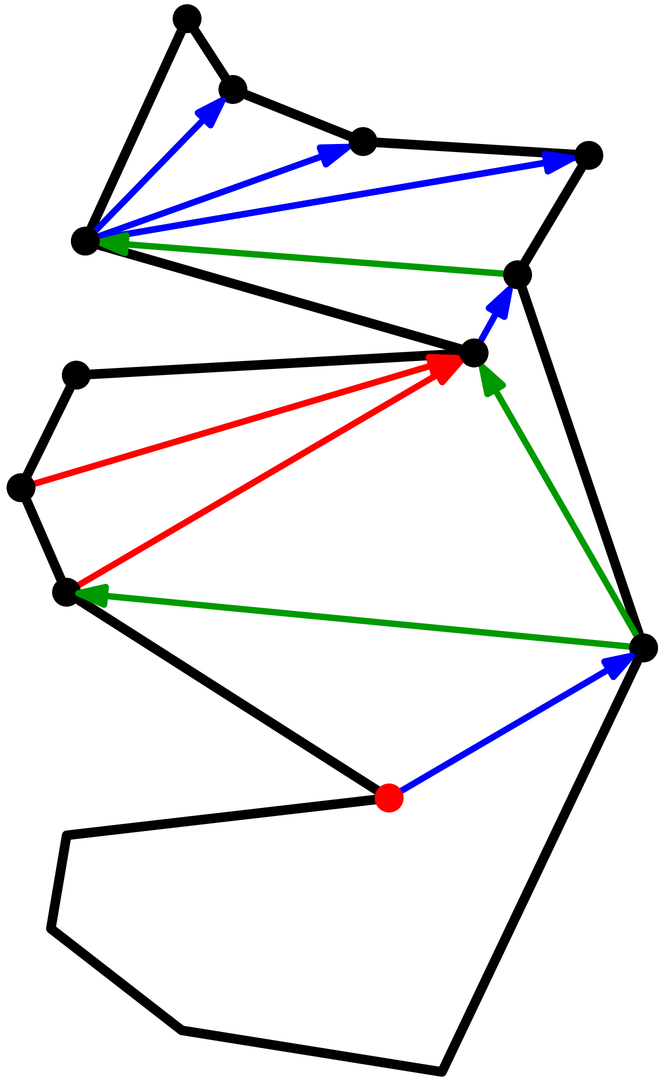

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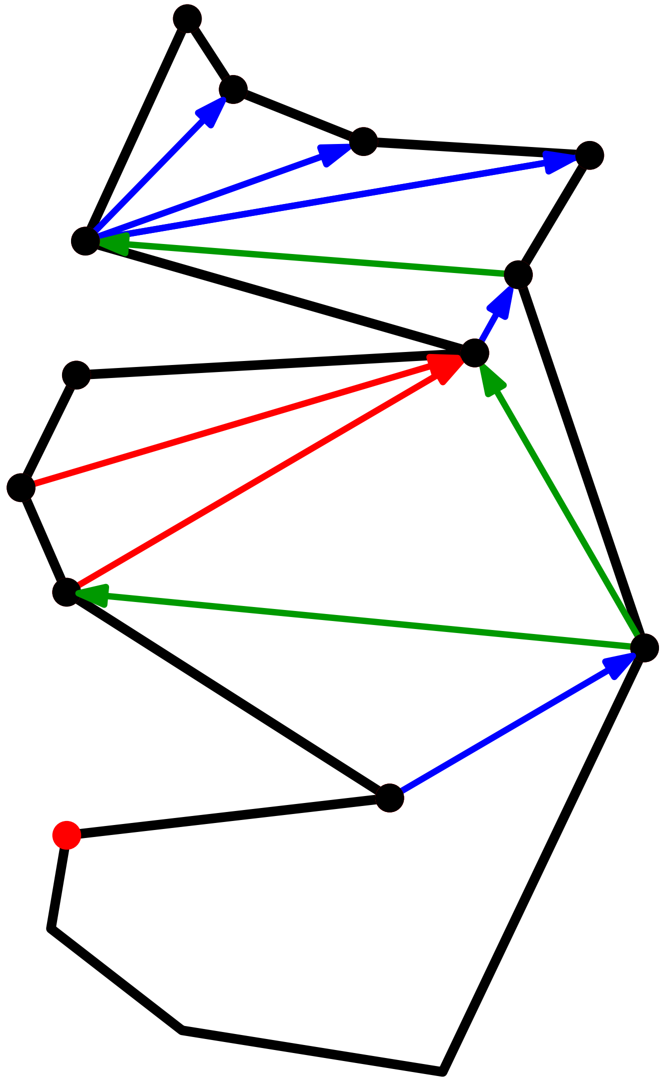

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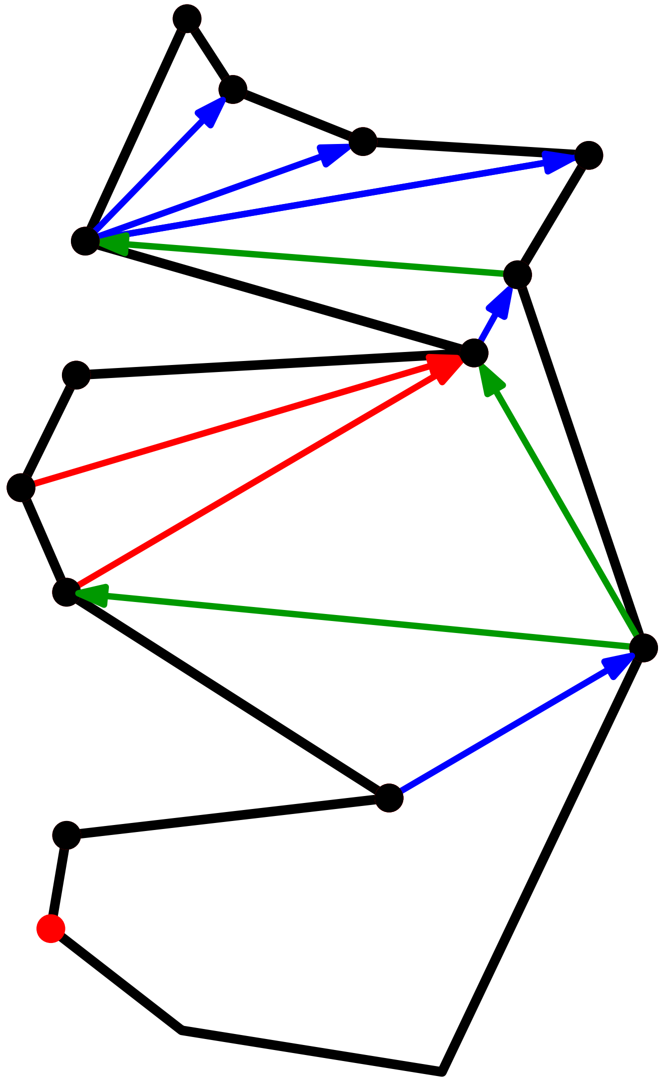

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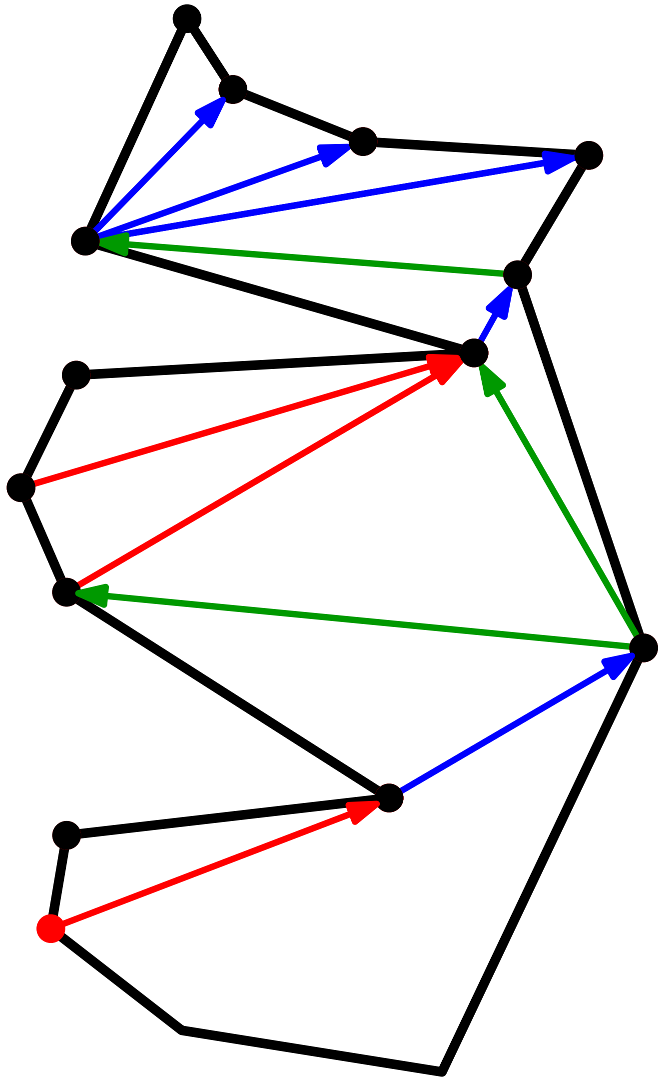

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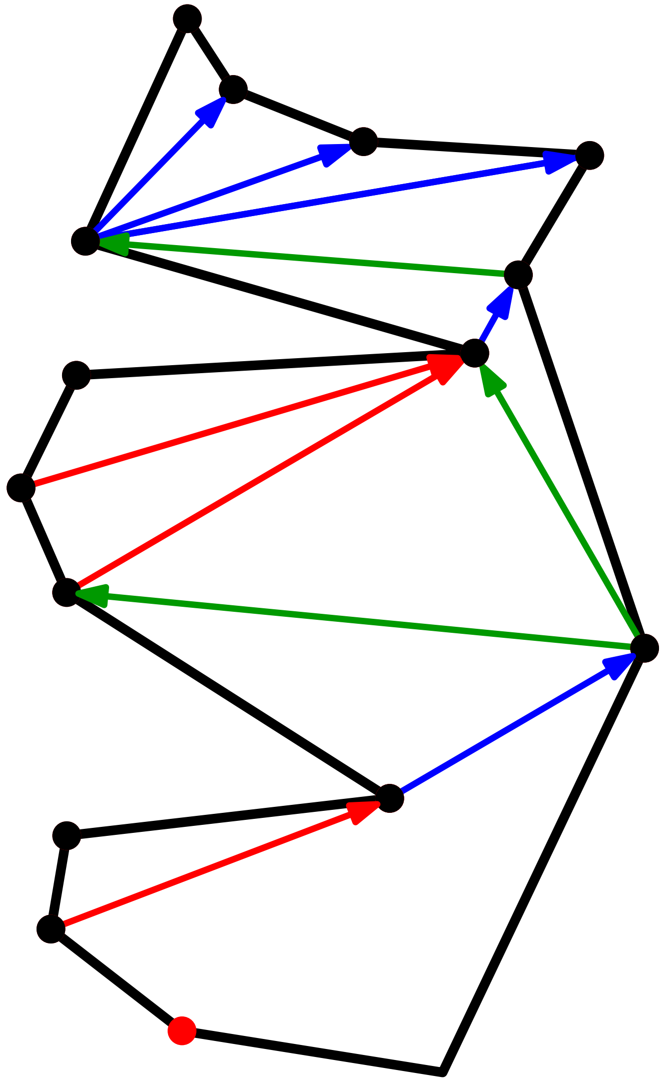

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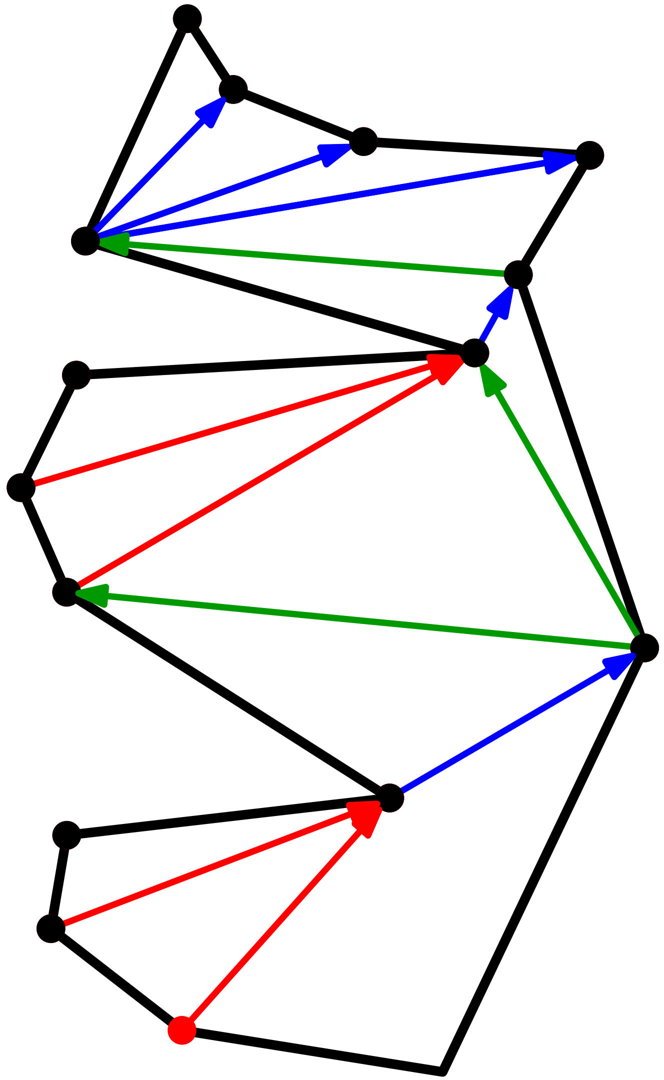

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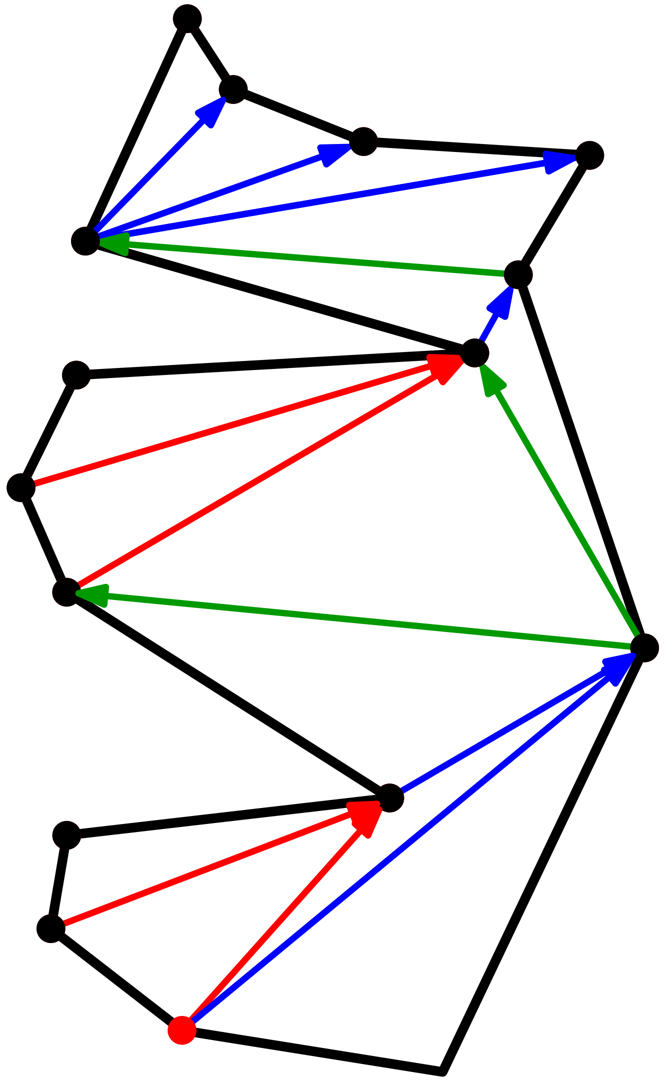

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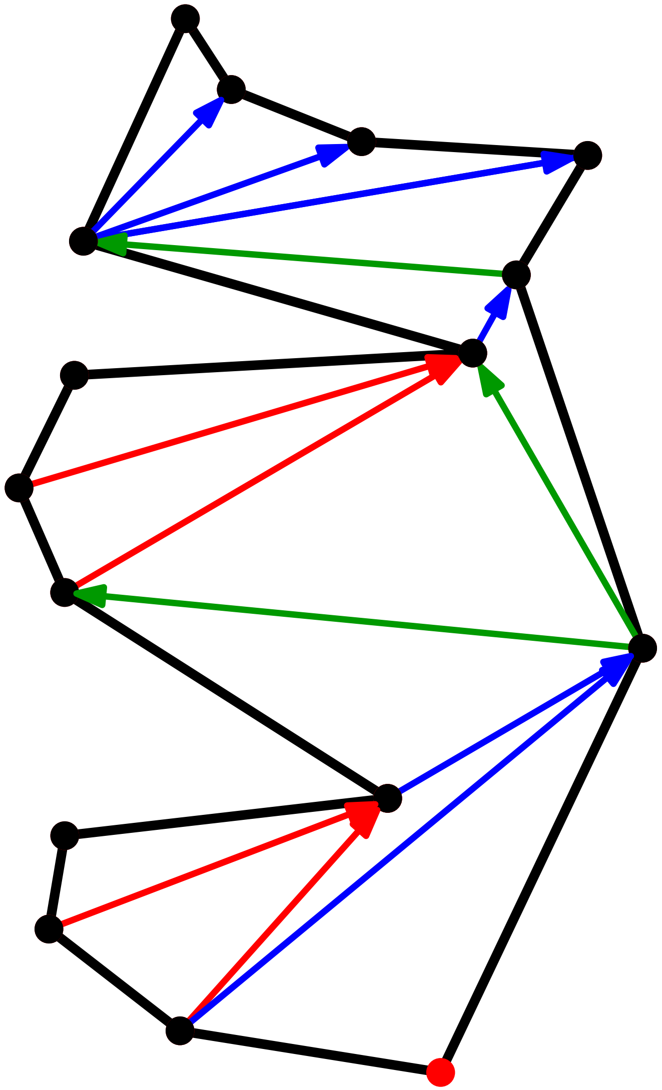

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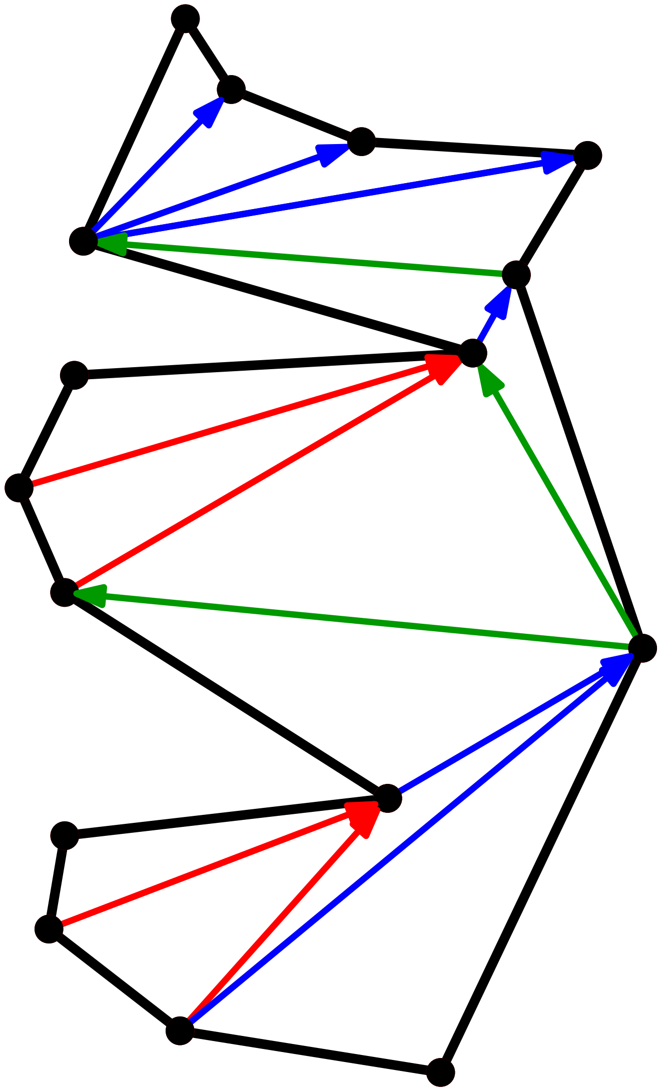

Invariant?

Triangulating a y-Monotone Polygon P

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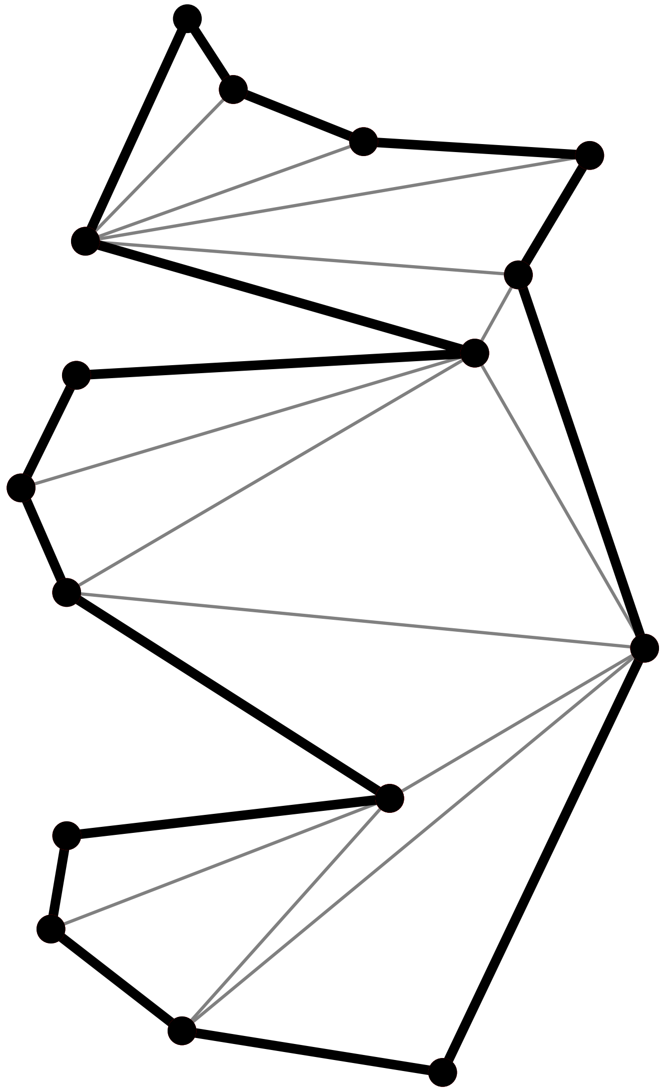

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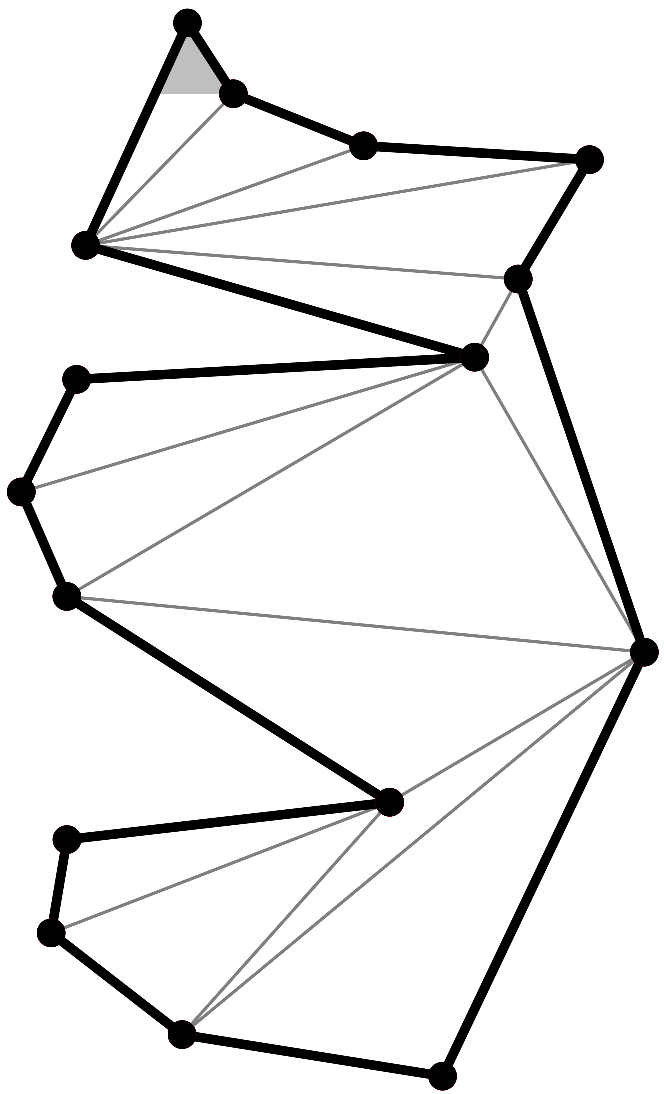
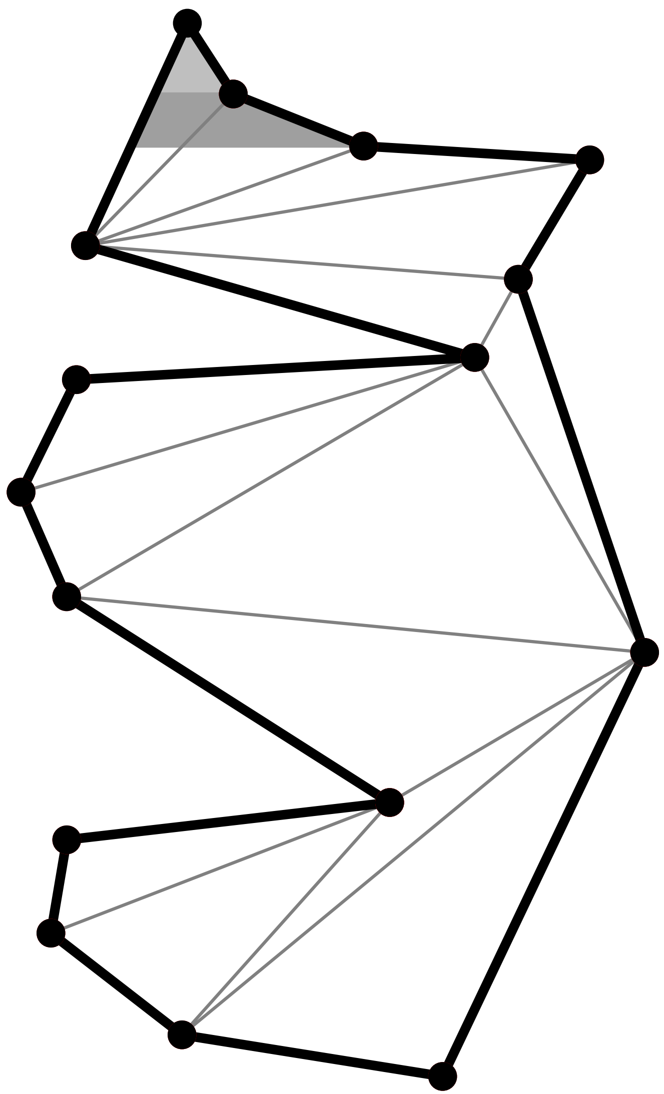


Invariant?

Triangulating a y-Monotone Polygon P

Approach: greedy, going from top to bottom



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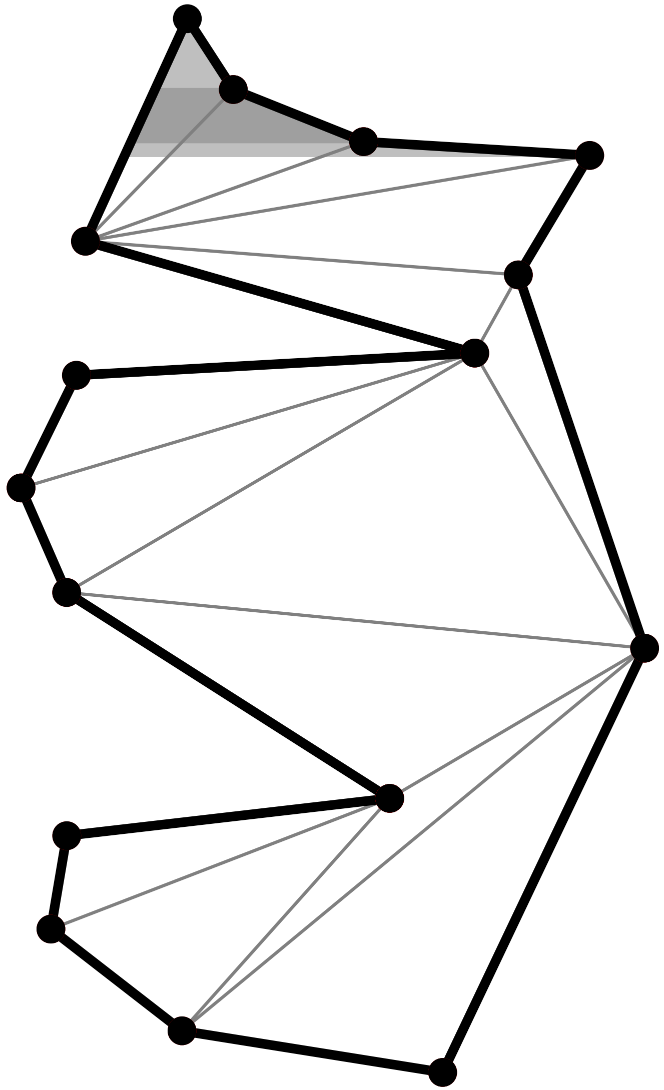

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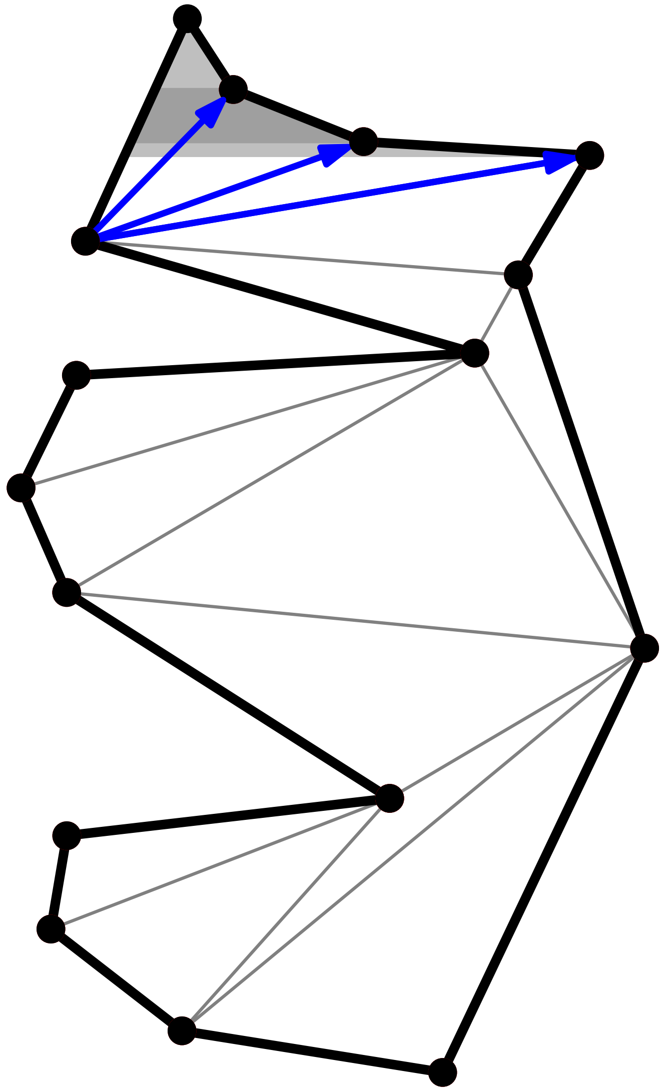

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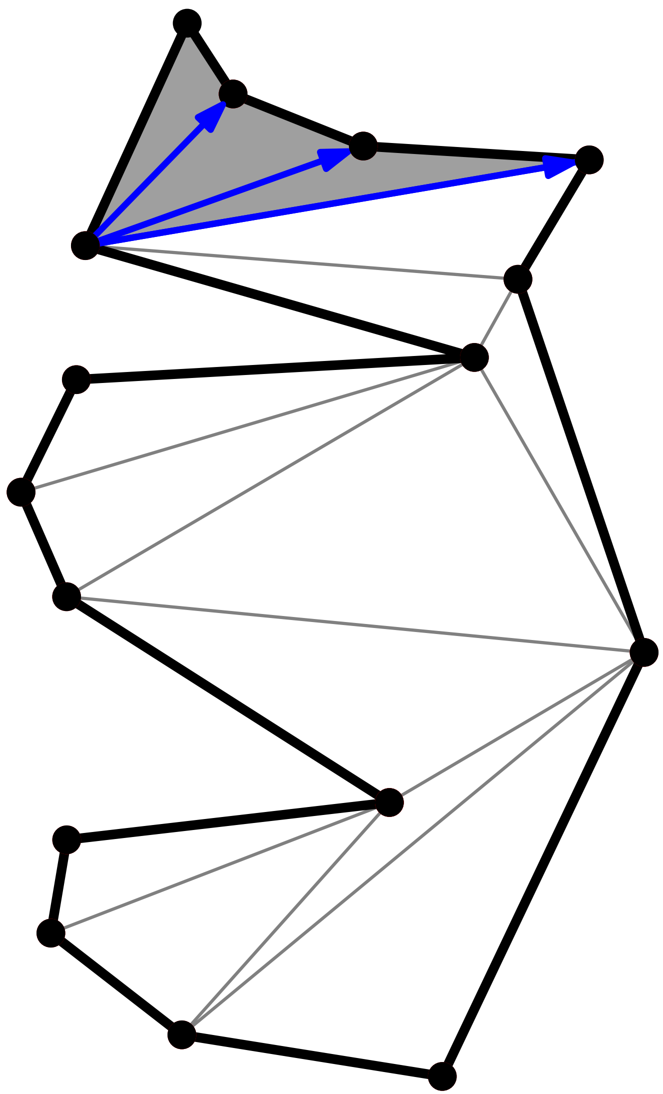
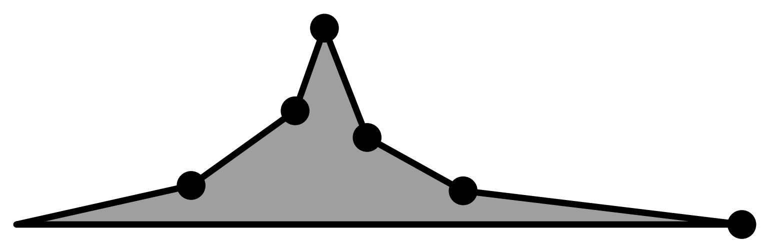


Invariant?

The part of P that we have seen but not yet triangulated is a *funnel*.



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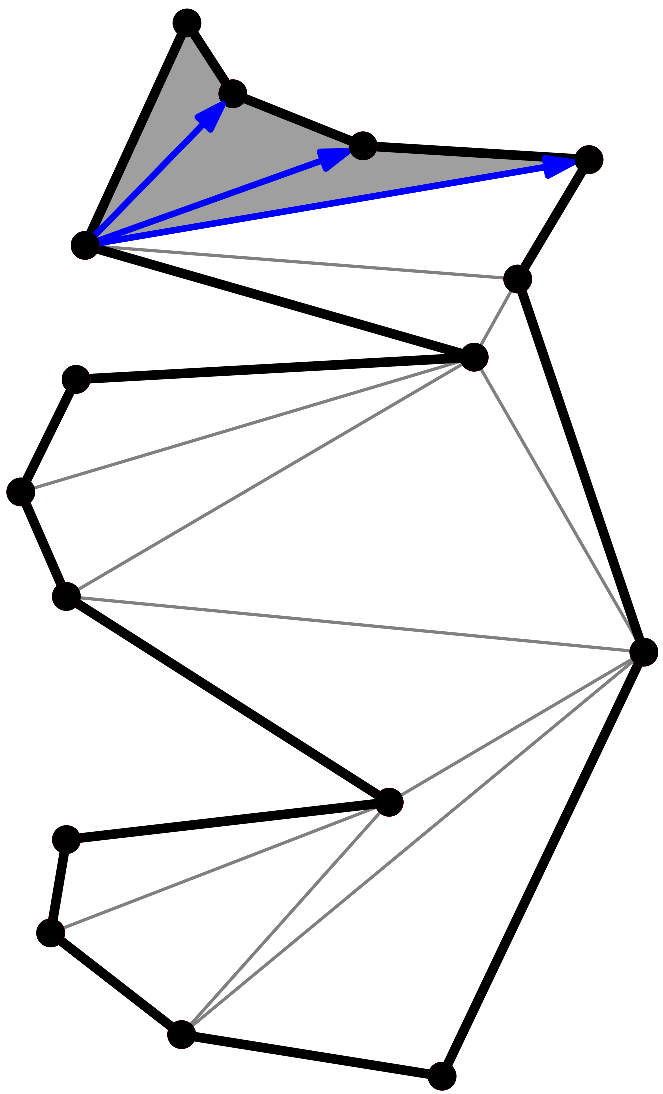

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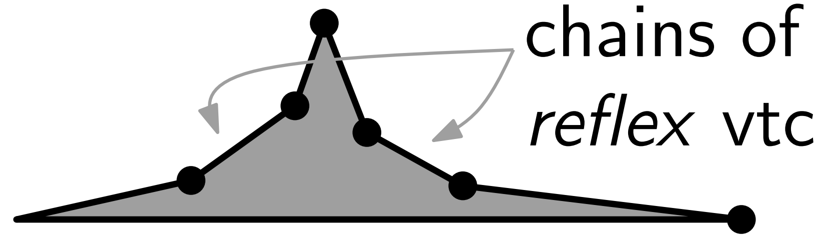

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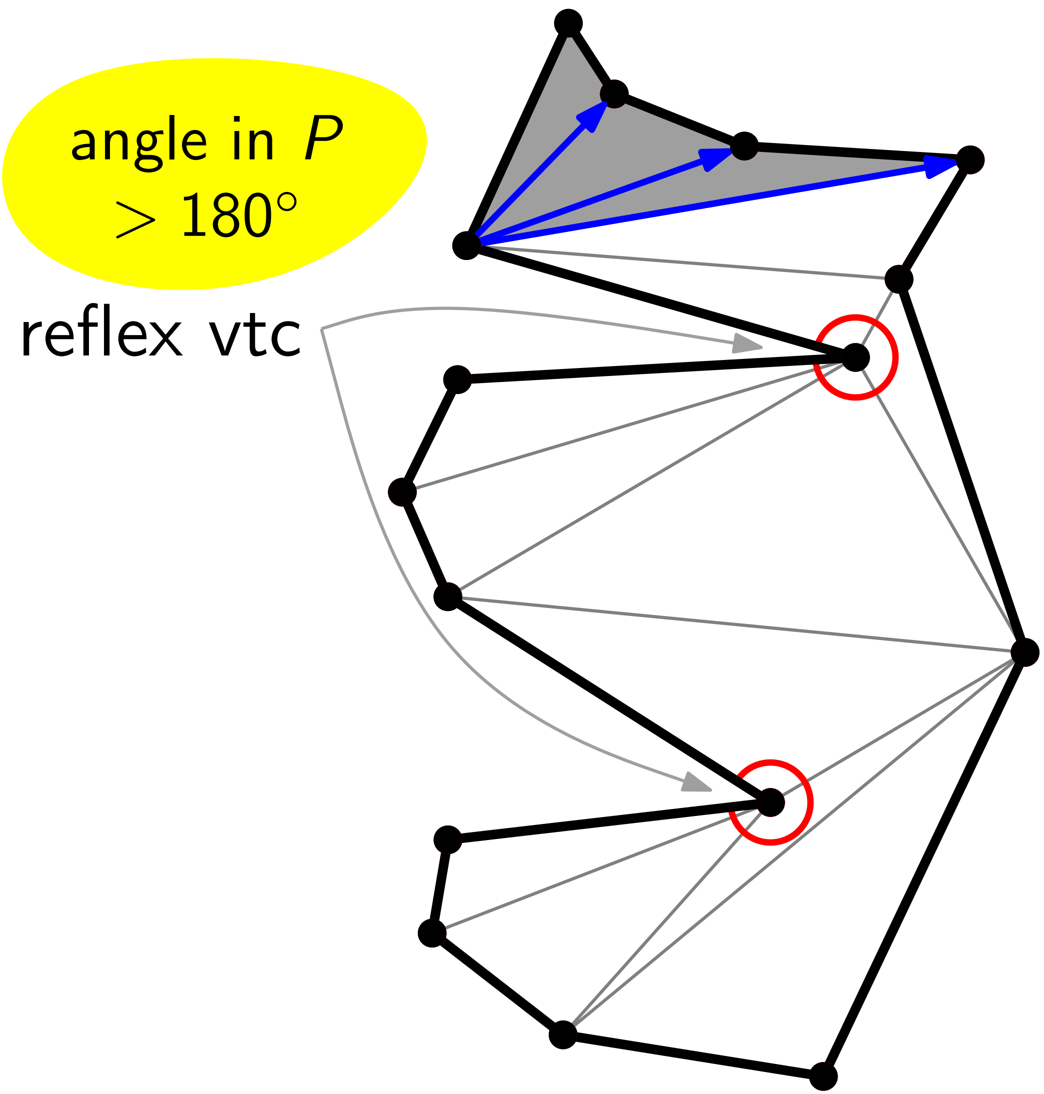

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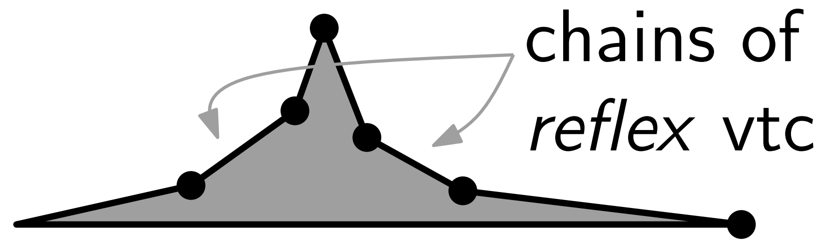

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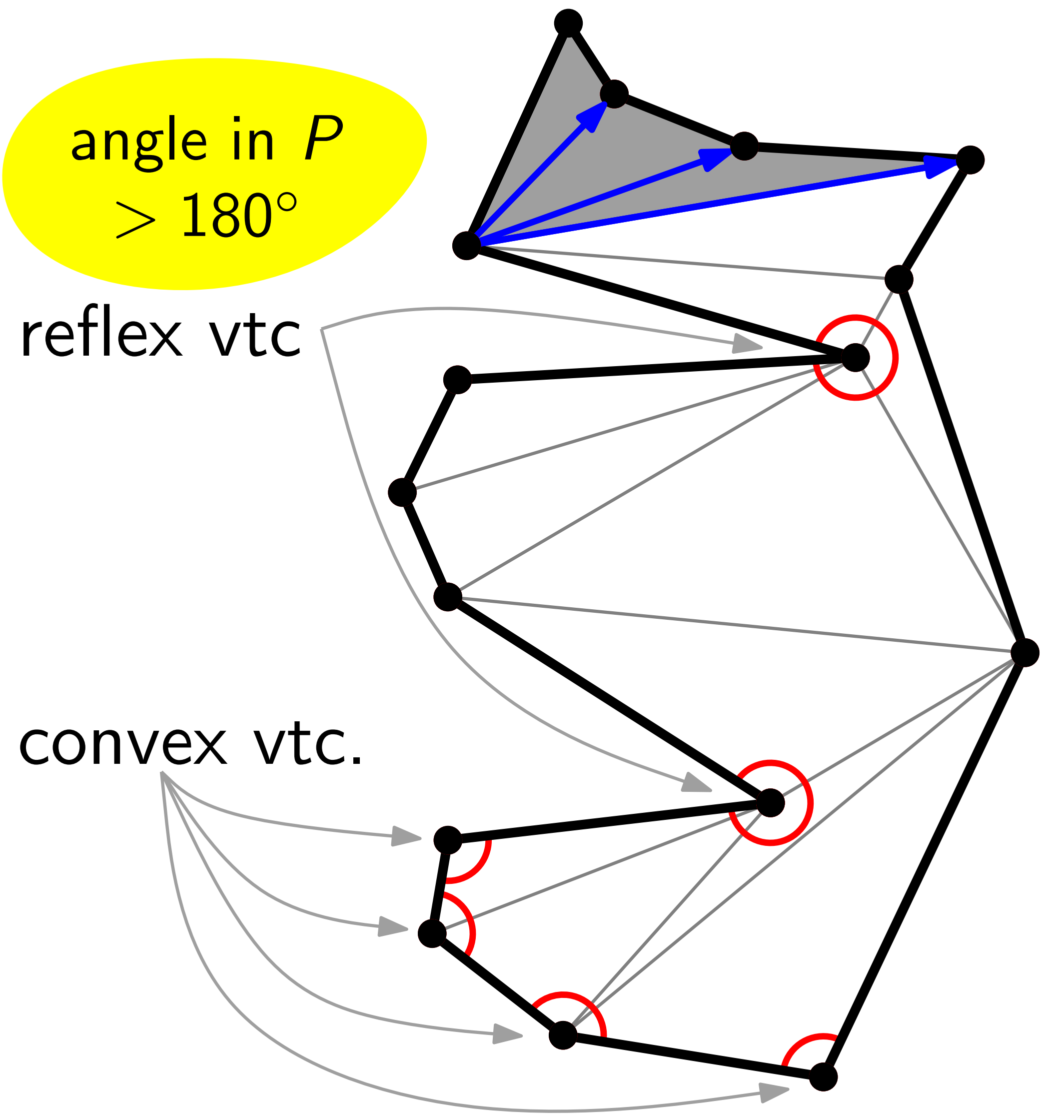

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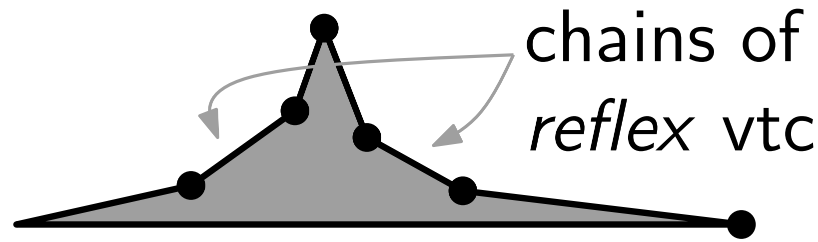

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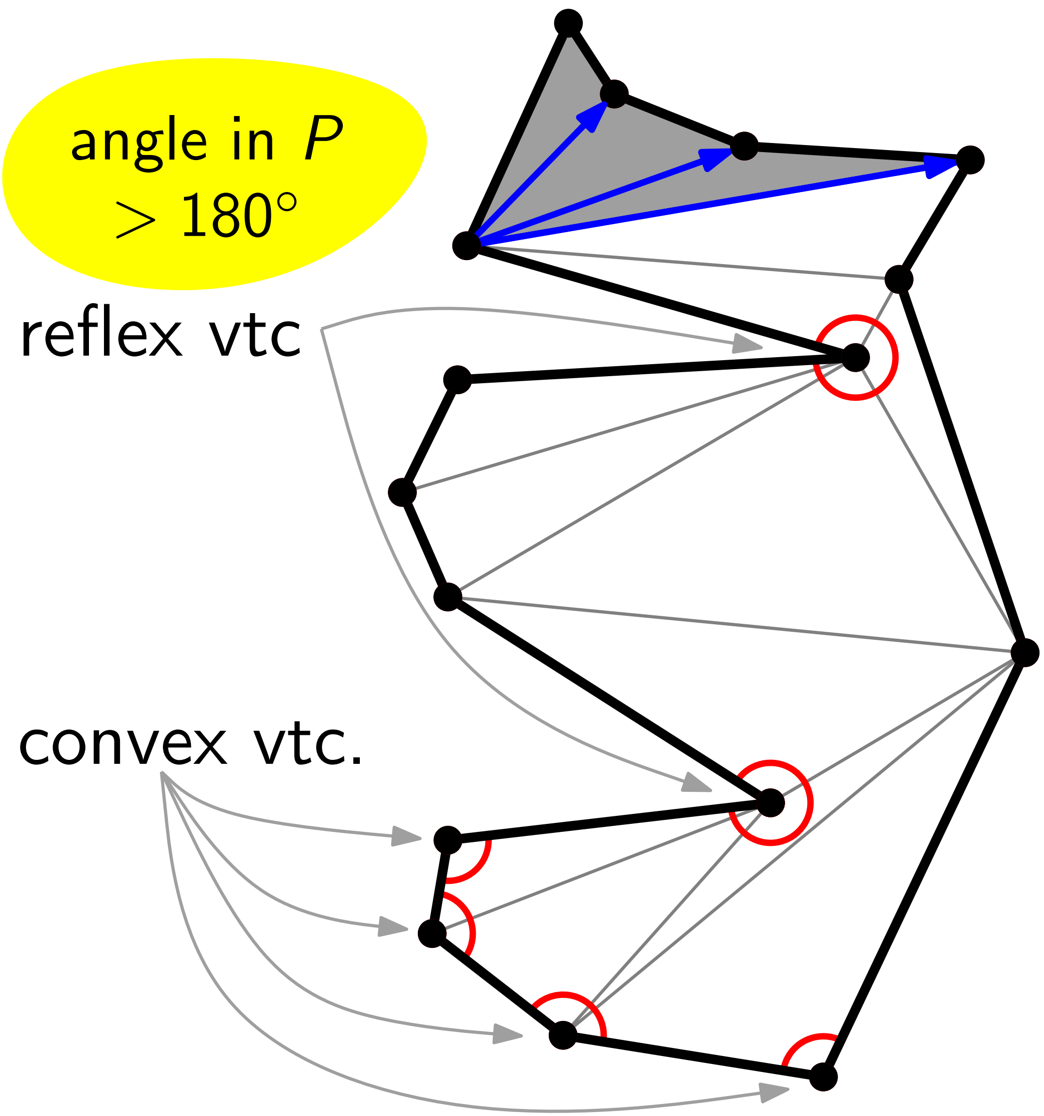

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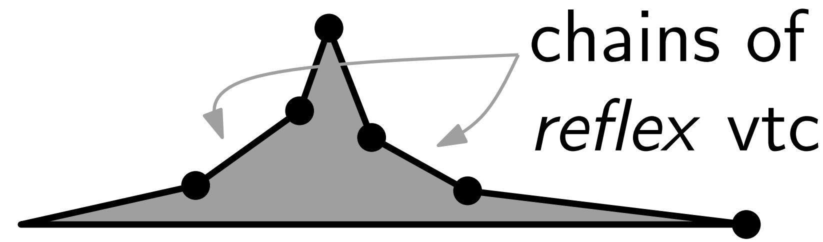

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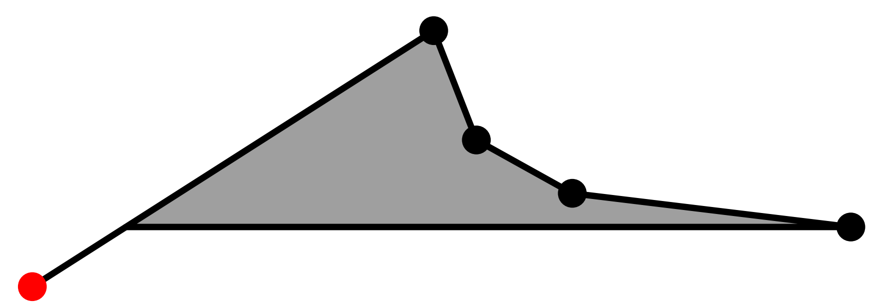

Triangulating a y-Monotone Polygon P

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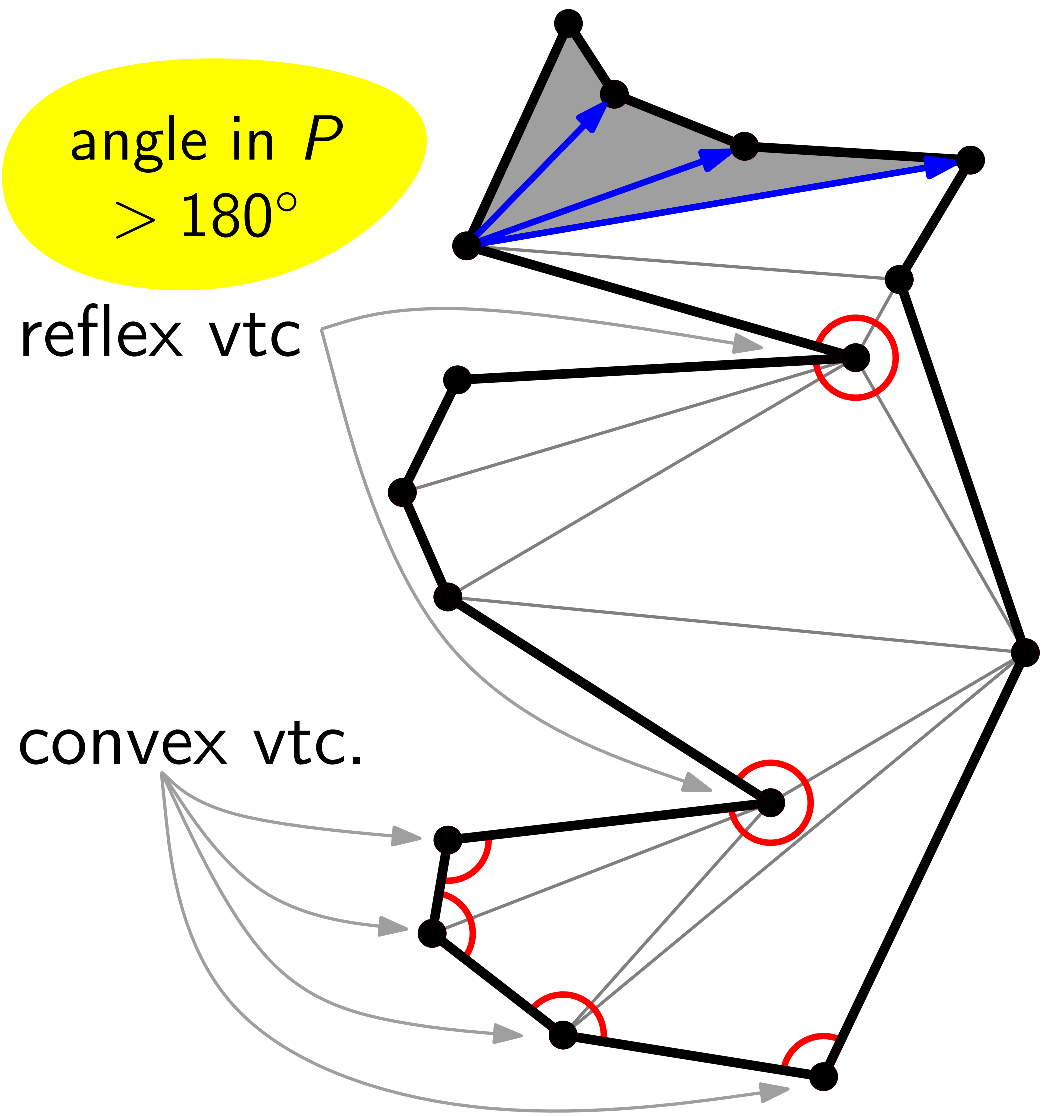

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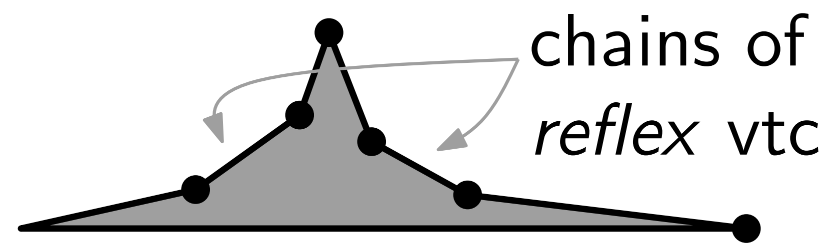

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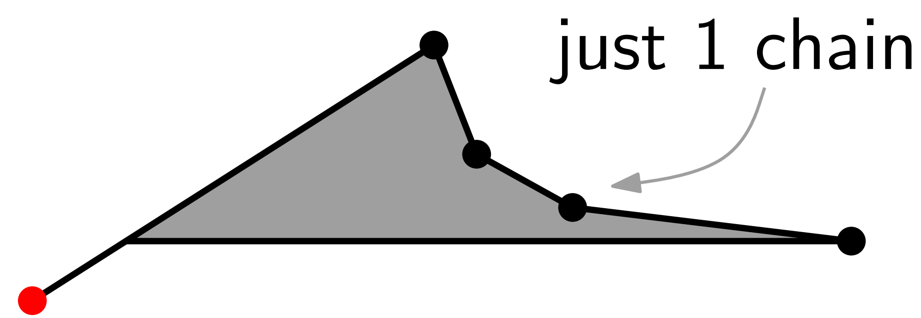

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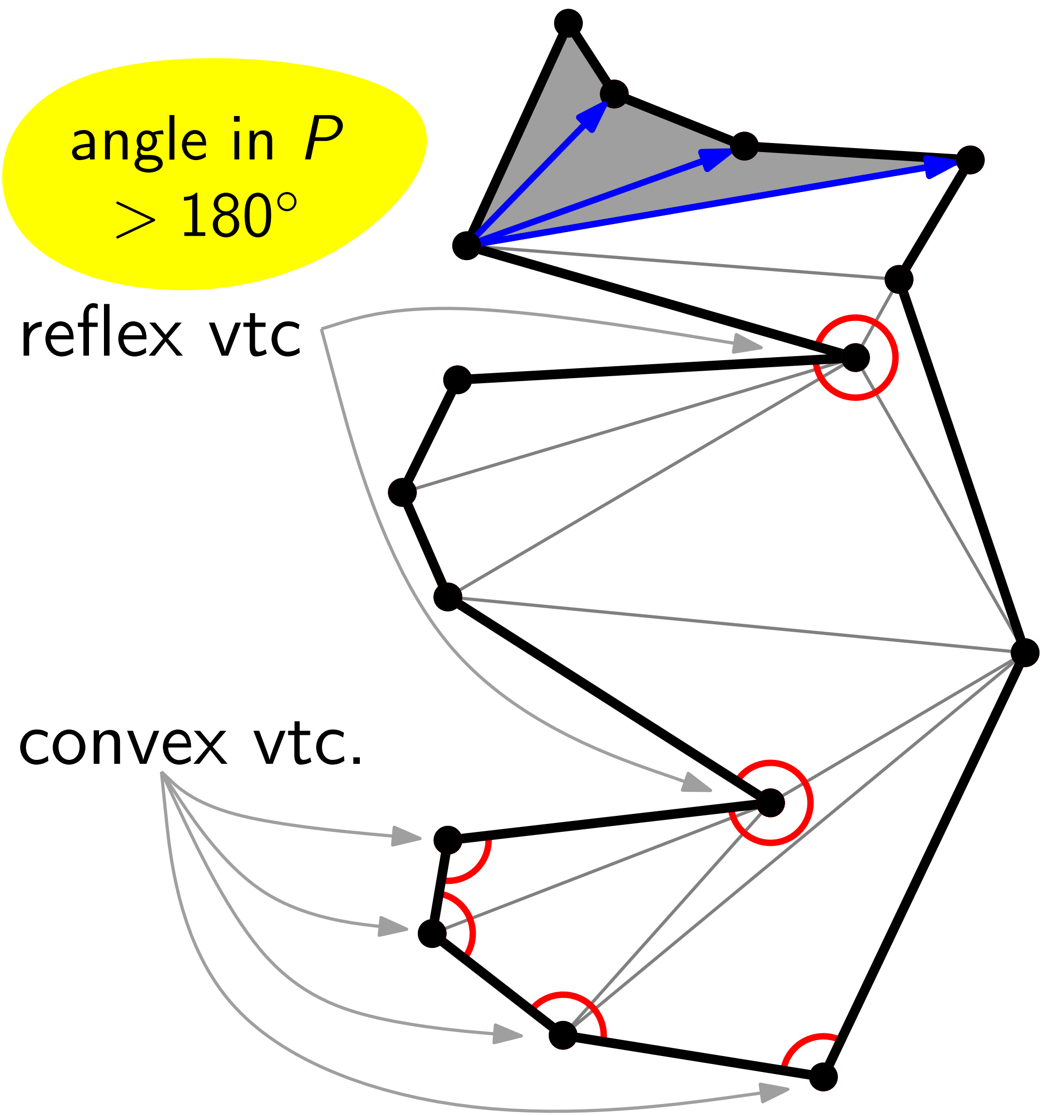

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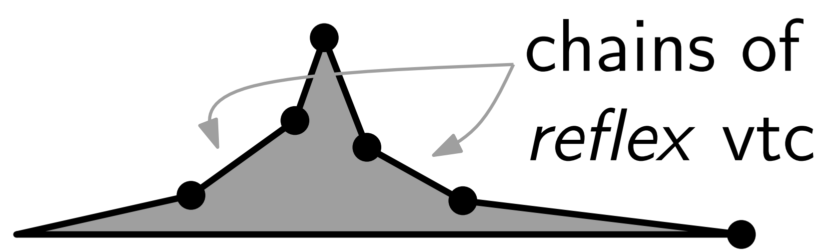

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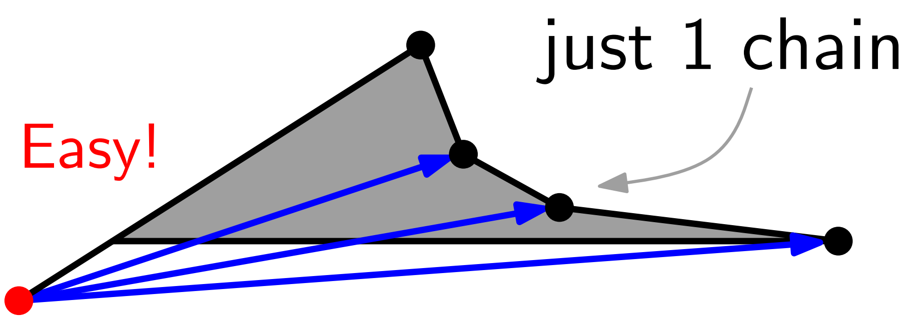

Algorithm

TriangulateMonotonePolygon(Polygon P as circular vertex list)

merge left and right chain $\rightarrow$ sequence $u_1, \ldots, u_n$ with $y_1 \geq \cdots \geq y_n$
Stack S; S.push(u_1); S.push(u_2)
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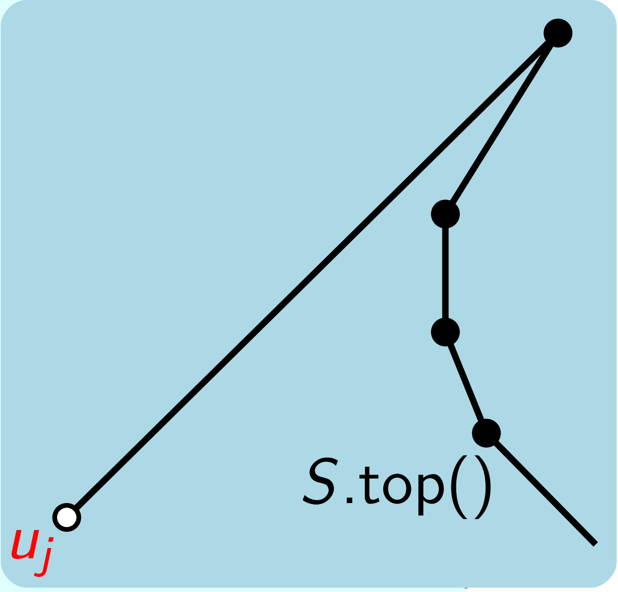

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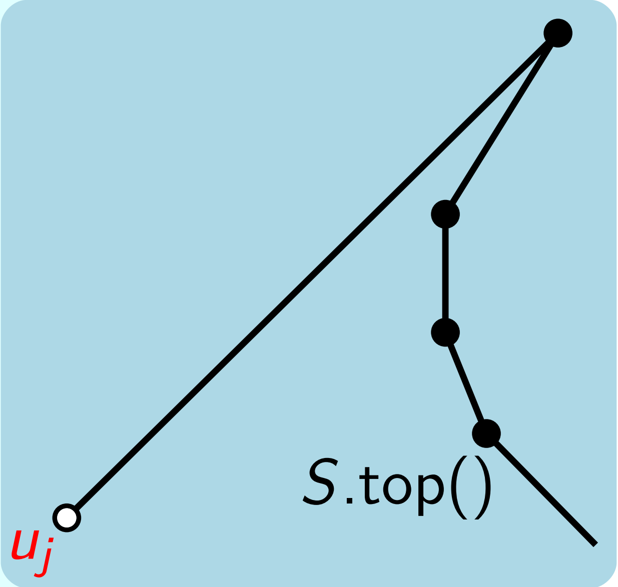

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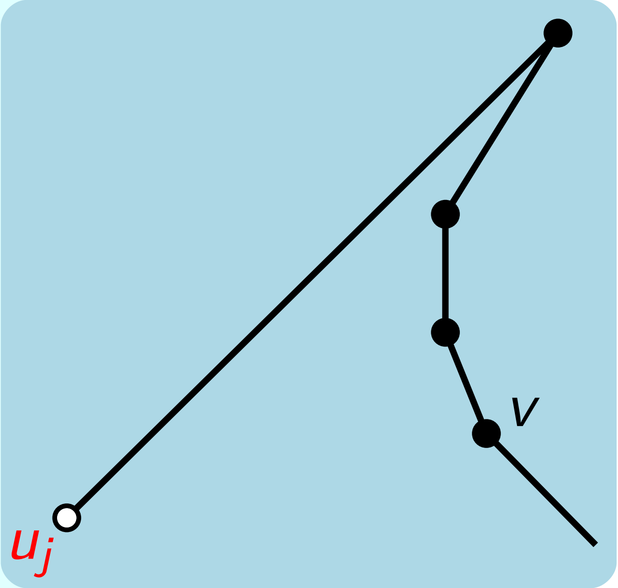

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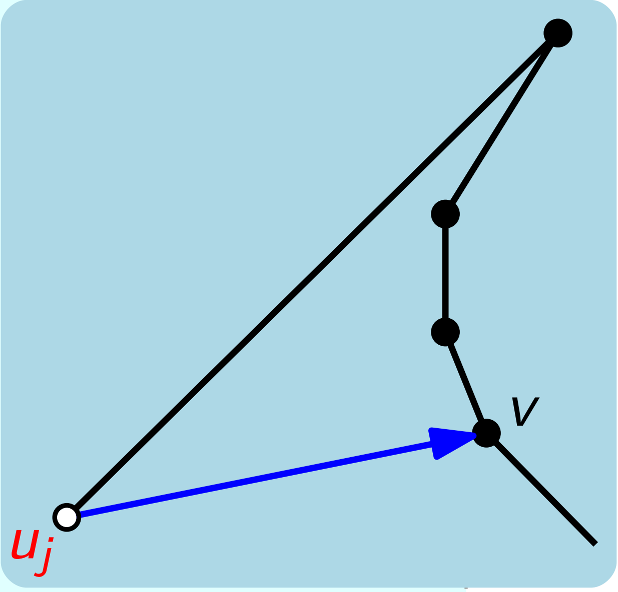

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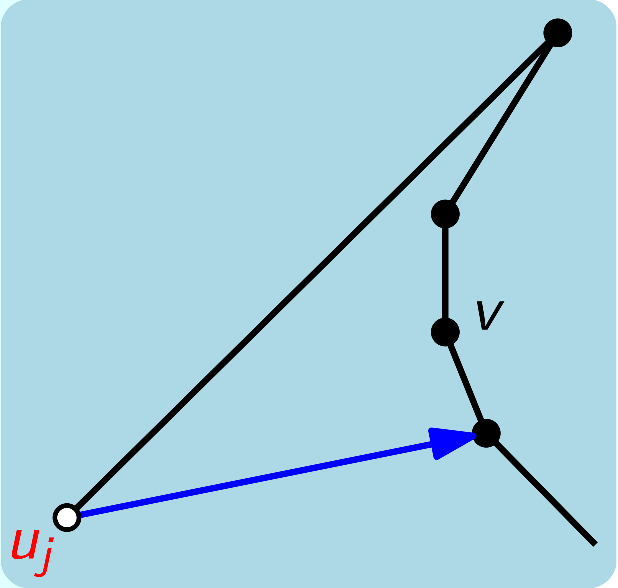

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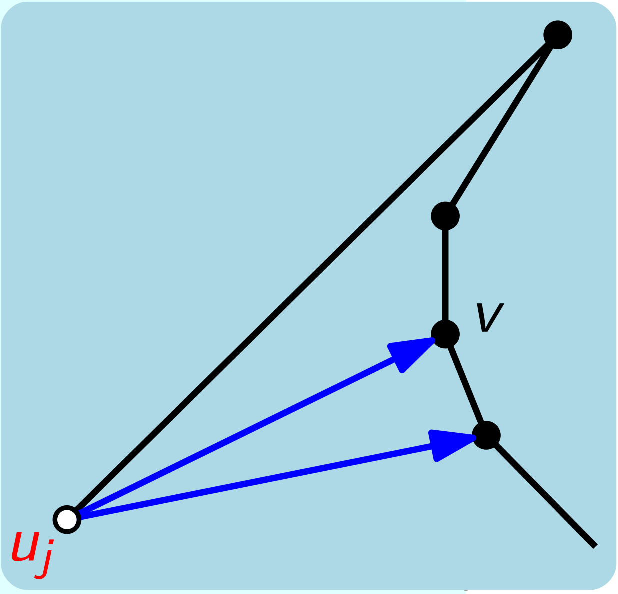

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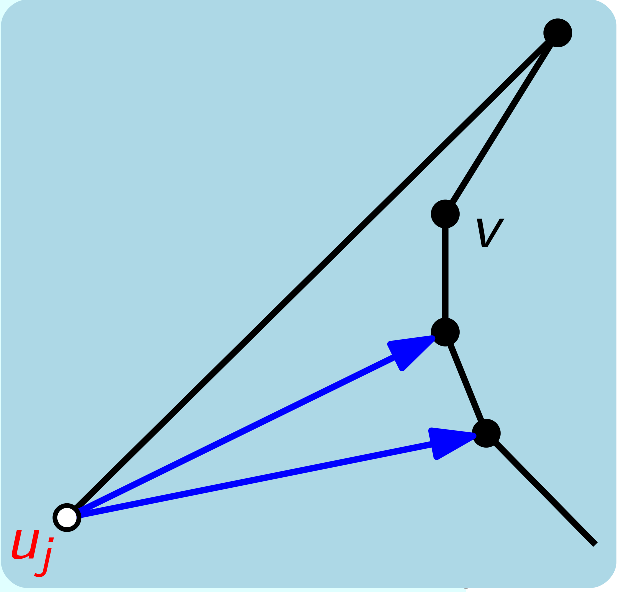

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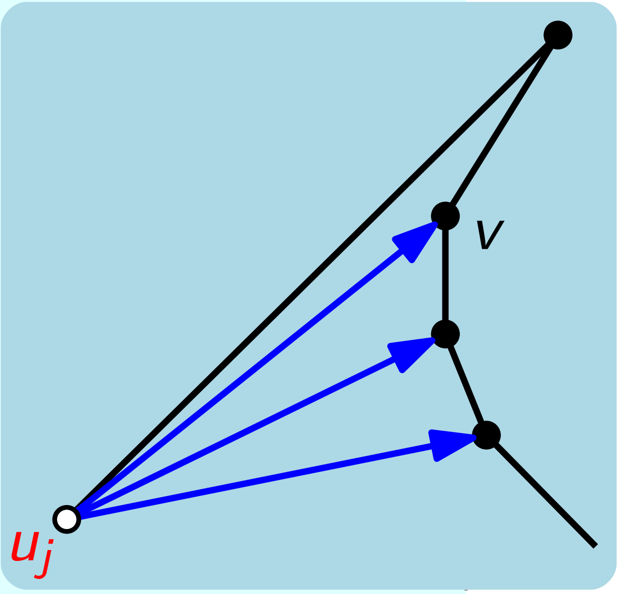

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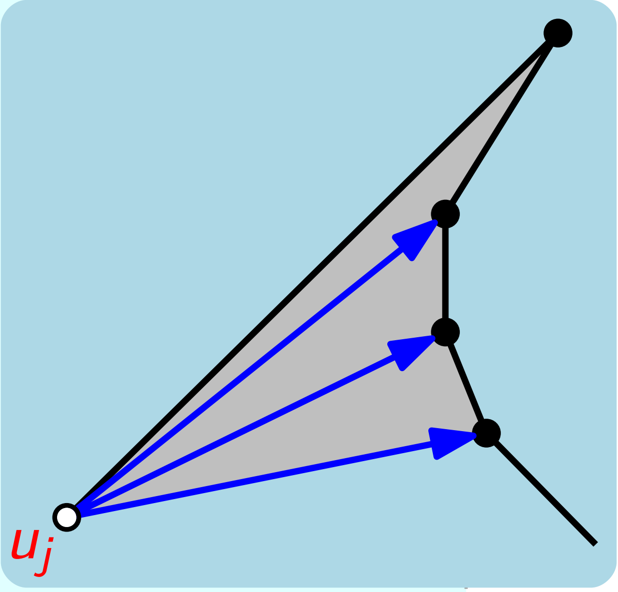

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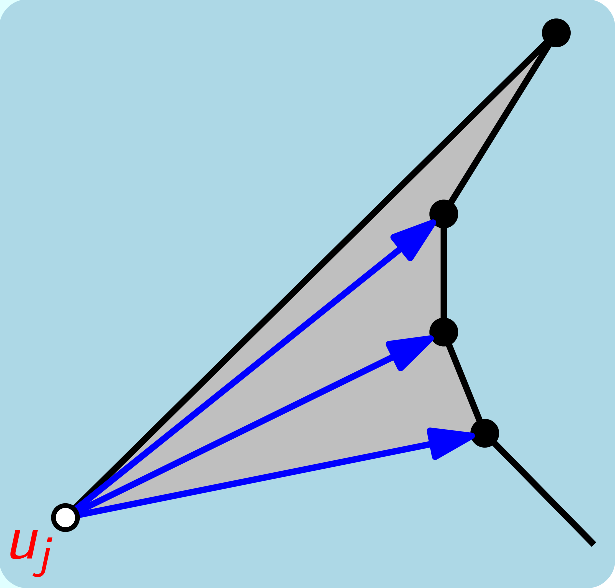

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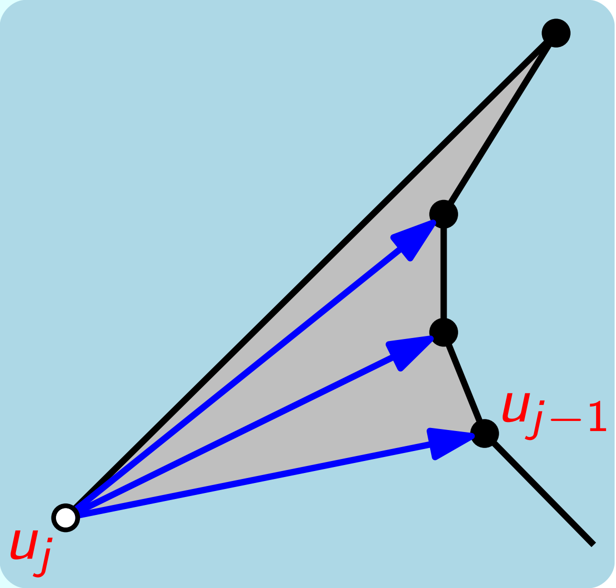

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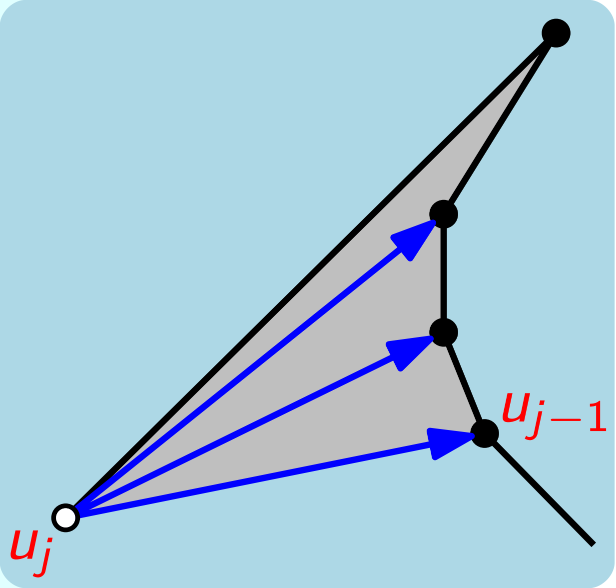

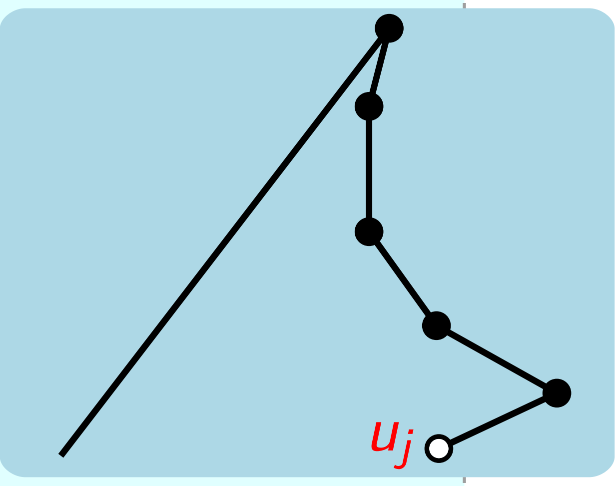

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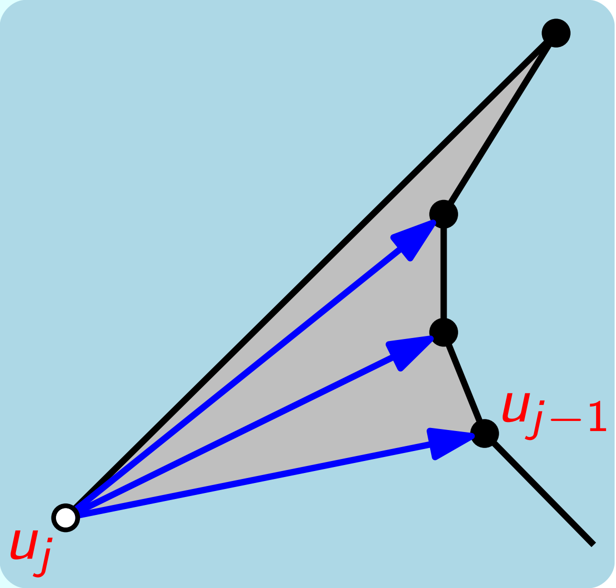

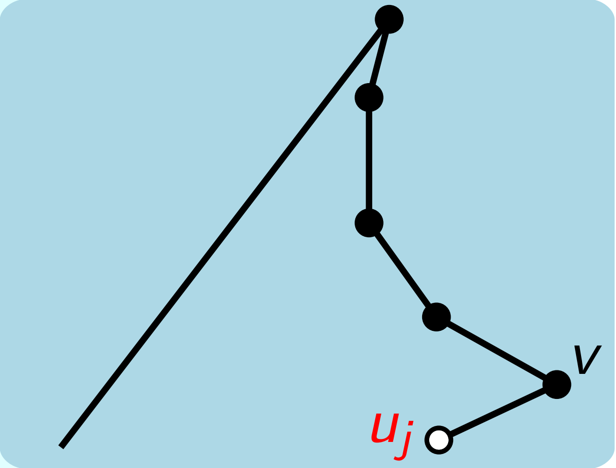

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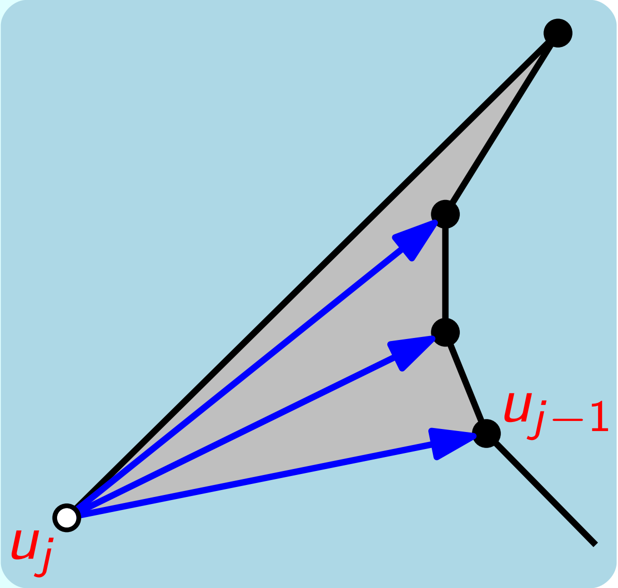

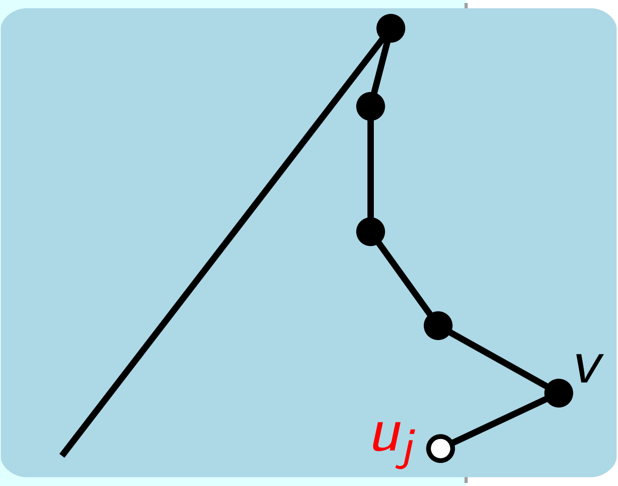

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```
for j ← 3 to n − 1 do
    if u_j and S.top() lie on different chains then
        while not S.empty() do
            v ← S.pop()
            if not S.empty() then draw diag. (u_j, v)
        S.push(u_{j−1}); S.push(u_j)
    else
        v ← S.pop()
        while not S.empty() and u_j sees S.top() do
            v ← S.pop()
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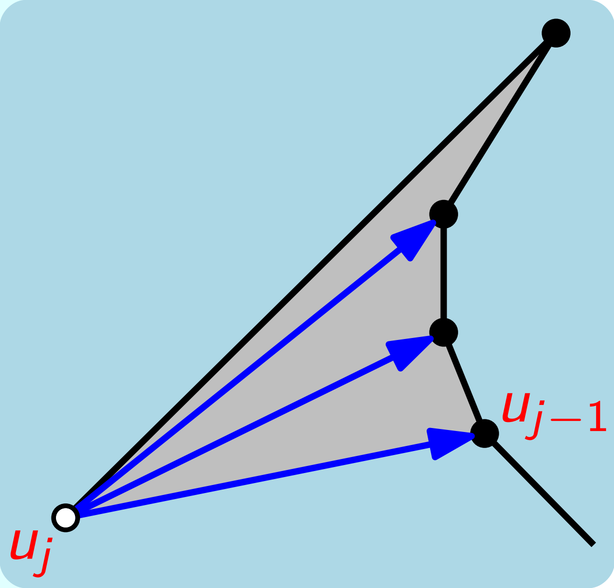

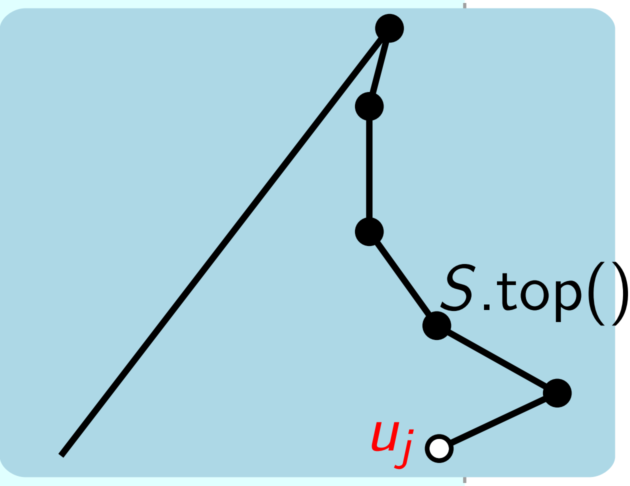

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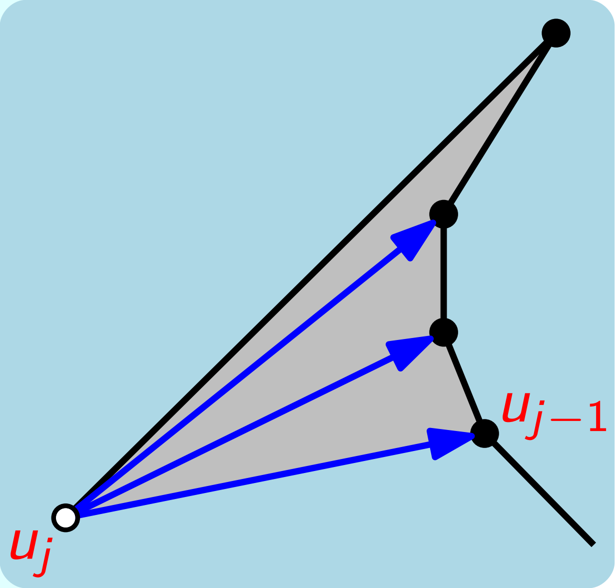

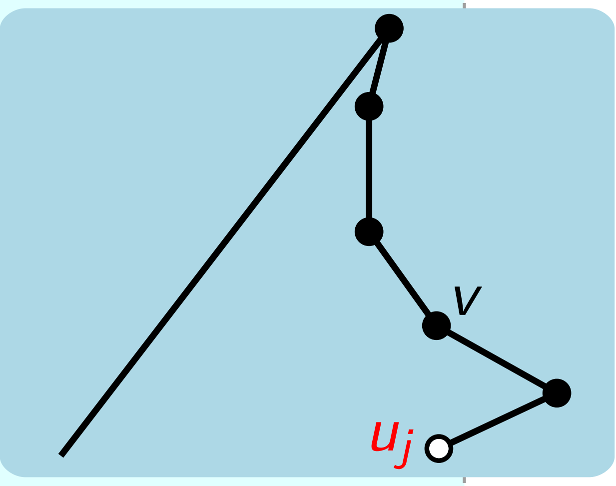

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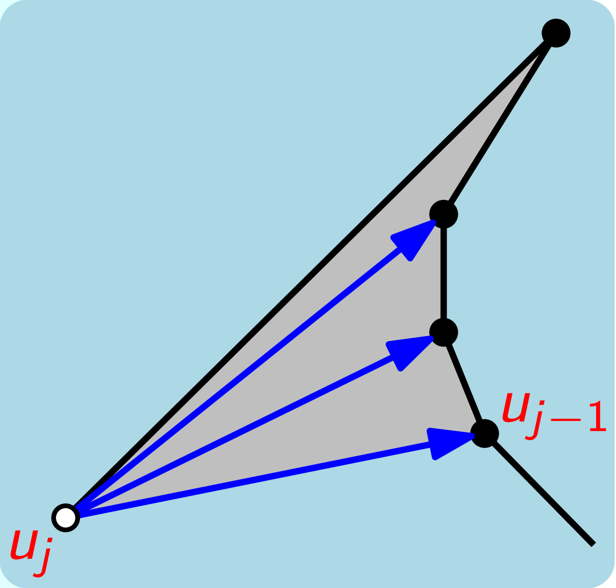

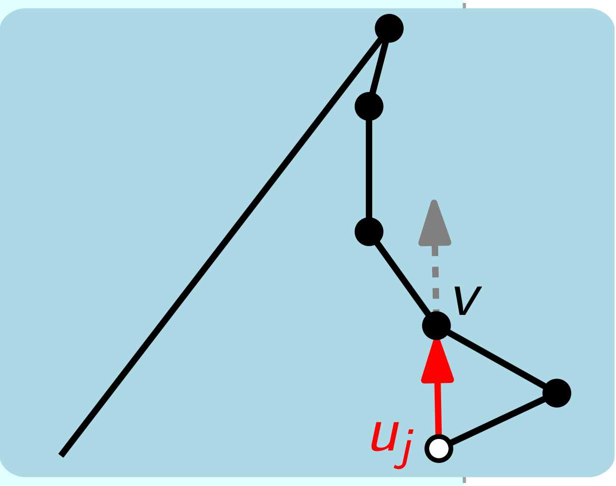

Algorithm

TriangulateMonotonePolygon(Polygon P as circular vertex list)

merge left and right chain $\rightarrow$ sequence $u_1, \dots, u_n$ with $y_1 \geq \cdots \geq y_n$
Stack S; S.push(u_1); S.push(u_2)
for $j \leftarrow 3$ **to** $n-1$ **do**
    if u_j and S.top() lie on different chains **then**
        while not S.empty() **do**
            $v \leftarrow S$.pop()
            if not S.empty() **then** draw diag. (u_j, v)
        S.push(u_{j-1}); S.push(u_j)
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        $v \leftarrow S$.pop()
        while not S.empty() **and** u_j sees S.top() **do**
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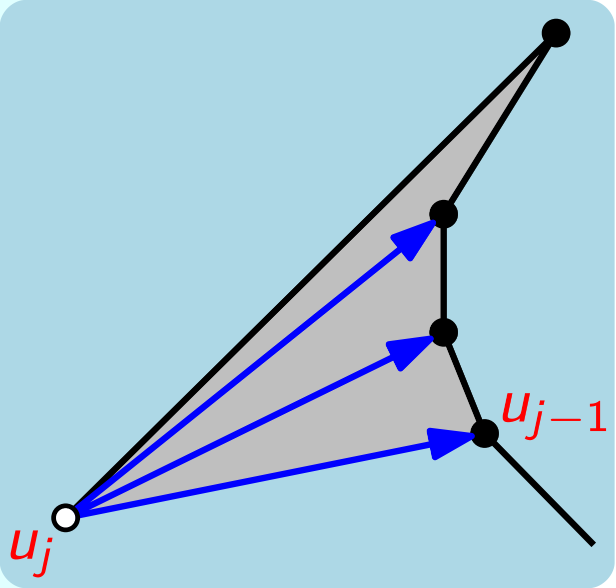

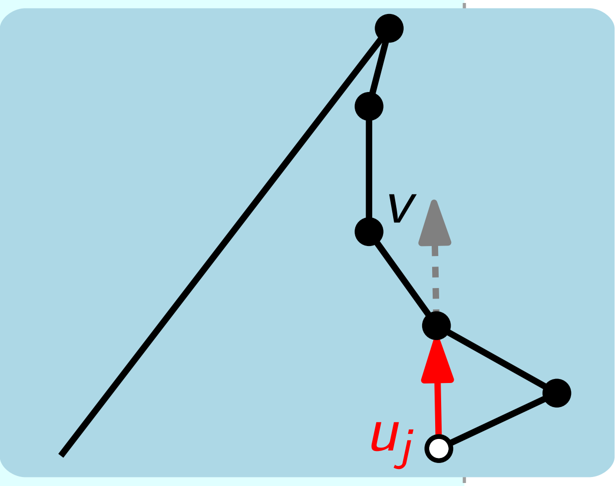

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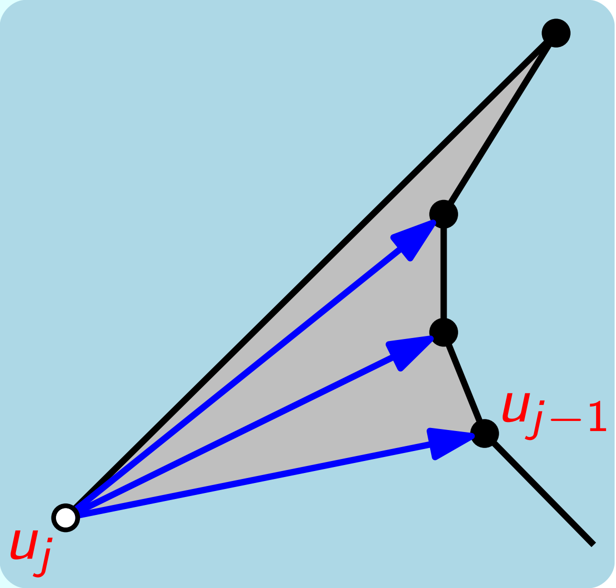

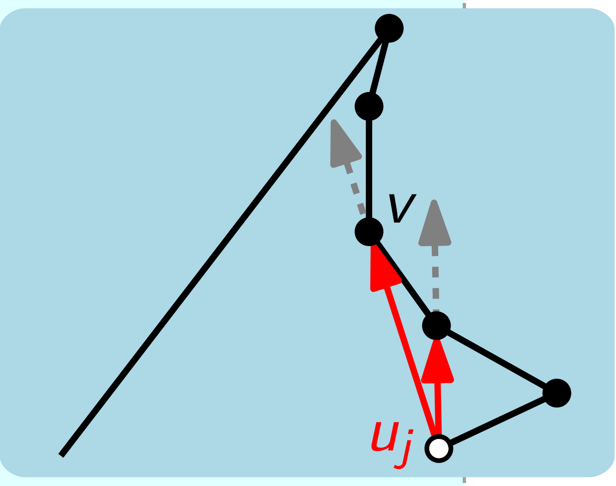

Algorithm

TriangulateMonotonePolygon(Polygon P as circular vertex list)

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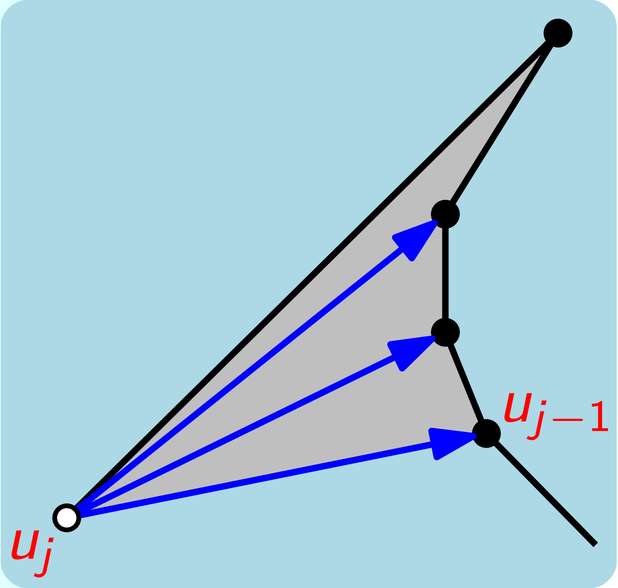

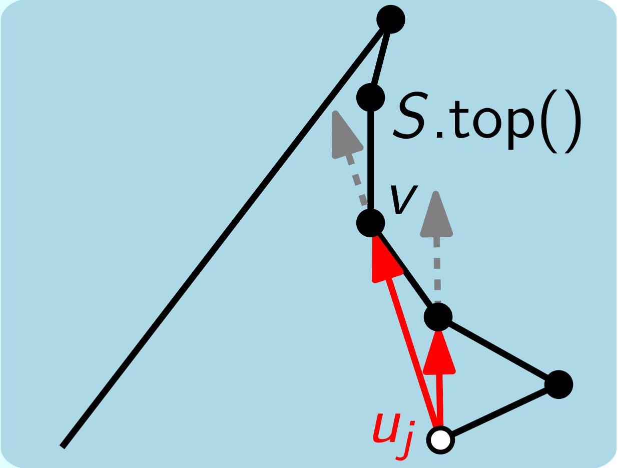

Algorithm

TriangulateMonotonePolygon(Polygon P as circular vertex list)

merge left and right chain → sequence $u_1, \ldots, u_n$ with $y_1 \geq \cdots \geq y_n$

Stack S; S.push(u_1); S.push(u_2)

for $j \leftarrow 3$ **to** $n-1$ **do**

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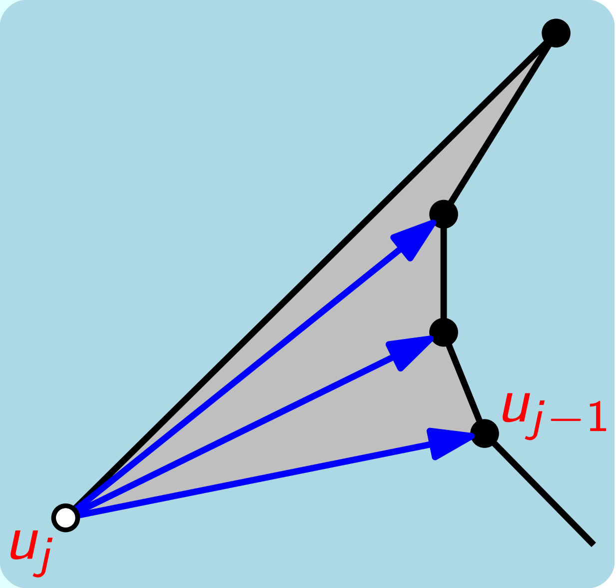
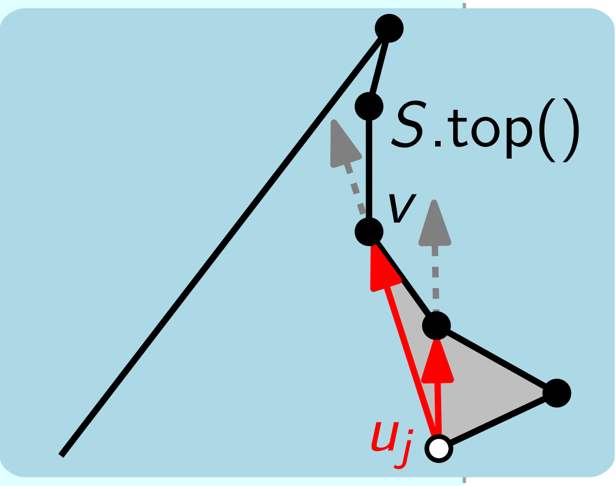
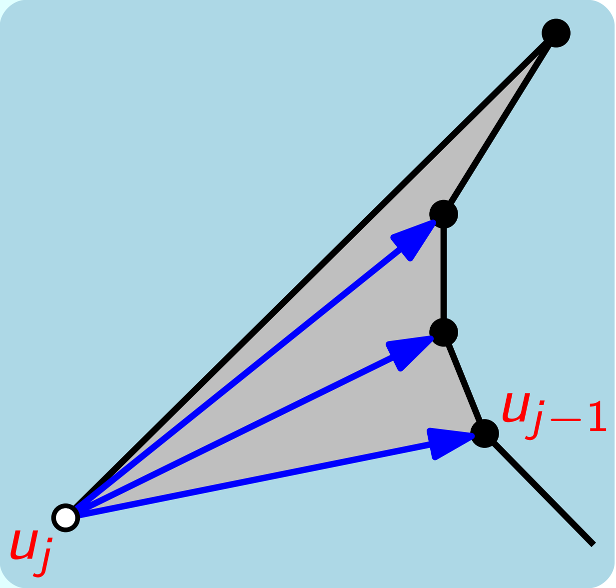


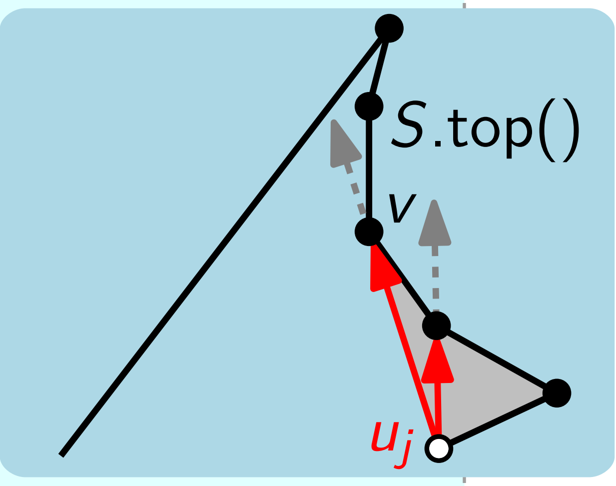


Algorithm

TriangulateMonotonePolygon(Polygon P as circular vertex list)

```
merge left and right chain → sequence u_1, ..., u_n with y_1 ≥ ··· ≥ y_n
Stack S; S.push(u_1); S.push(u_2)
for j ← 3 to n − 1 do
    if u_j and S.top() lie on different chains then
        while not S.empty() do
            v ← S.pop()
            if not S.empty() then draw diag. (u_j, v)
        S.push(u_{j−1}); S.push(u_j)
    else
        v ← S.pop()
        while not S.empty() and u_j sees S.top() do
            v ← S.pop()
            draw diagonal (u_j, v)
        S.push(v); S.push(u_j)
```





Algorithm

```
TriangulateMonotonePolygon(Polygon P as circular vertex list)
merge left and right chain → sequence u_1, ..., u_n with y_1 ≥ ··· ≥ y_n
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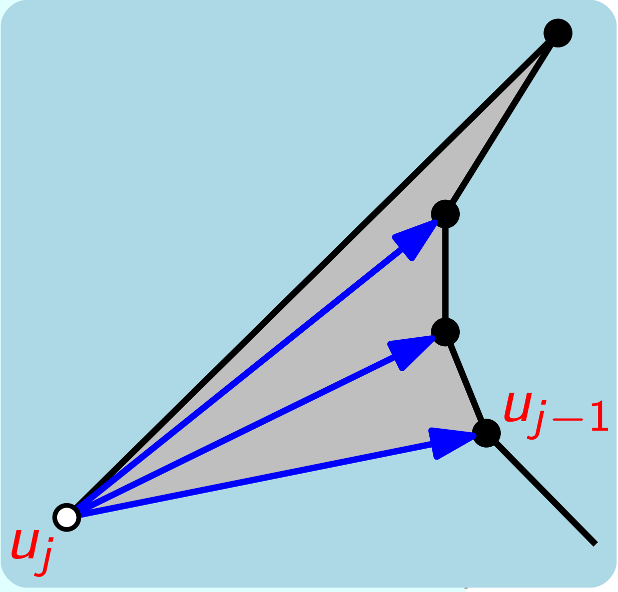
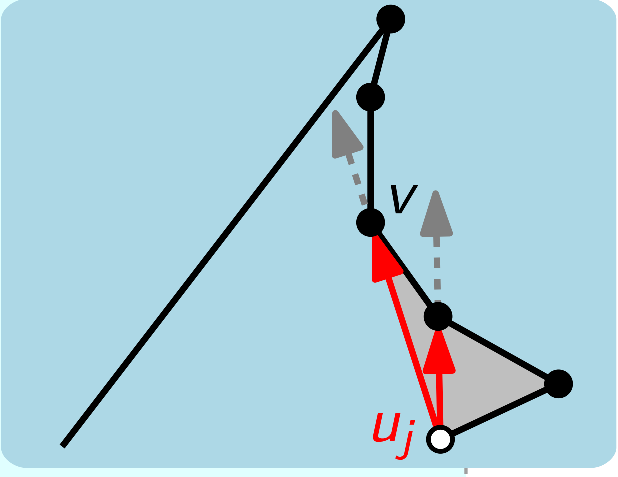
            v ← S.pop()
            draw diagonal (u_j, v)
        S.push(v); S.push(u_j)
```





Algorithm

TriangulateMonotonePolygon(Polygon P as circular vertex list)

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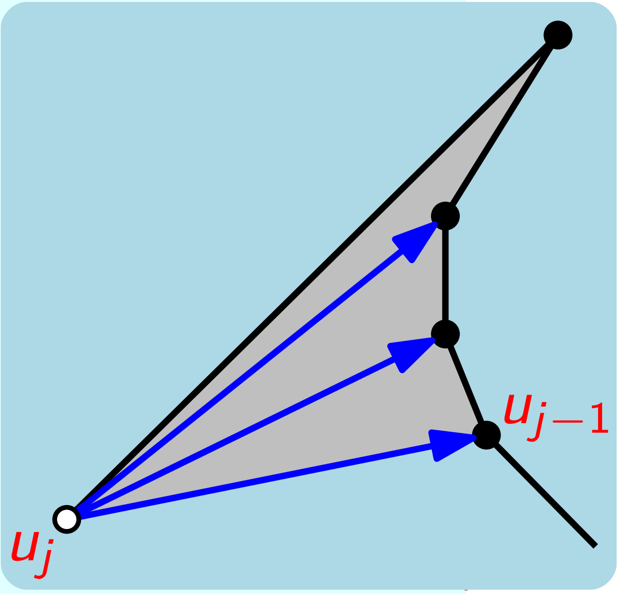
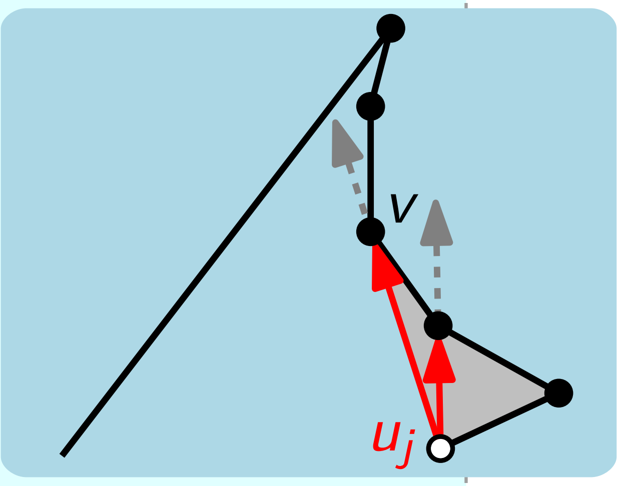
        while not S.empty() **and** u_j sees S.top() **do**
            $v \leftarrow S$.pop()
            draw diagonal (u_j, v)
        S.push(v); S.push(u_j)
draw diagonals from u_n to all vtc on S except first and last one





Algorithm

Running time?

TriangulateMonotonePolygon(Polygon P as circular vertex list)

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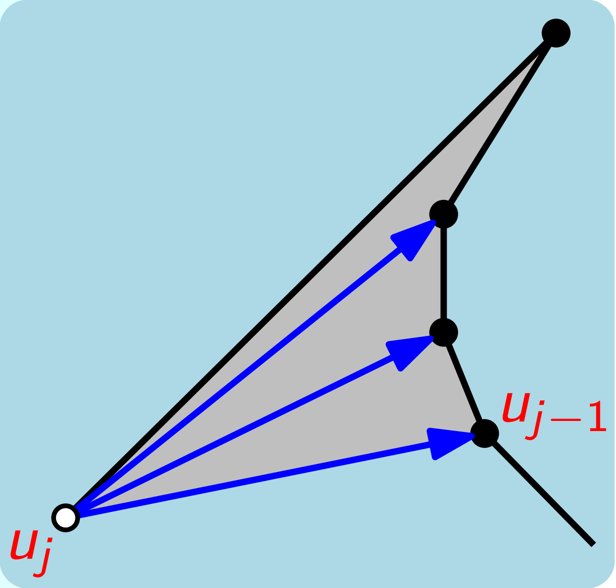

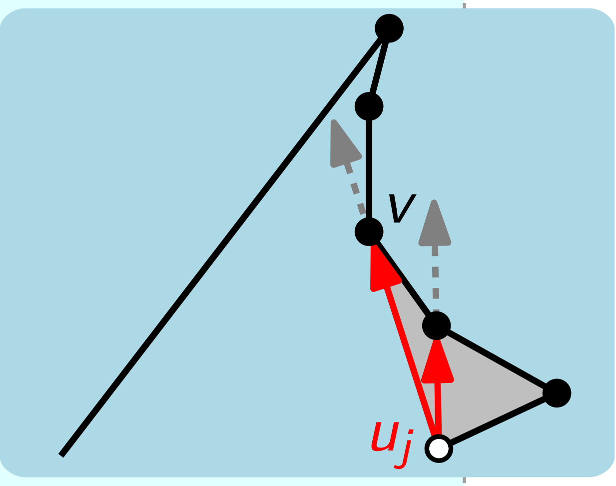

Algorithm

Running time? $\Theta(n)$

TriangulateMonotonePolygon(Polygon P as circular vertex list)

merge left and right chain $\rightarrow$ sequence $u_1, \ldots, u_n$ with $y_1 \geq \cdots \geq y_n$

Stack S; S.push(u_1); S.push(u_2)

for $j \leftarrow 3$ **to** $n - 1$ **do**

  if u_j and S.top() lie on different chains **then**

    while not S.empty() **do**

      $v \leftarrow S$.pop()

      if not S.empty() **then** draw diag. (u_j, v)

    S.push(u_{j-1}); S.push(u_j)

  else

    $v \leftarrow S$.pop()

    while not S.empty() **and** u_j sees S.top() **do**

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      draw diagonal (u_j, v)

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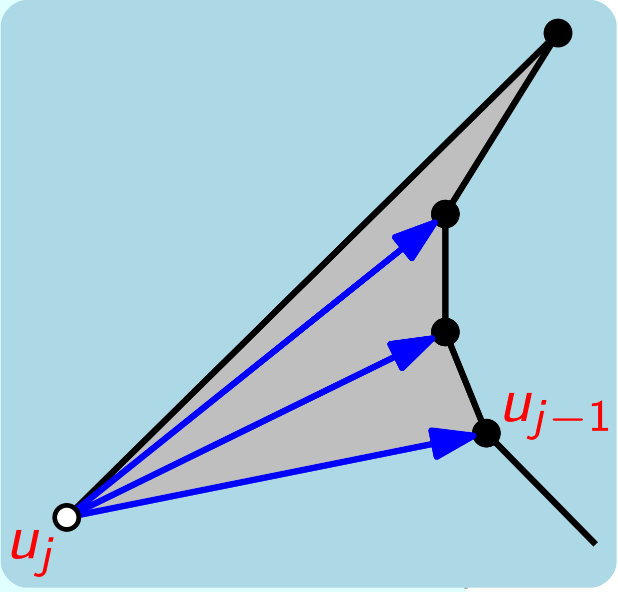
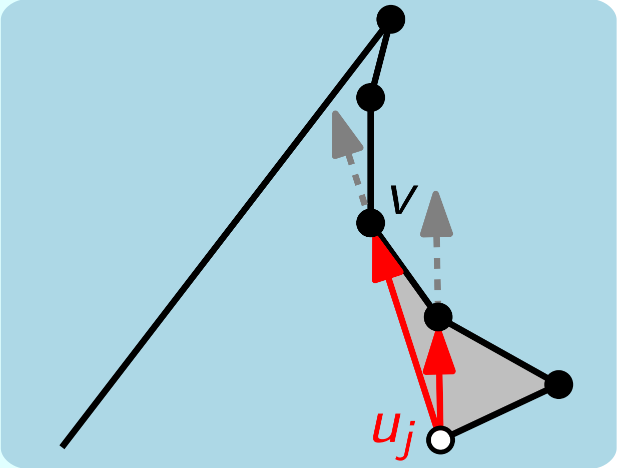
draw diagonals from u_n to all vtc on S except first and last one

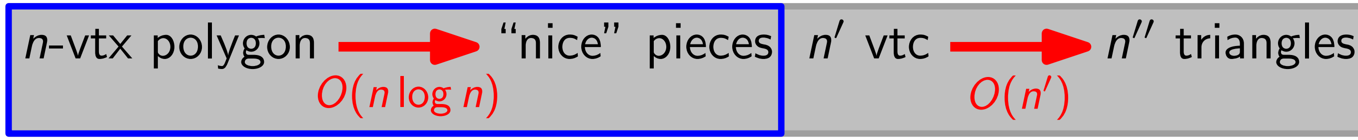




Summary

n-vtx polygon → (with $O(n \log n)$) “nice” pieces, n' vtc → (with $O(n')$) n'' triangles

Summary



Lemma. A simple polygon with n vertices can be subdivided into y-monotone polygons in $O(n \log n)$ time.

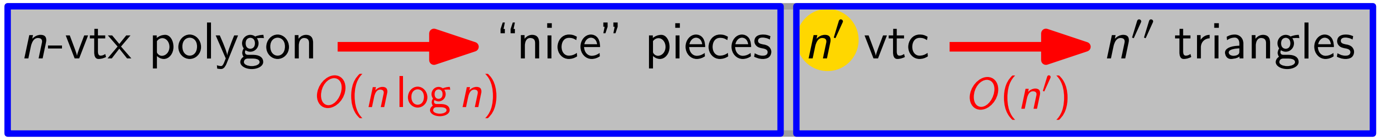
Summary

n-vtx polygon → "nice" pieces ($O(n \log n)$) | n' vtc → n'' triangles ($O(n')$)

Lemma. (old) A simple polygon with n vertices can be subdivided into y-monotone polygons in $O(n \log n)$ time.

Lemma. (new) A y-monotone polygon with n vertices can be triangulated in $O(n)$ time.

Summary



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Lemma. (new) A y-monotone polygon with n vertices can be triangulated in $O(n)$ time.

Lemma. (home-work) Subdividing a simple polygon with n vertices by drawing d (pairwise non-crossing) diagonals yields $d + 1$ simple polygons of total complexity $O(n)$.

Summary

n-vtx polygon → "nice" pieces ($O(n \log n)$) | n' vtc → n'' triangles ($O(n')$)

Lemma. *old*
A simple polygon with n vertices can be subdivided into y-monotone polygons in $O(n \log n)$ time.

Lemma. *new*
A y-monotone polygon with n vertices can be triangulated in $O(n)$ time.

Lemma. *home-work*
Subdividing a simple polygon with n vertices by drawing d (pairwise non-crossing) diagonals yields $d + 1$ simple polygons of total complexity $O(n)$.

Theorem. A simple polygon with n vertices can be triangulated in $O(n \log n)$ time.

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Is this it?

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Is this it? Tarjan & van Wyk [1988]:

Summary

n-vtx polygon → "nice" pieces: $O(n \log n)$ | n' vtc → n'' triangles: $O(n')$

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Tarjan & van Wyk [1988]: $O(n \log \log n)$

Clarkson, Tarjan, van Wyk [1989]:

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Tarjan & van Wyk [1988]: $O(n \log \log n)$

Clarkson, Tarjan, van Wyk [1989]: $O(n \log^* n)$

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Chazelle [1991]:

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Clarkson, Tarjan, van Wyk [1989]:	$O(n \log^* n)$
Chazelle [1991]:	$O(n)$

Summary

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Is this it?

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Clarkson, Tarjan, van Wyk [1989]: $O(n \log^* n)$

Chazelle [1991]: $O(n)$

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Is this it?

Tarjan & van Wyk [1988]:	$O(n \log \log n)$	Kirkpatrick, Klawe, Tarjan [1992]
Clarkson, Tarjan, van Wyk [1989]:	$O(n \log^* n)$	Seidel [1991]: *randomized*
Chazelle [1991]:	$O(n)$	