Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems

Institute for Informatics

Computational Geometry

Winter term 2016/17

Line-Segment Intersection

or

Map Overlay

Lecture #2

Joachim Spoerhase *Chair for Informatics I*

Chapter 2

Line-Segment Intersection

or

Map Overlay


Dawson
Whitehorse
Echo Bay
Ft. Simpson
Yellowknife
Uranium City
Prince Rupert
Dawson Creek
Lynn Lake
Edmonton
Vancouver
Victoria
Saskatoon
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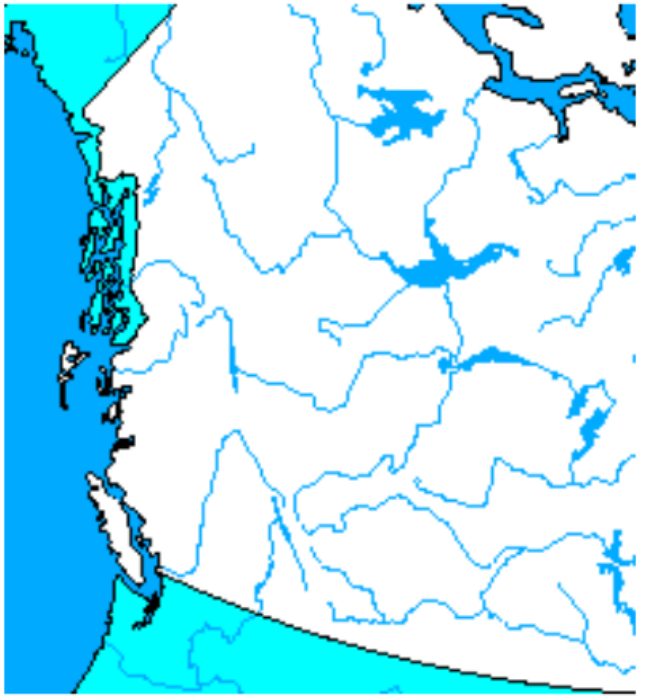

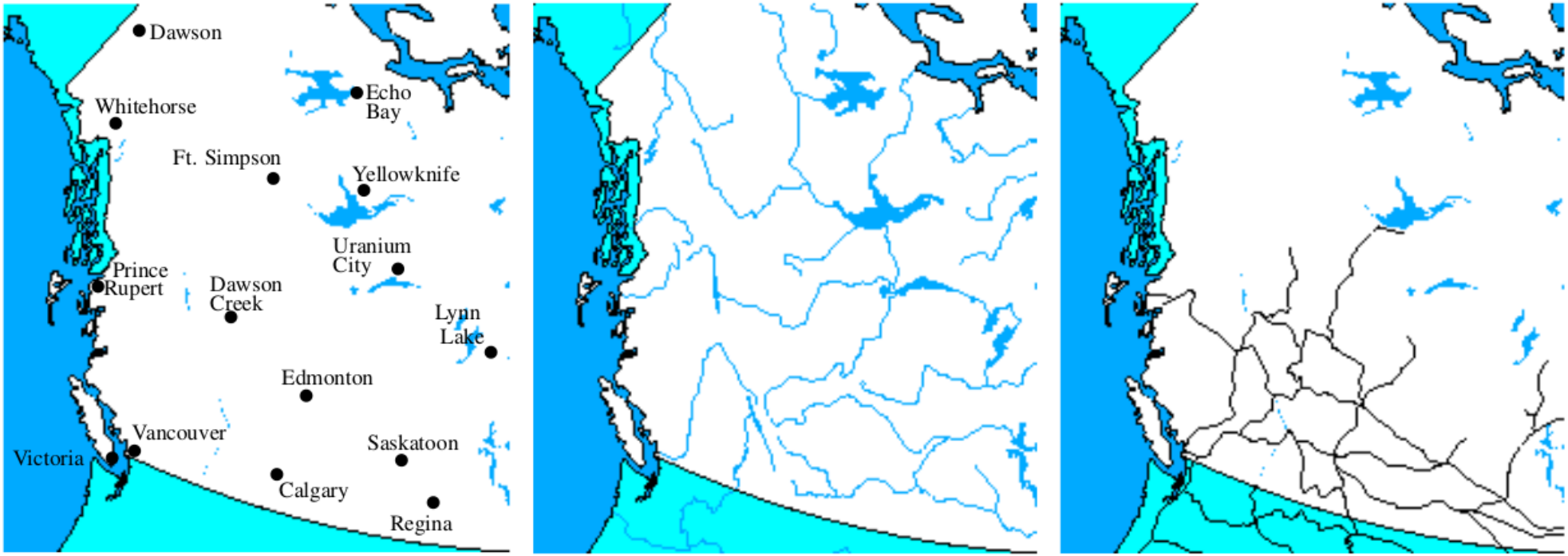
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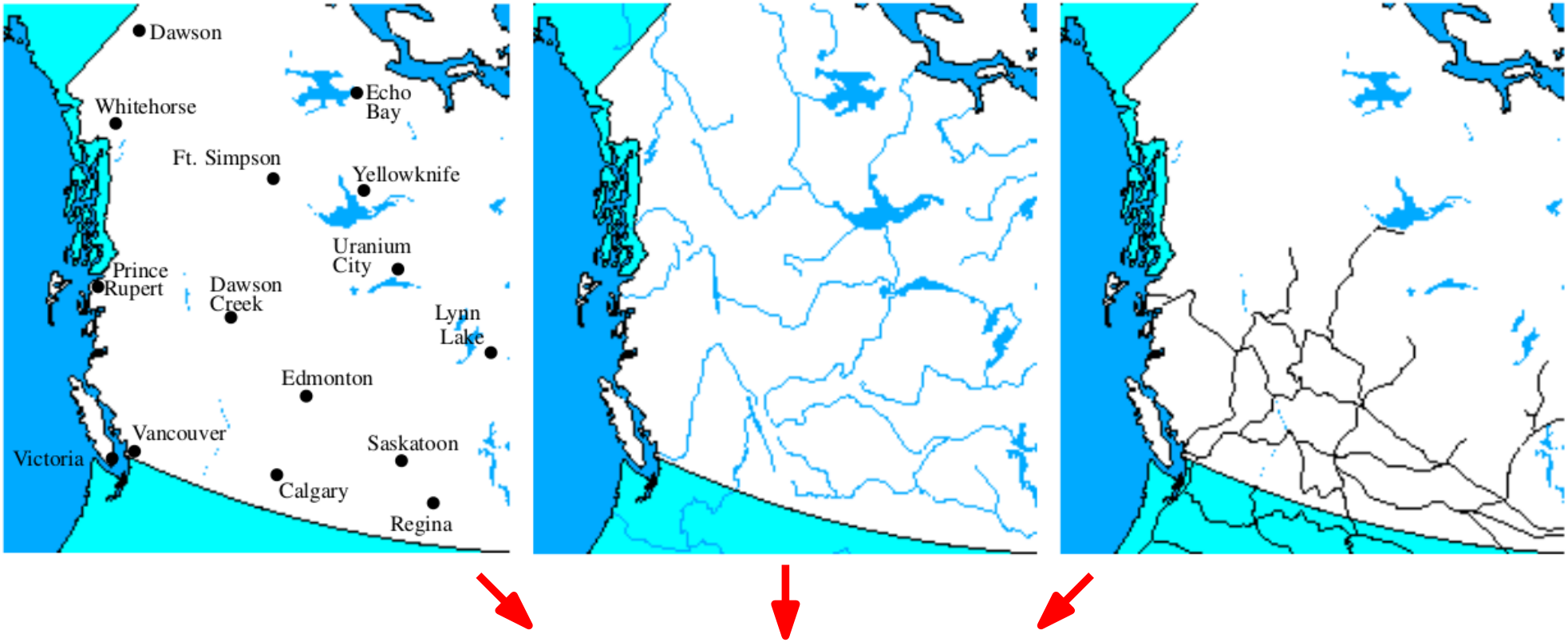
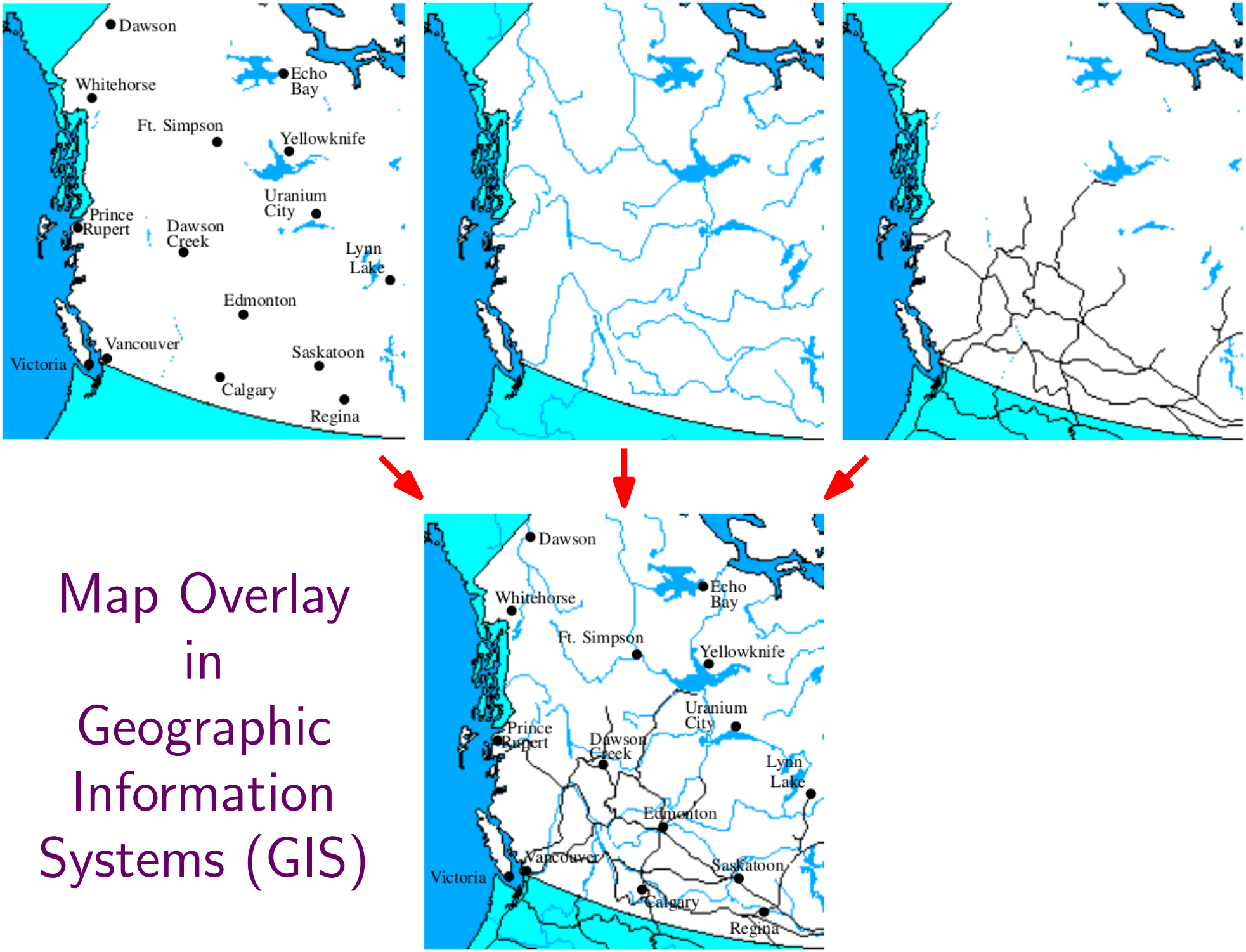


Map Overlay in Geographic Information Systems (GIS)



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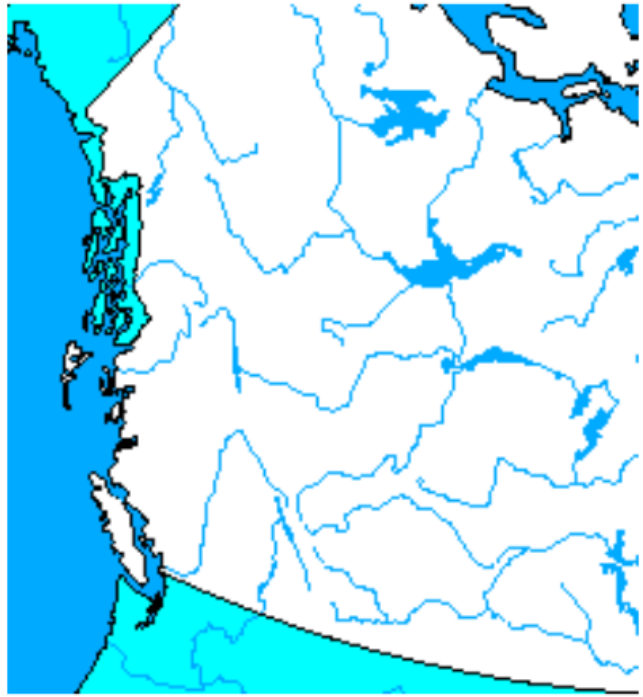

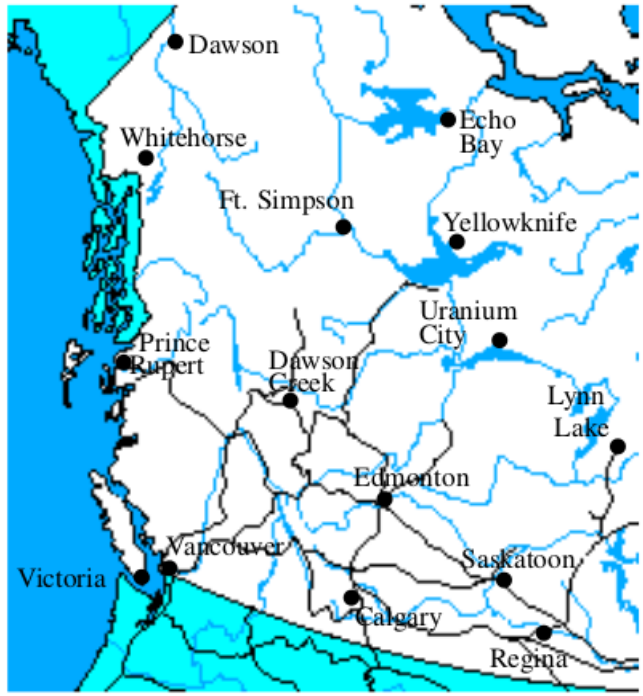

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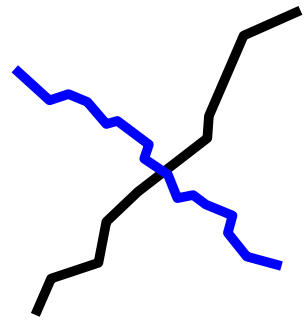

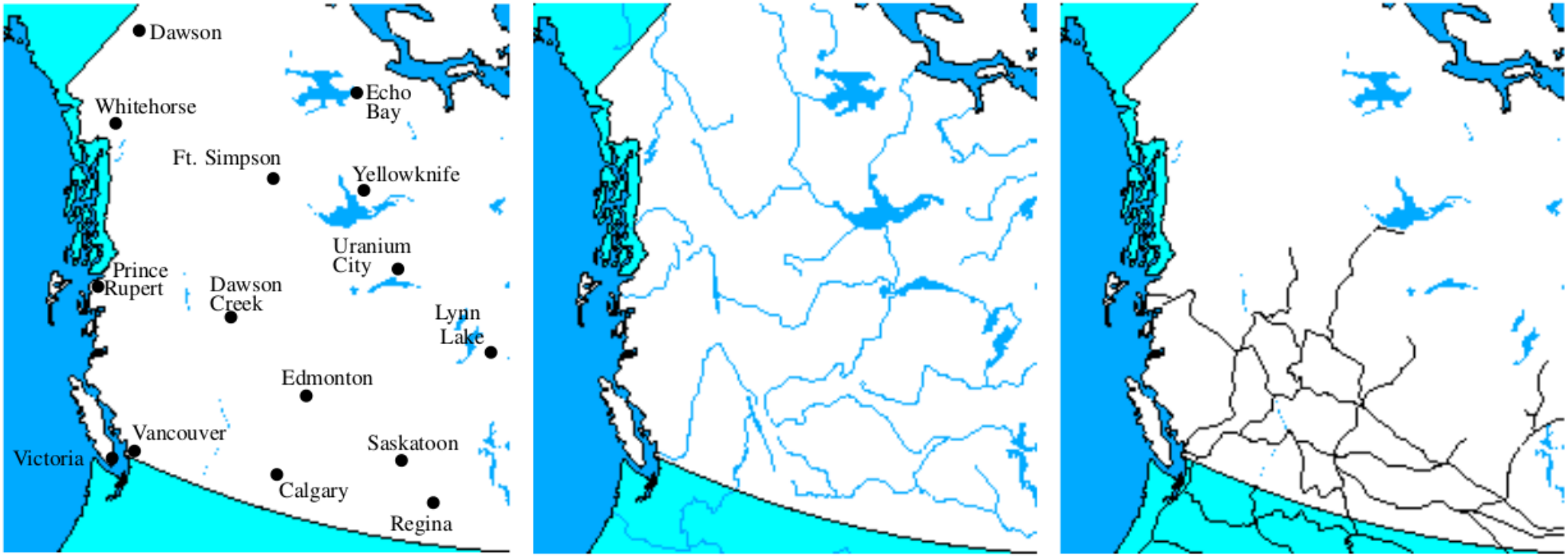

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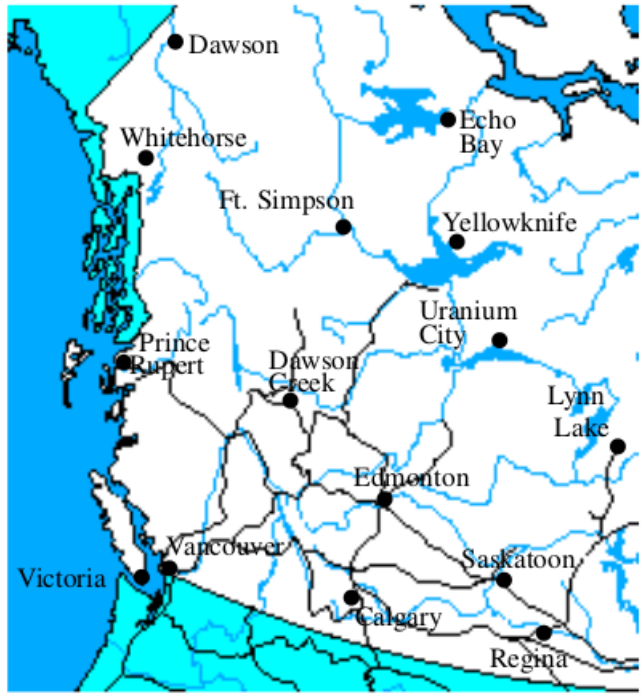

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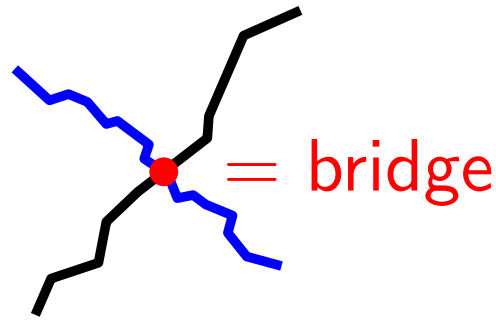

Line-Segment Intersection

Definition:

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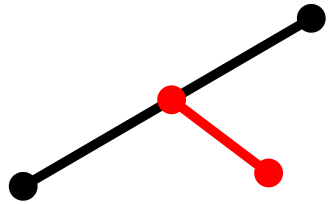
Definition: Is  an intersection?

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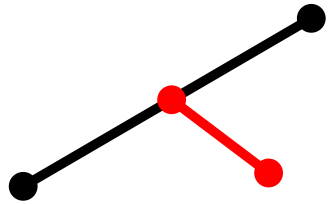
Definition: Is  an intersection?

Answer: Depends. . .

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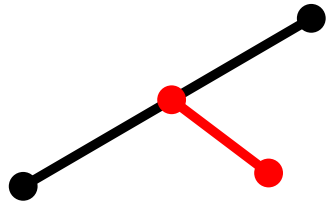
Definition: Is  an intersection?

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Problem: Given a set S of n *closed* non-overlapping line segments in the plane, compute...

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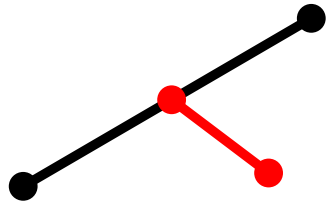
Definition: Is  an intersection?

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- all points where at least two segments intersect and
- for each such point report all segments that contain it.

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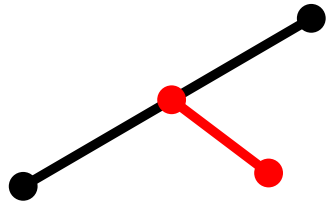
Definition: Is  an intersection?

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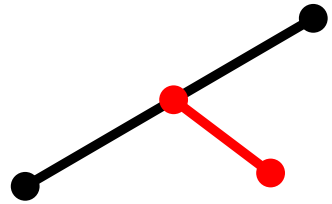
Definition: Is  an intersection?

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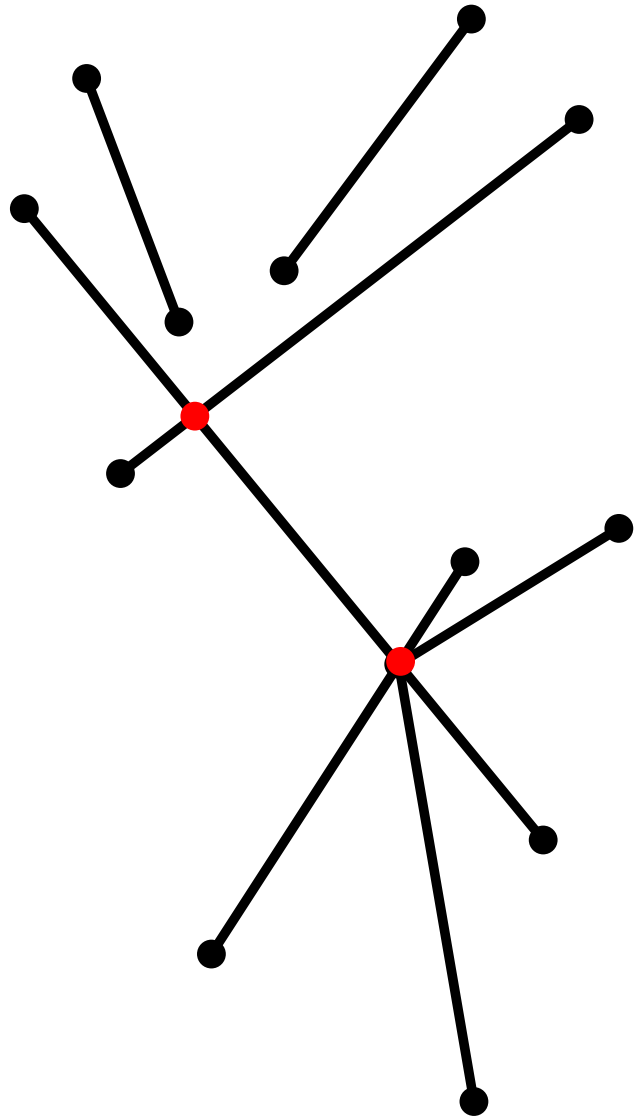
- all points where at least two segments intersect and
- for each such point report all segments that contain it.

Task: Discuss with your neighbor: how would *you* do it?

Example



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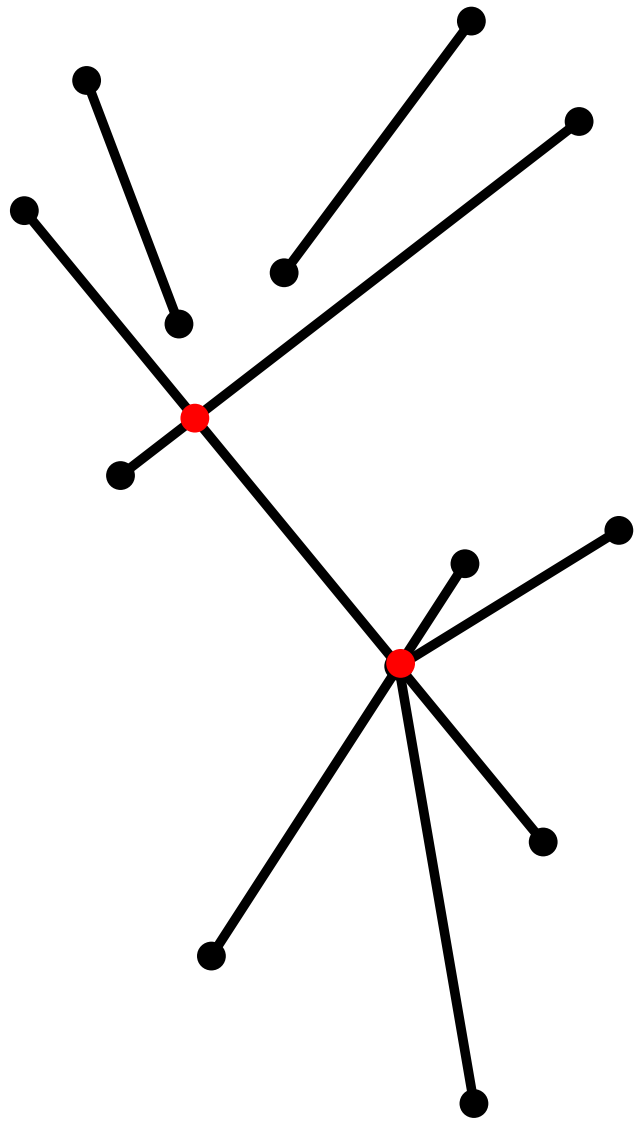

Brute Force?

$O(n^2)$ ... can we do better?

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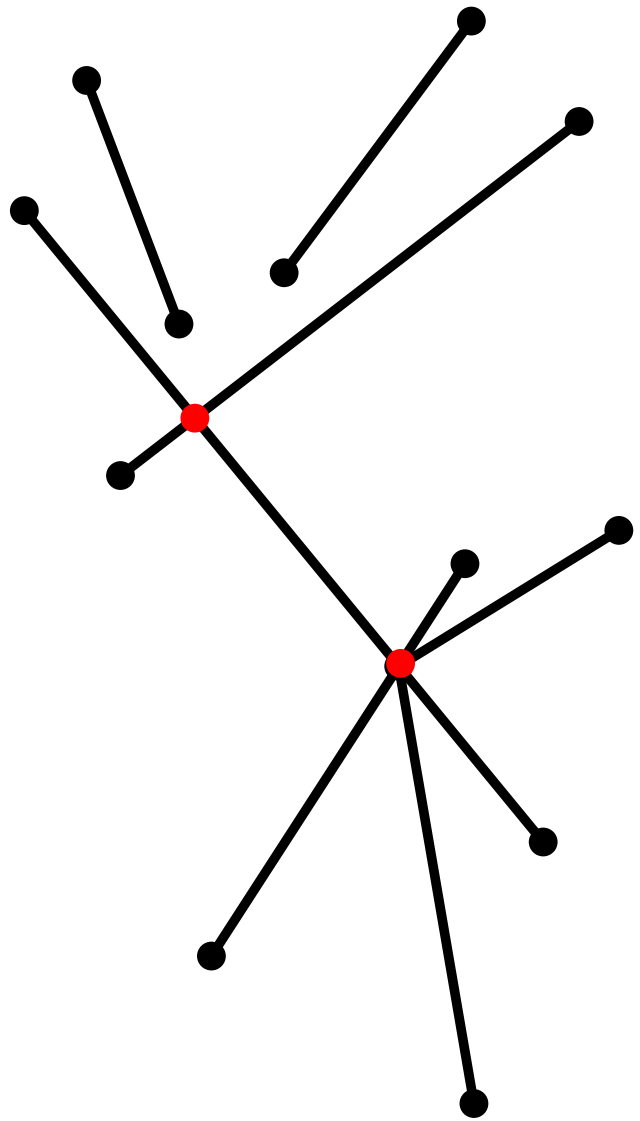
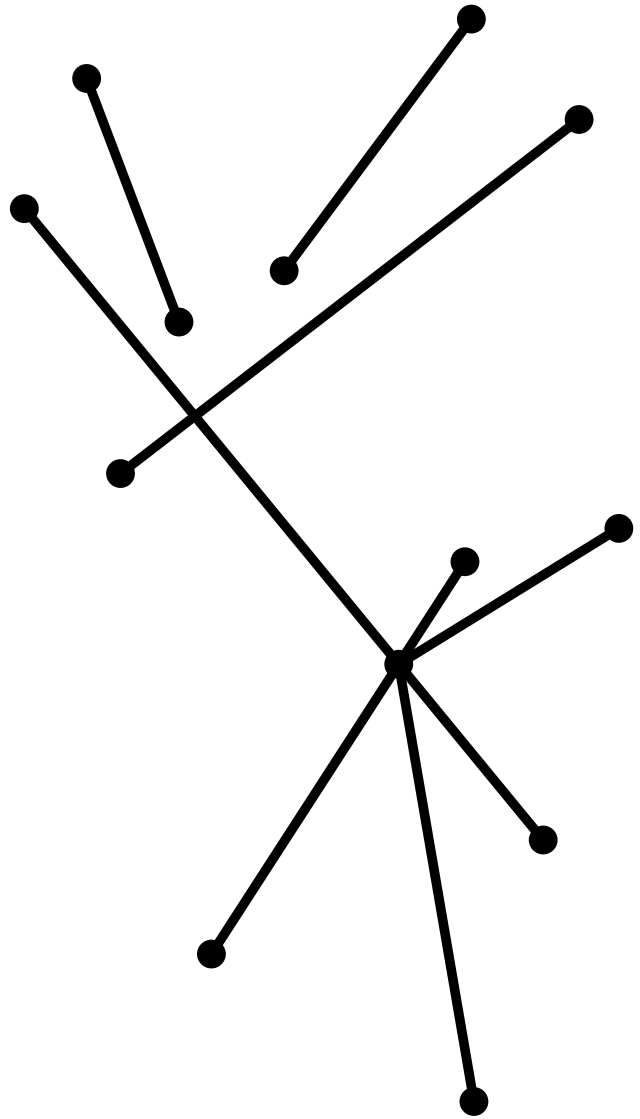


Brute Force?

$O(n^2)$ ... can we do better?

Idea: Process Segments top-to-bottom using a "sweep-line"

Sweep-Line Algorithm



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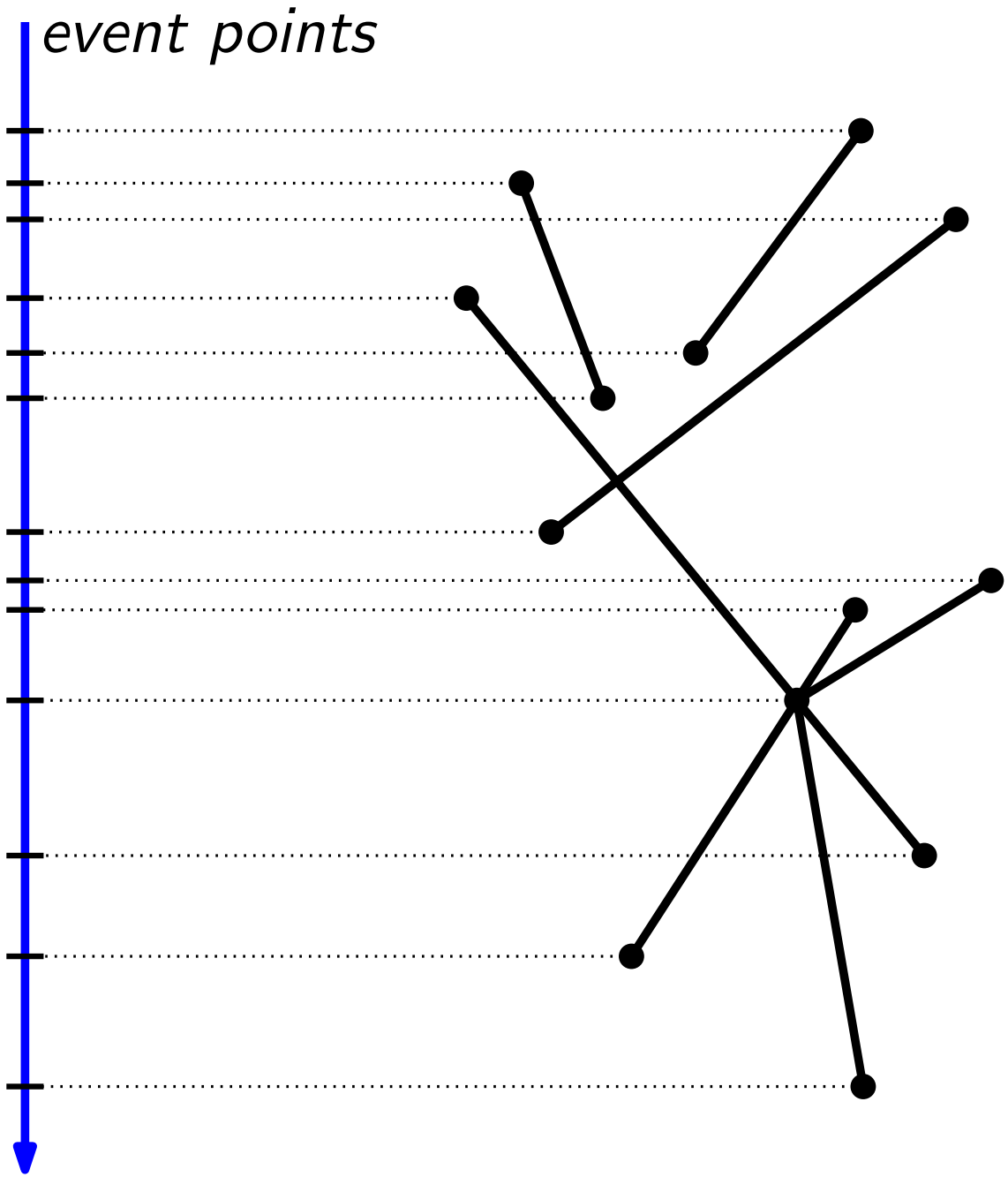
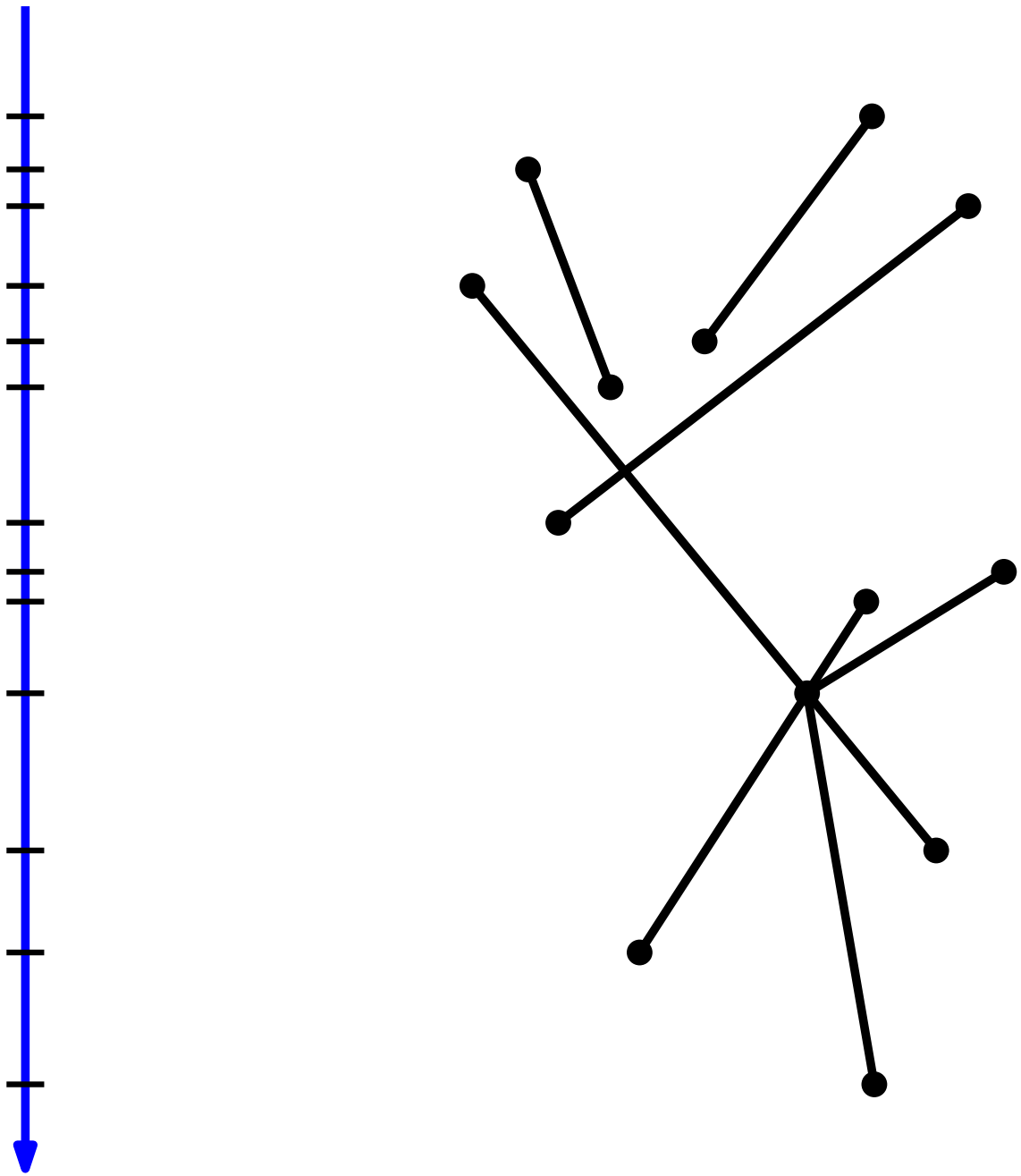


Which active segments should be compared?

Sweep-Line Algorithm



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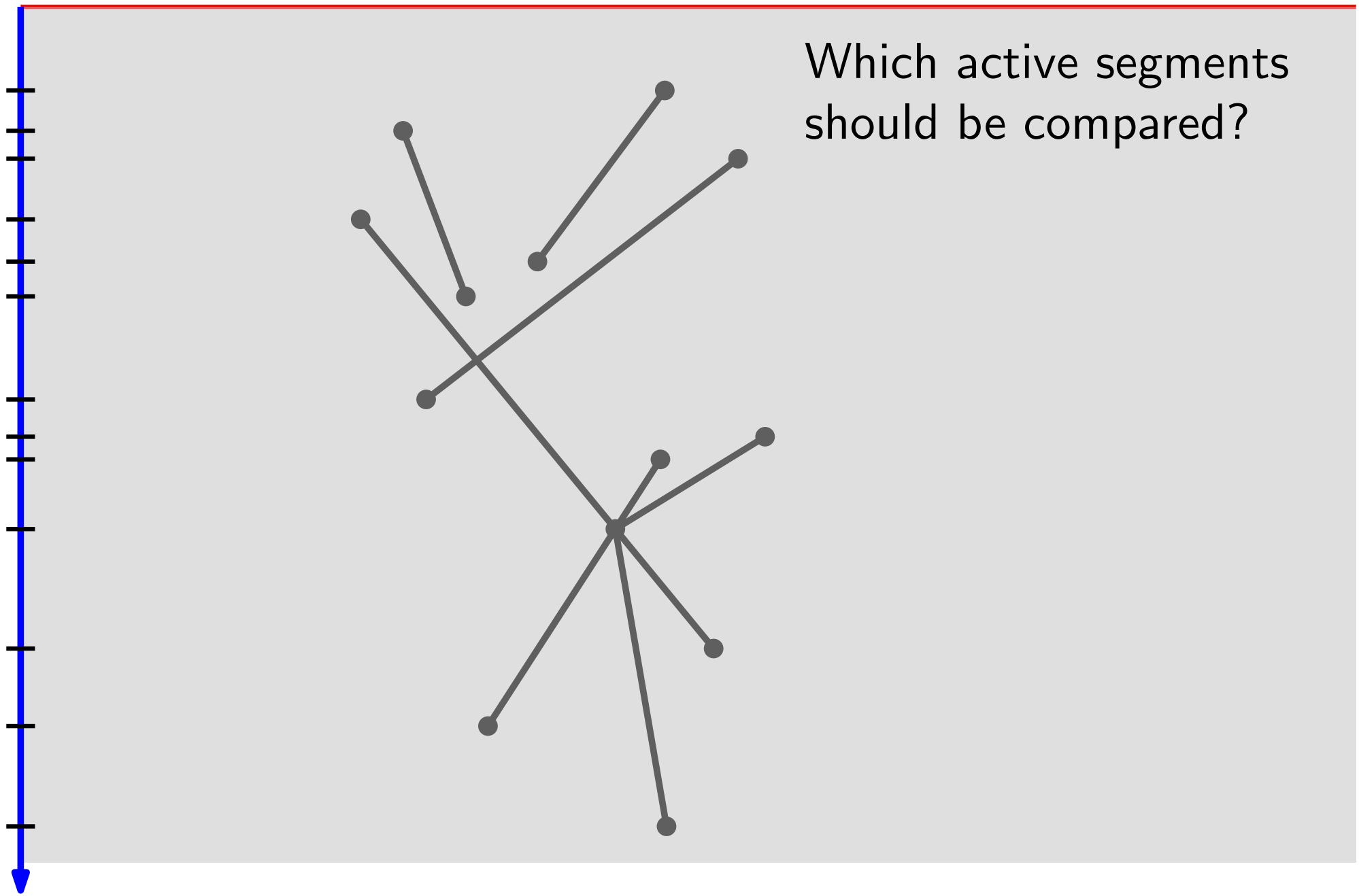

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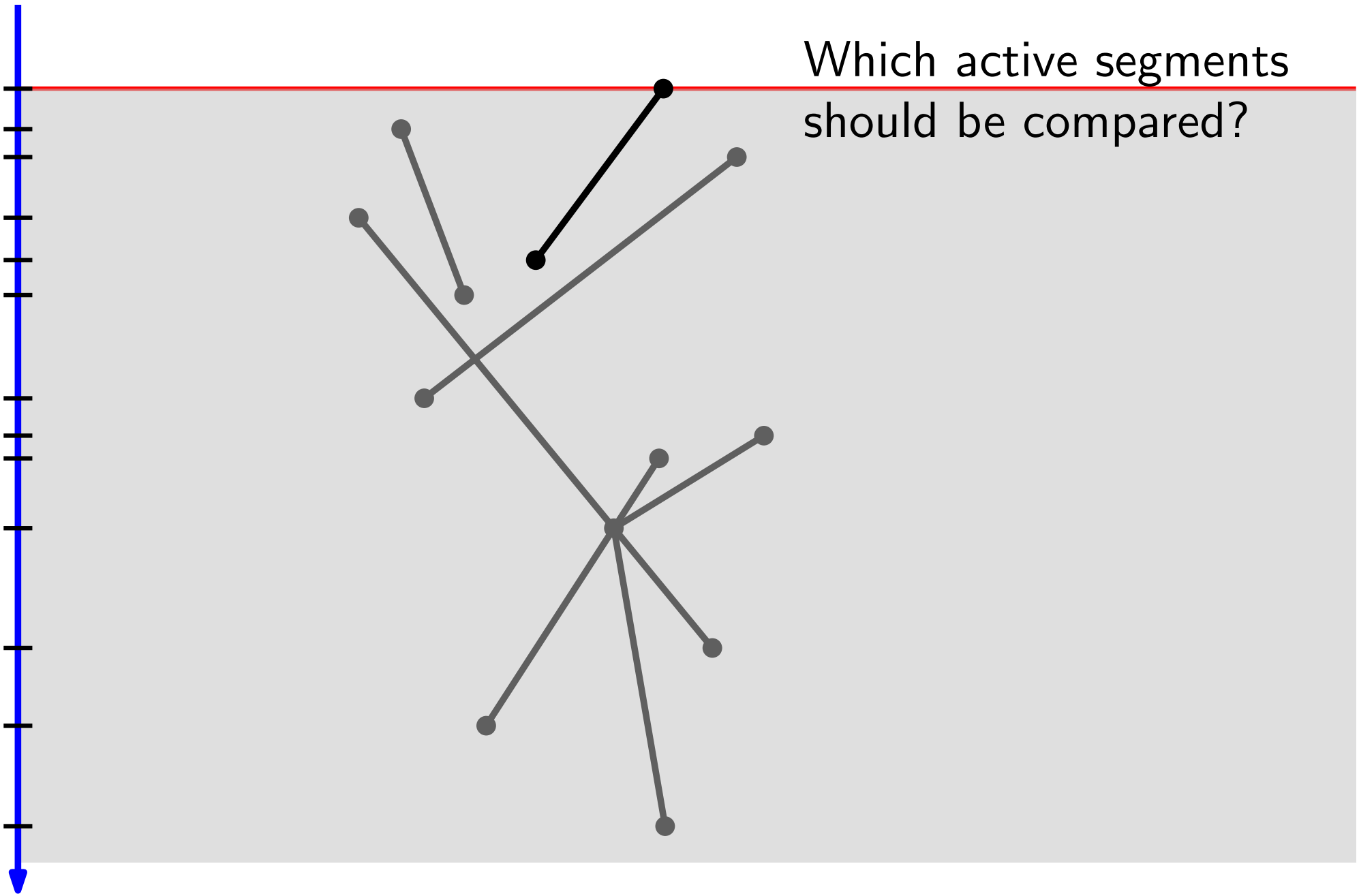

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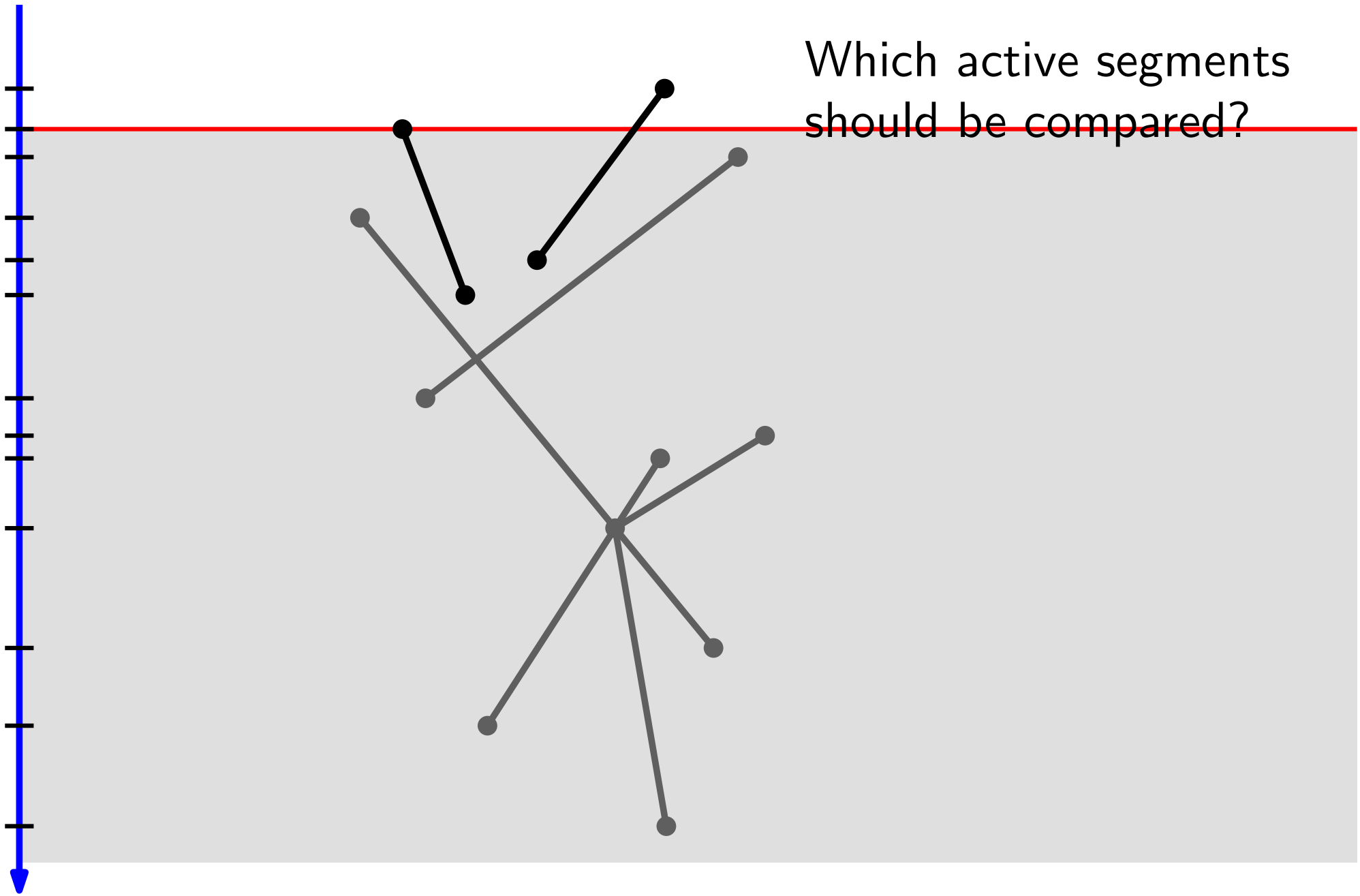

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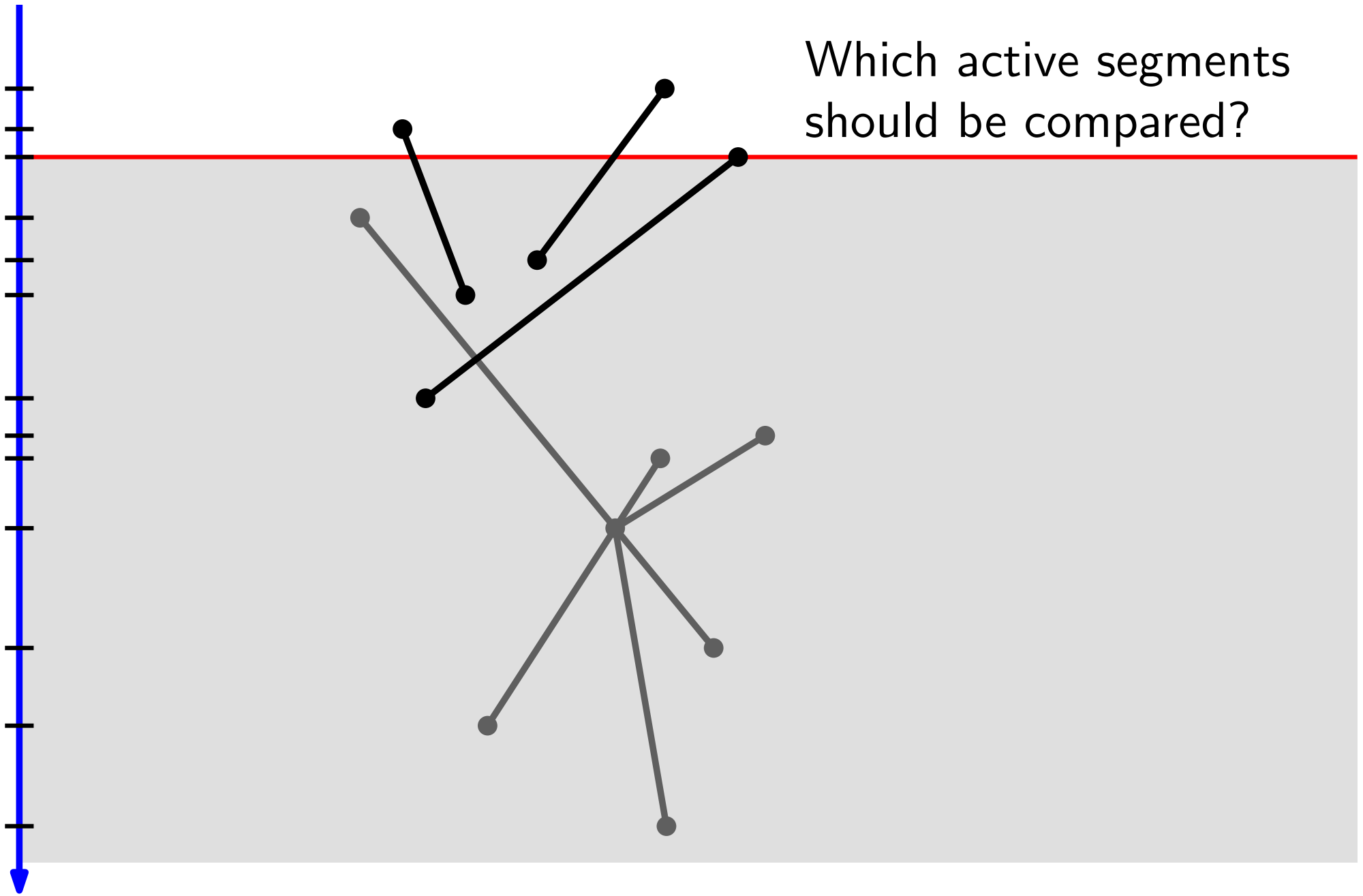

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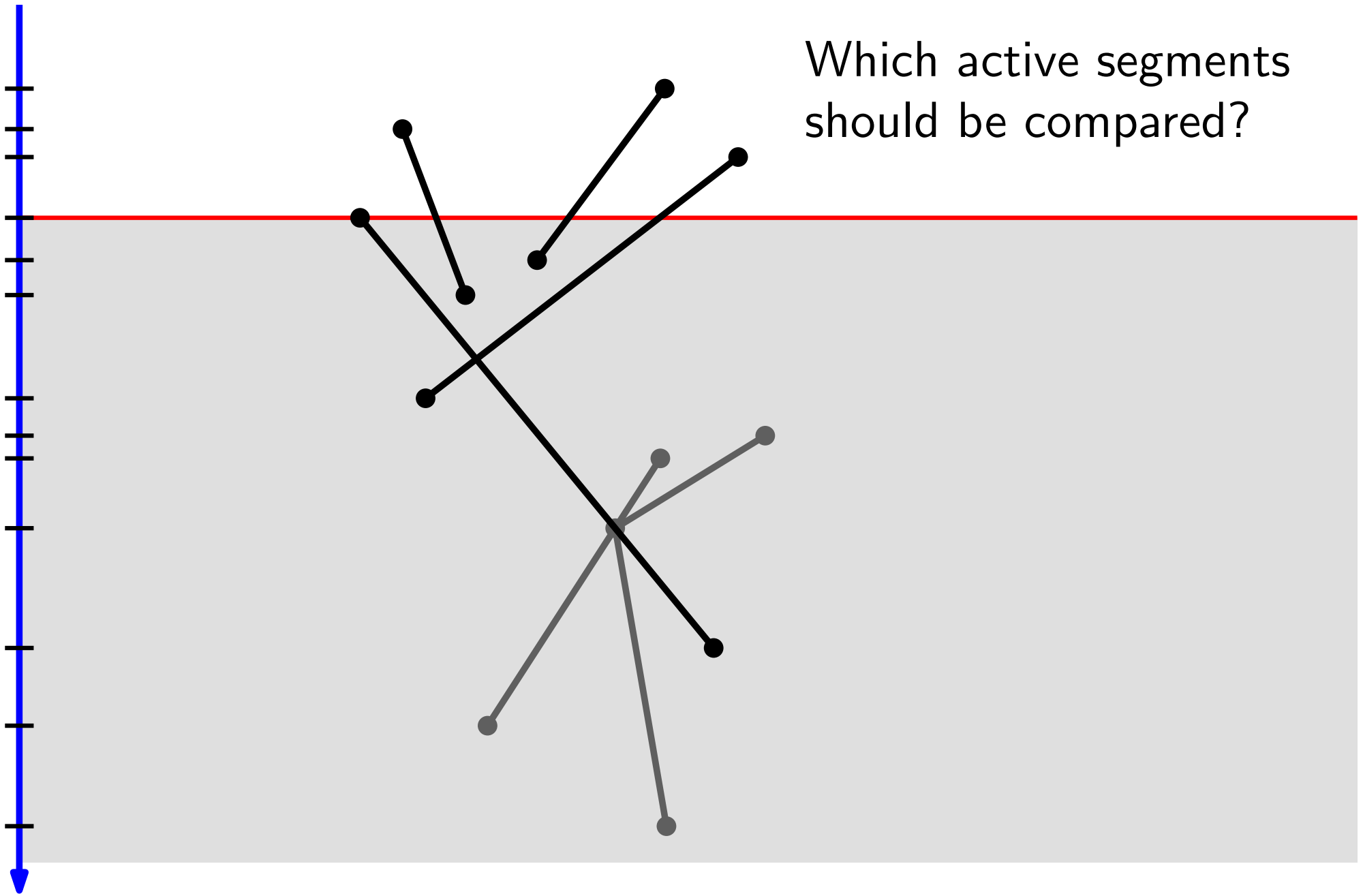

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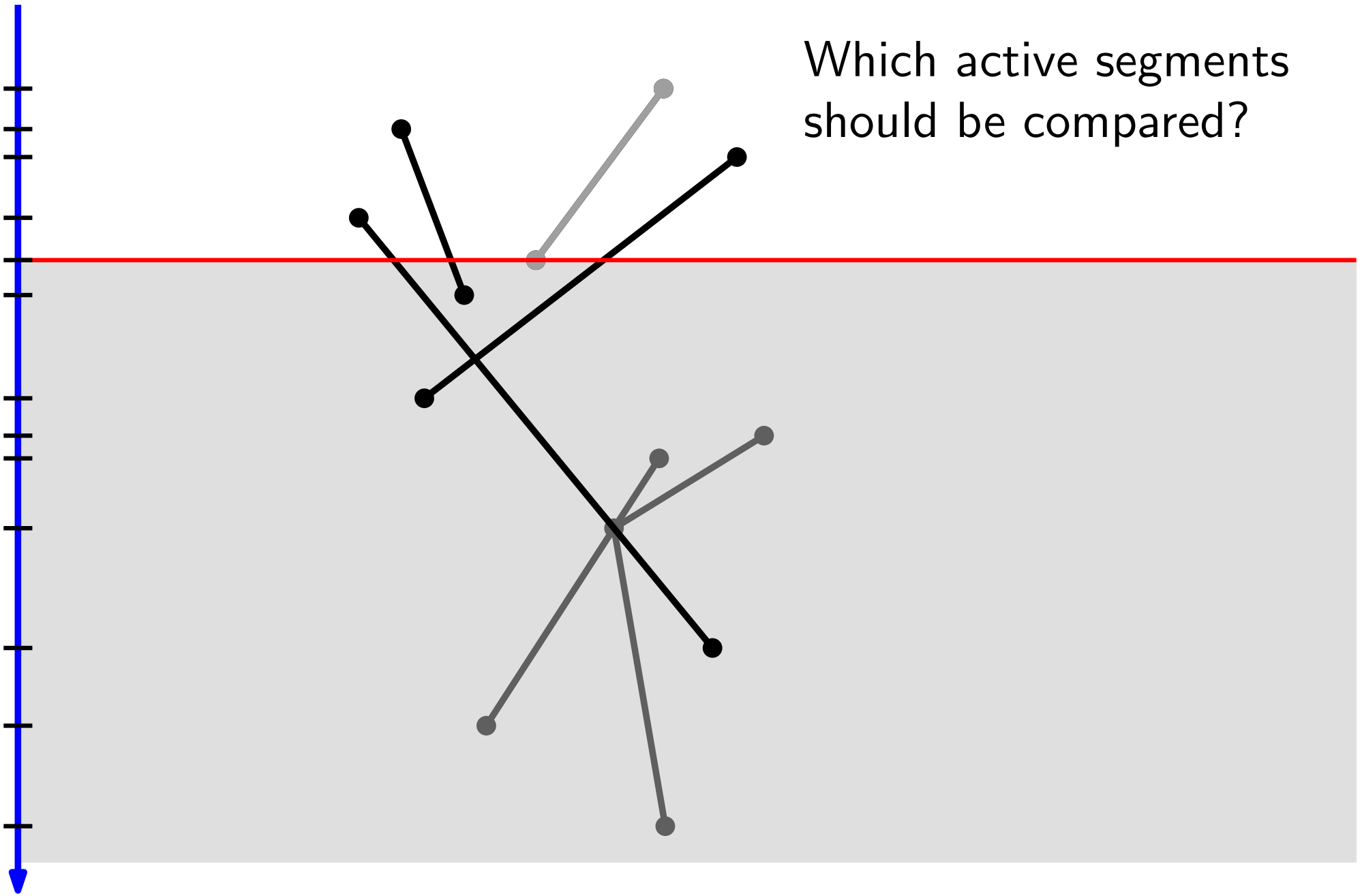

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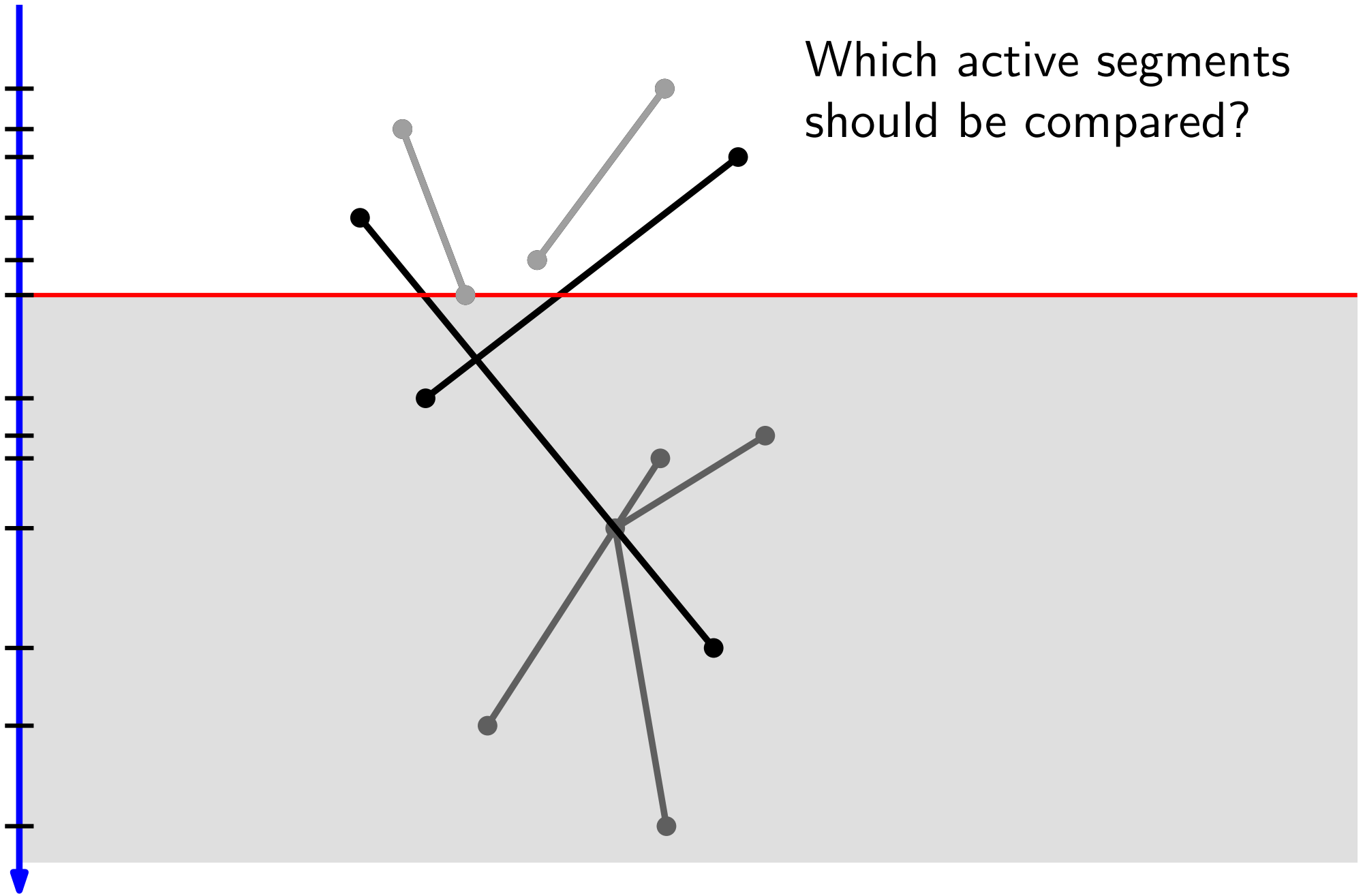

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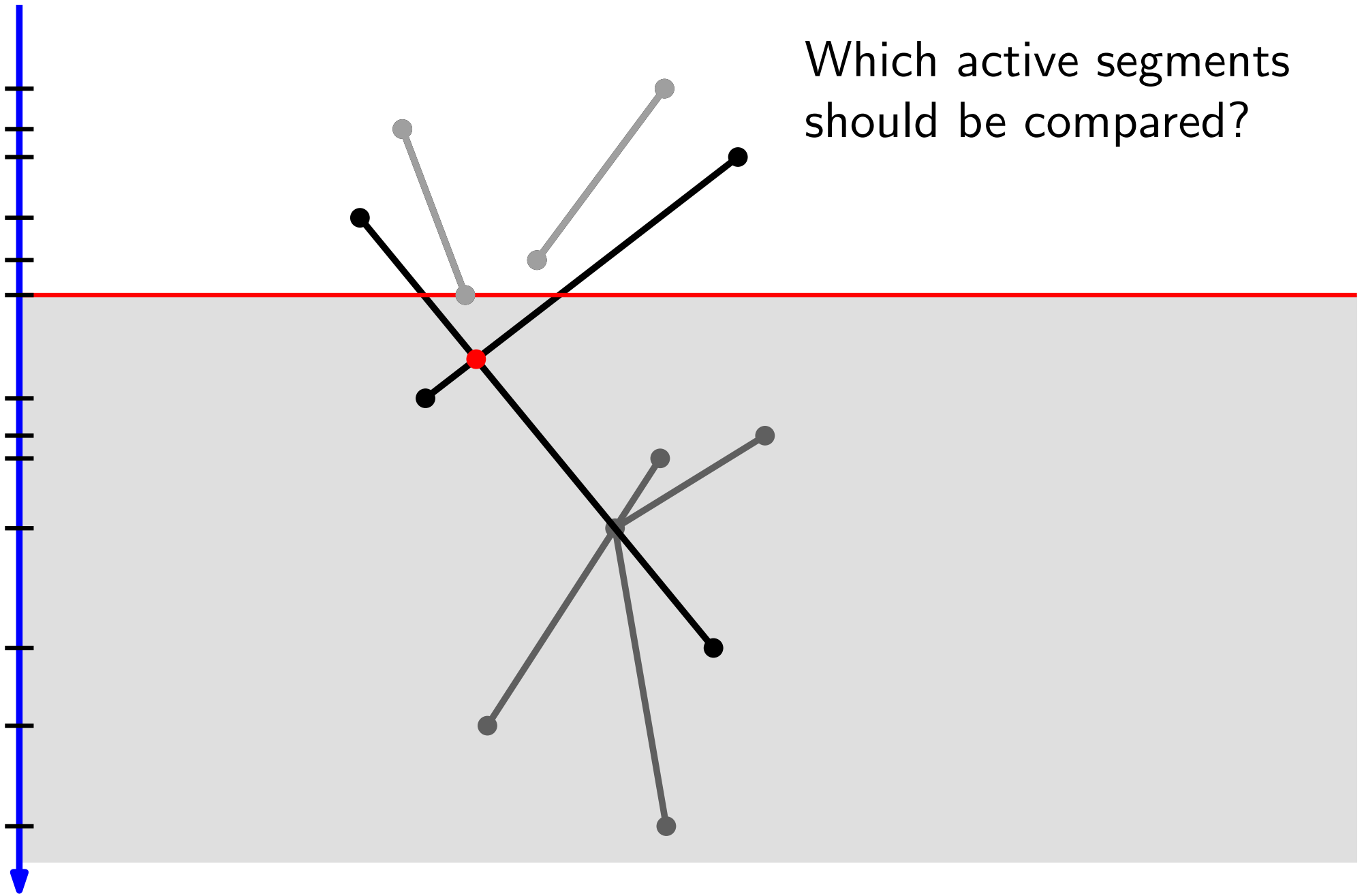

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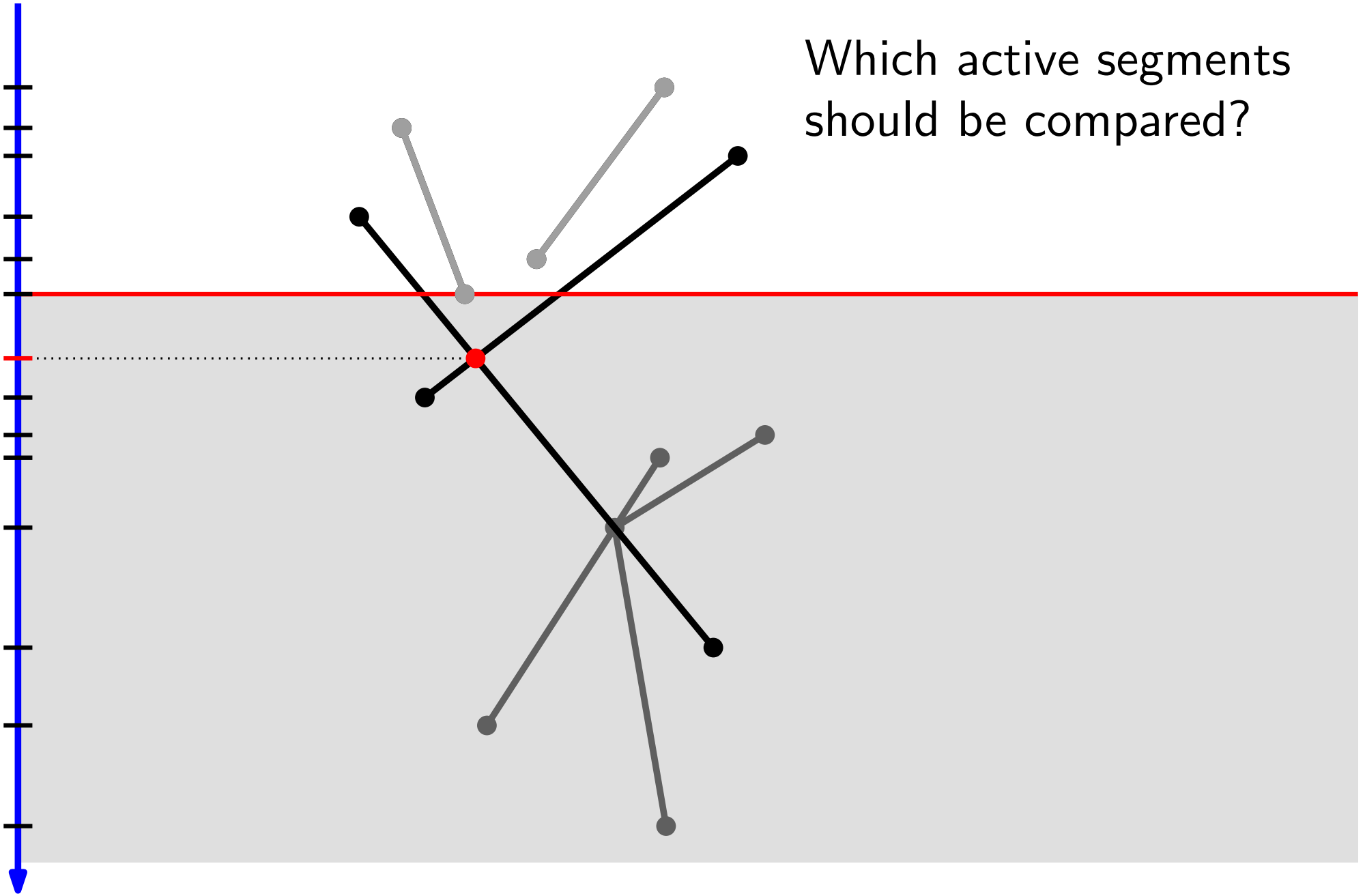

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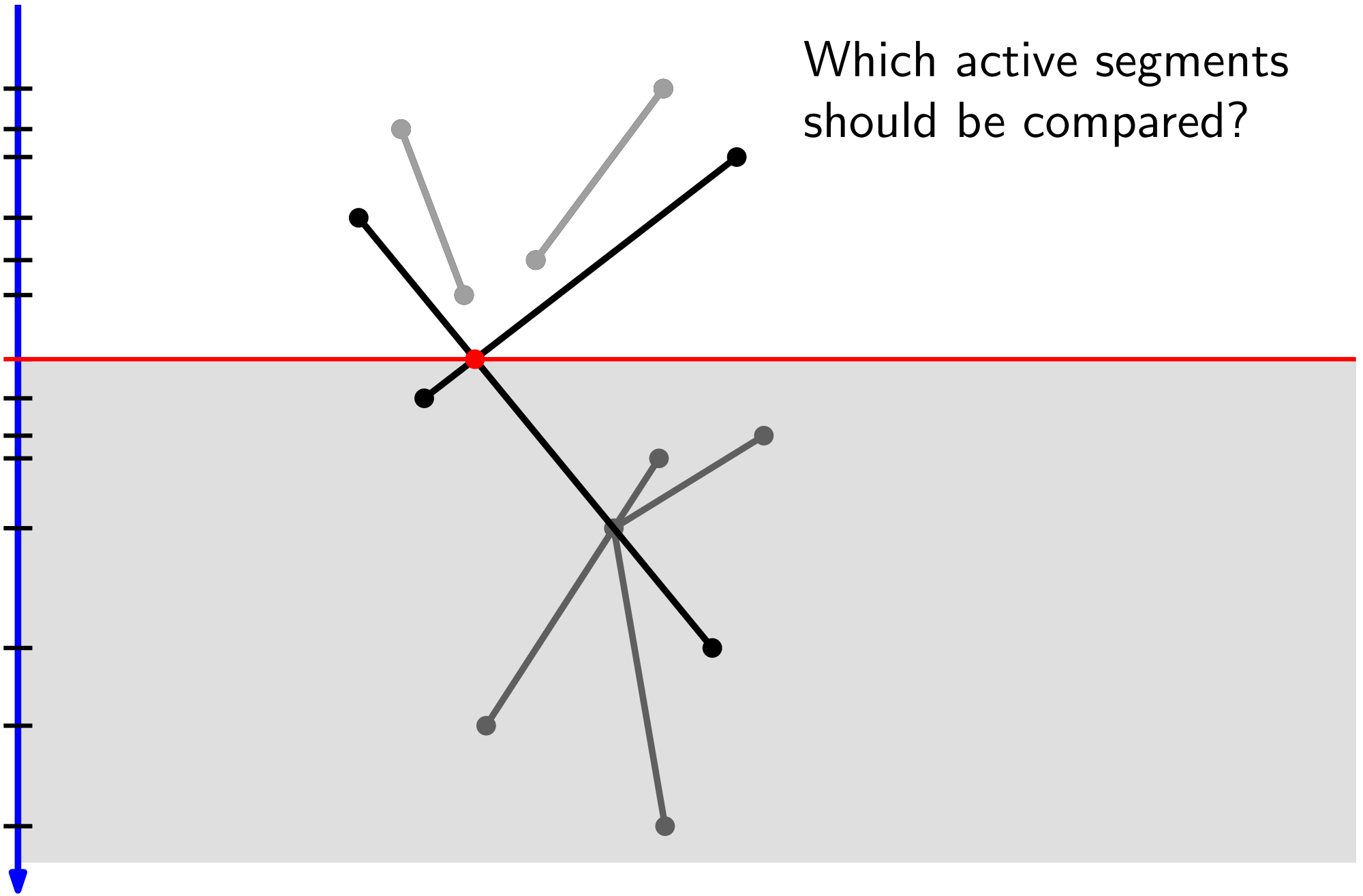

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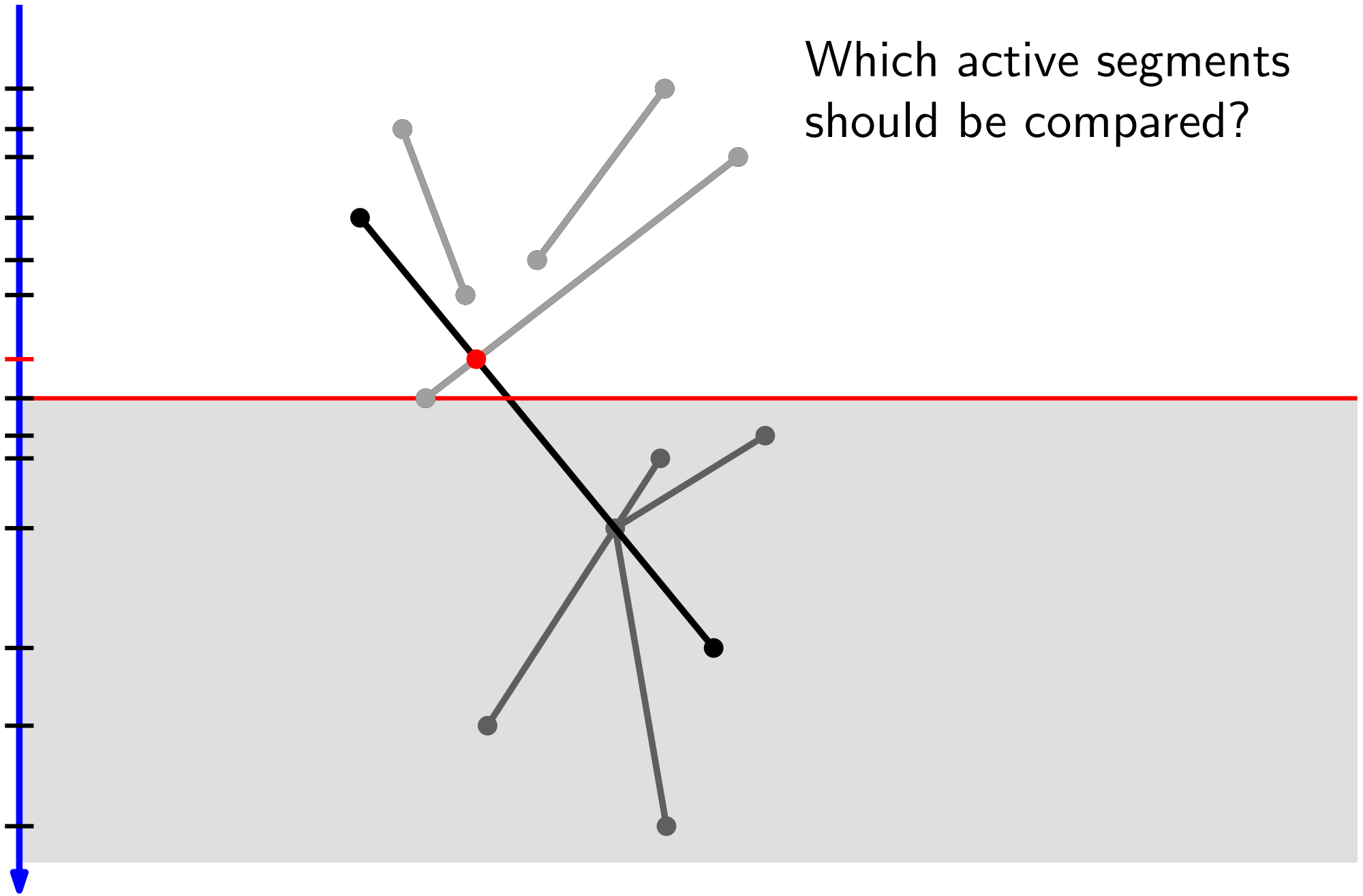

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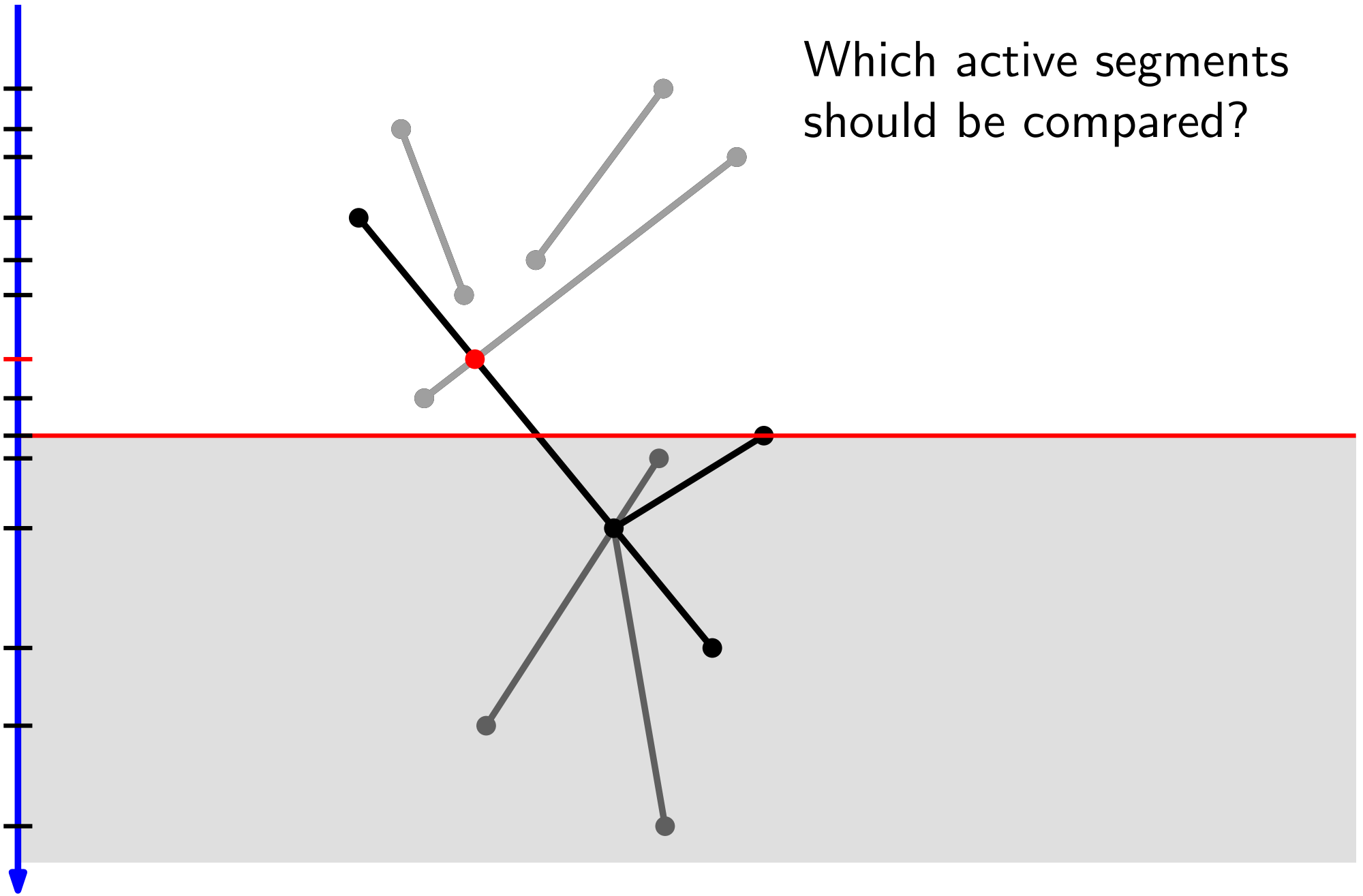

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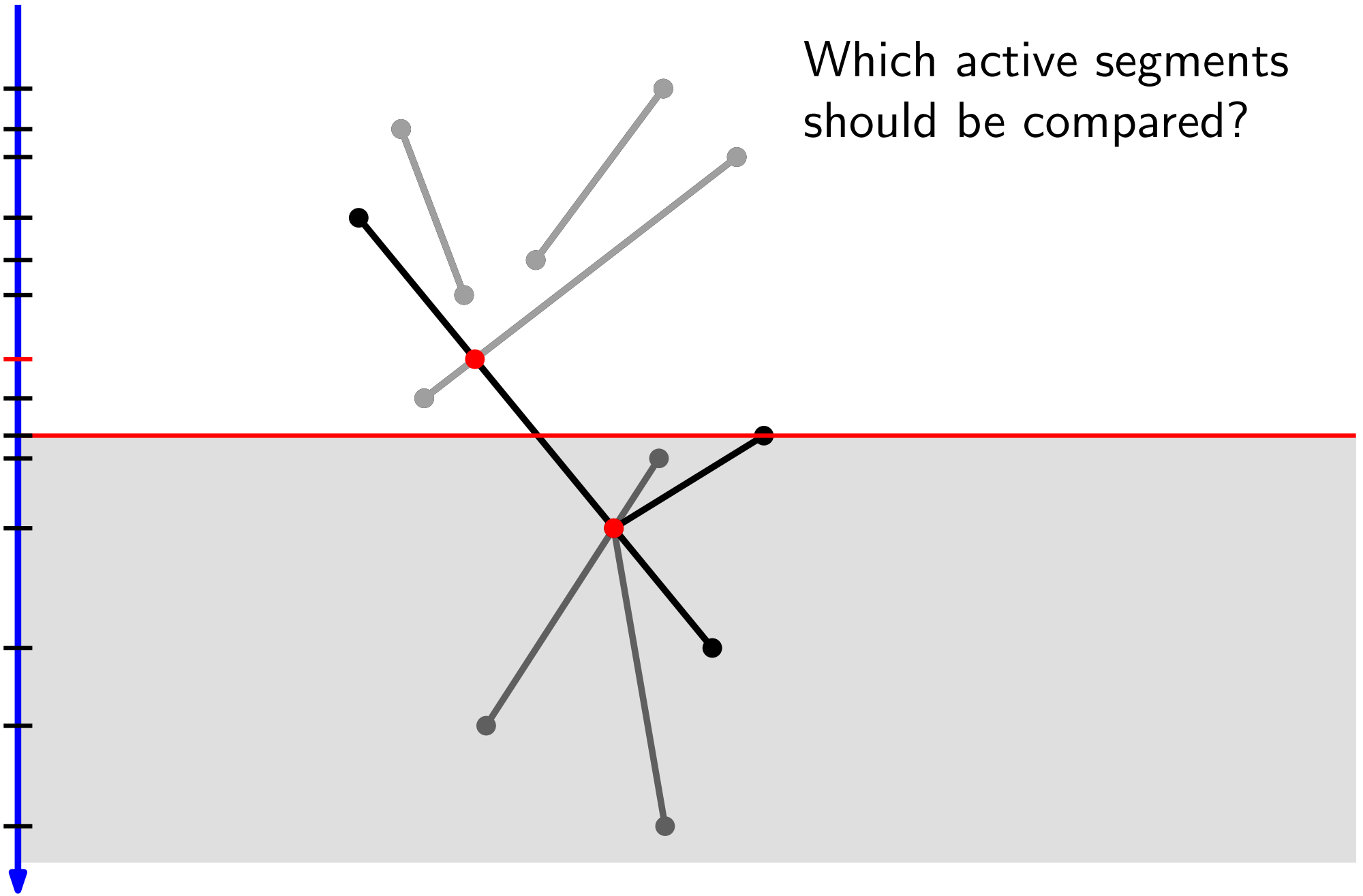

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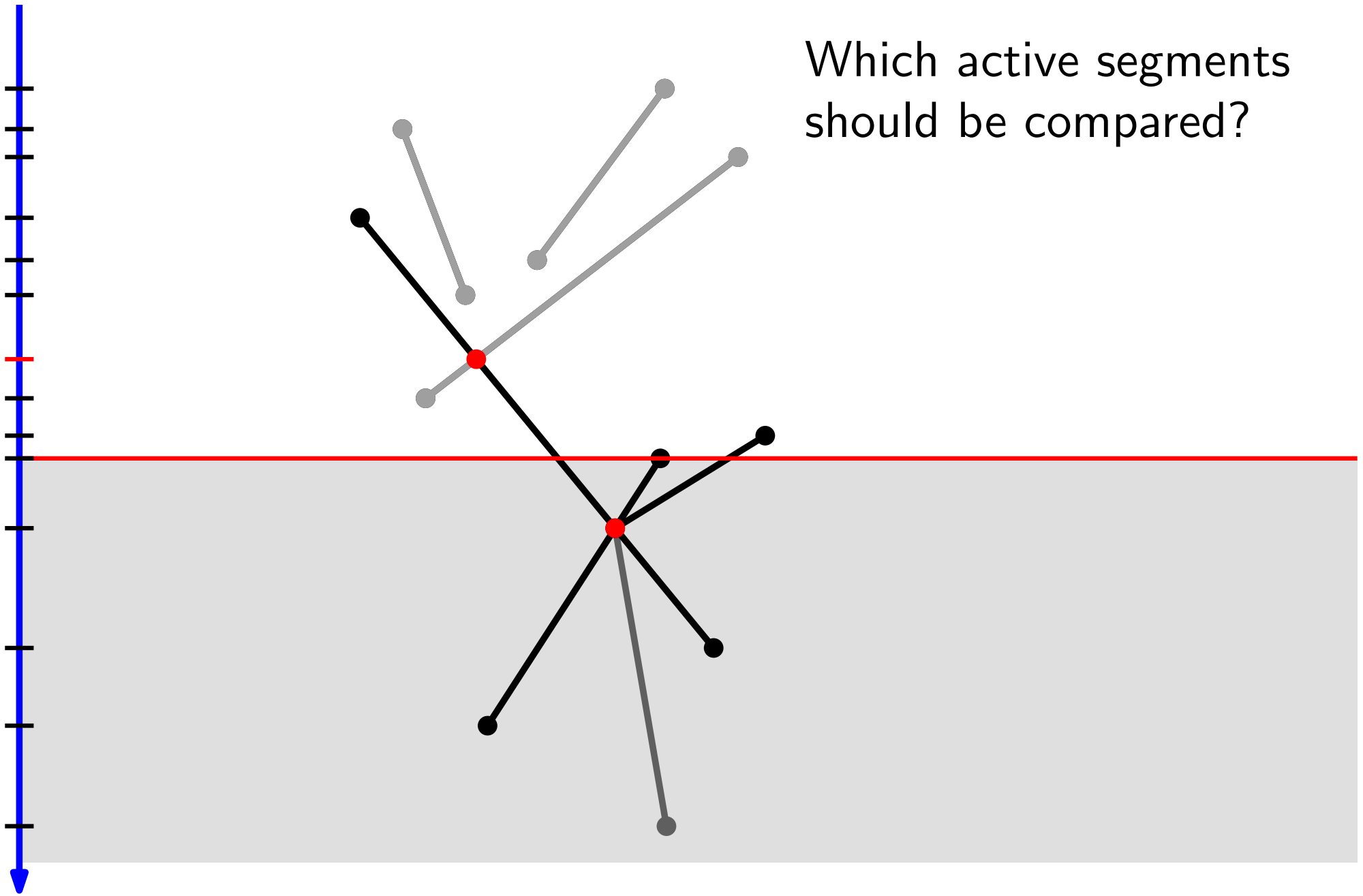

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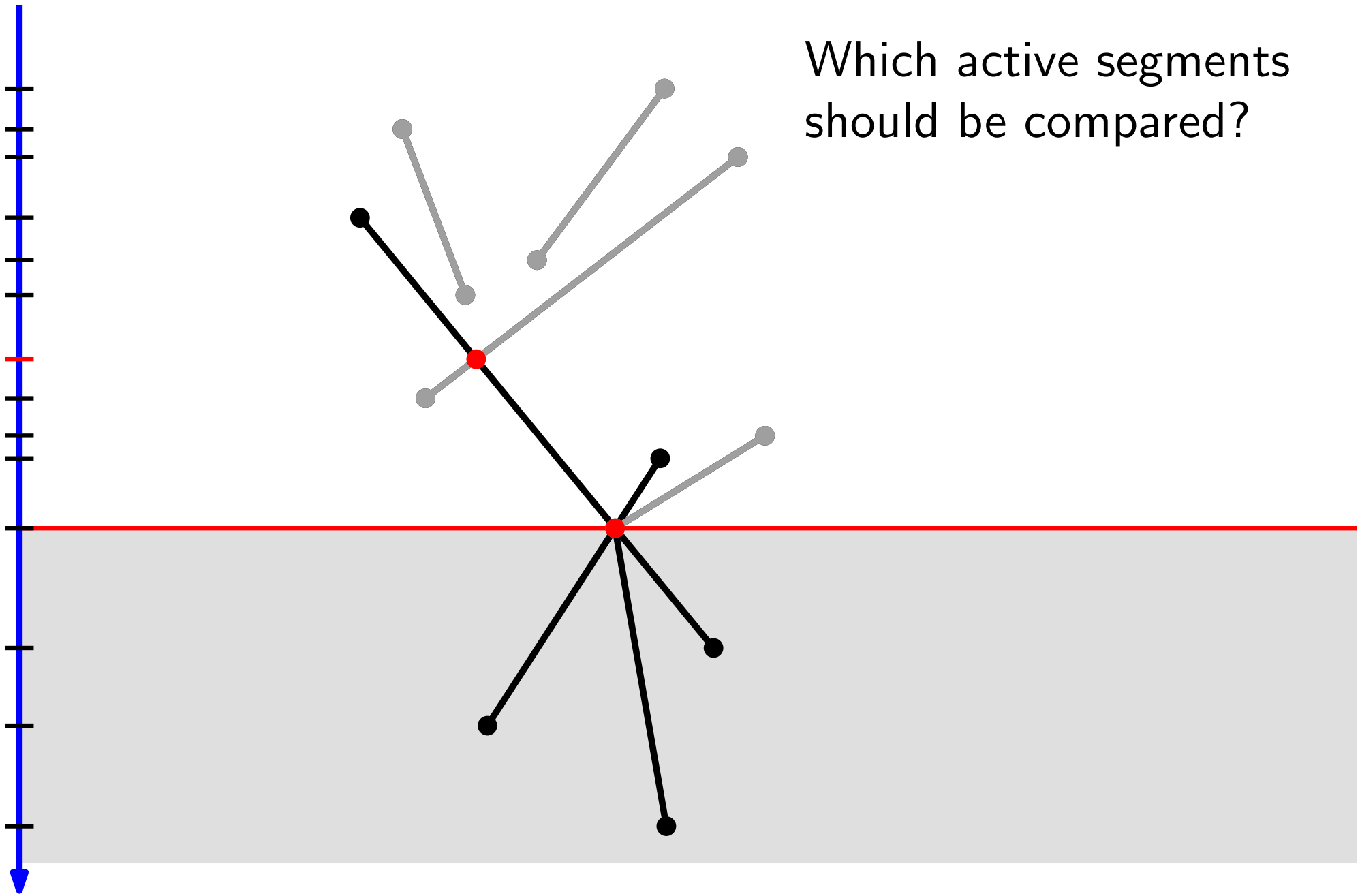

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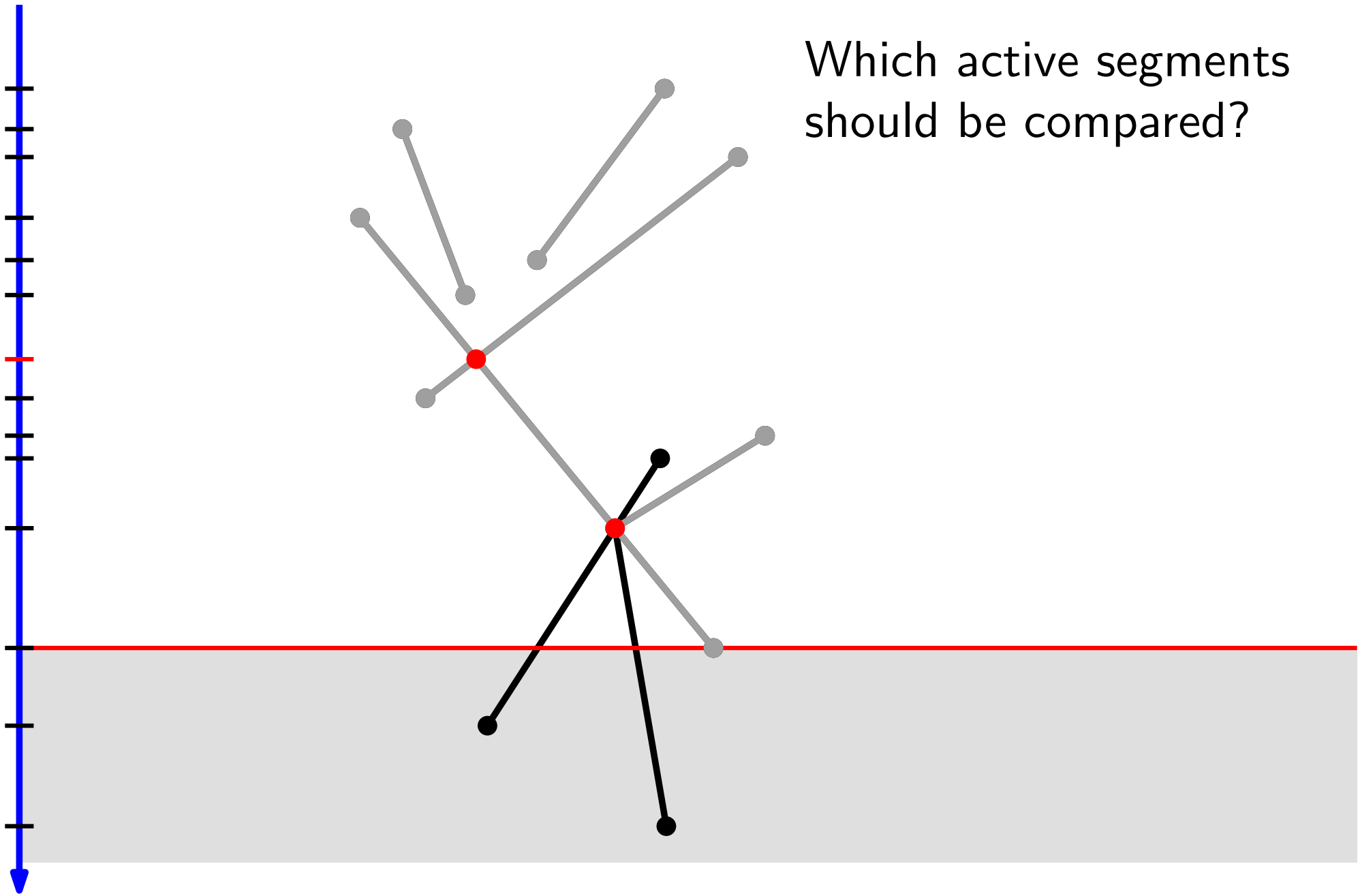

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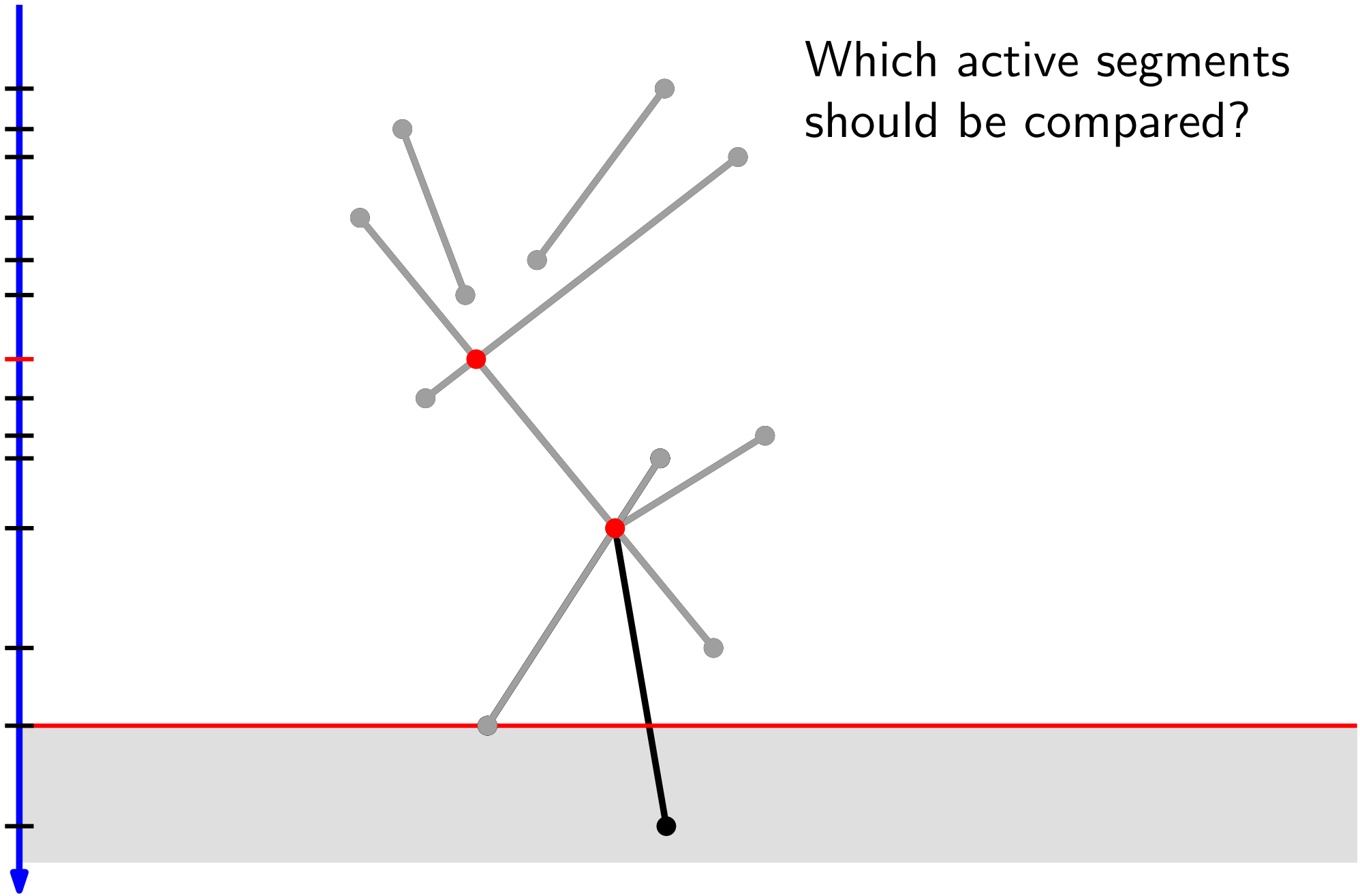

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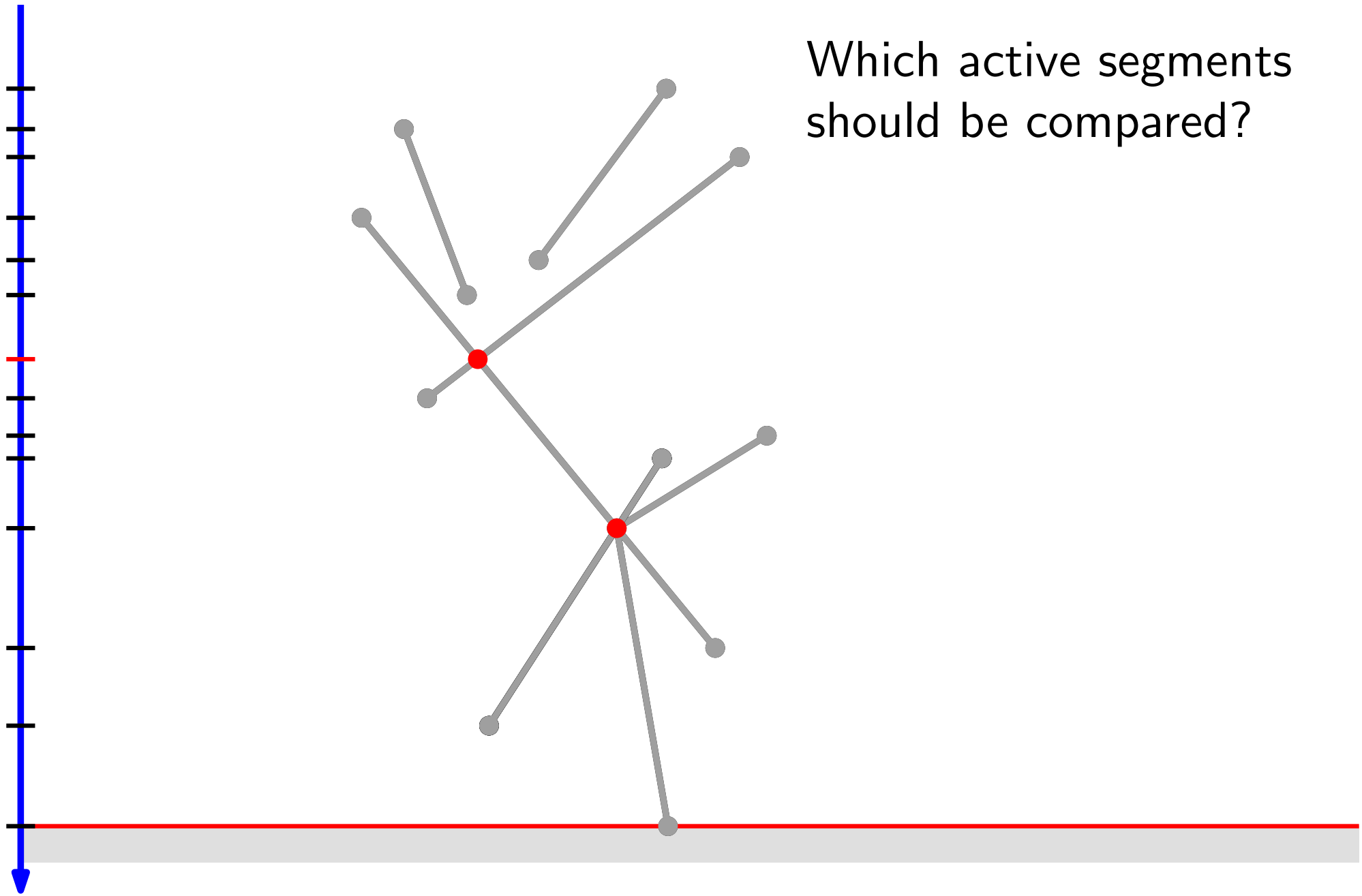

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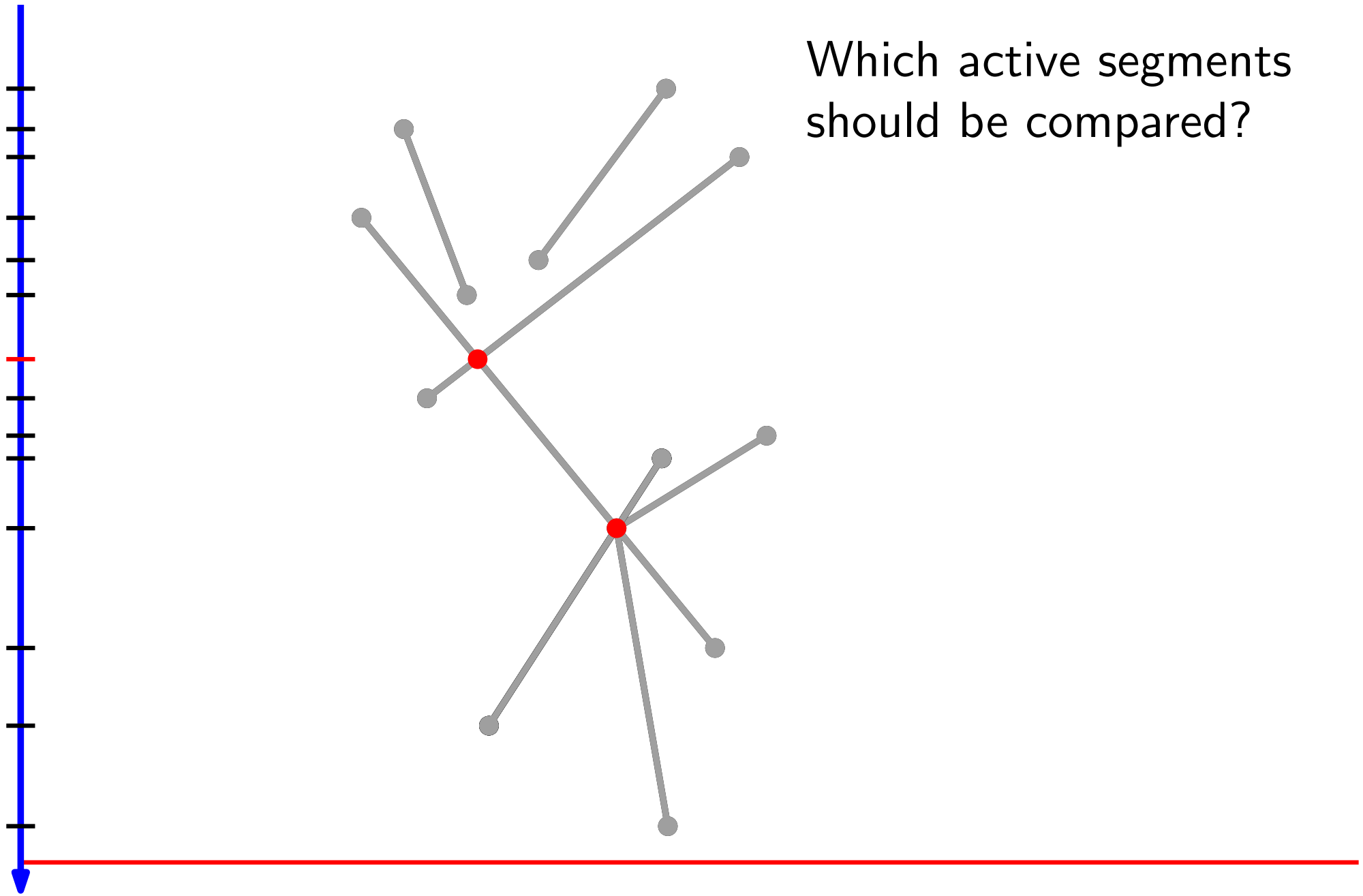

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Data Structures

1) event (-point) queue $\mathcal{Q}$

2) (sweep-line) status $\mathcal{T}$

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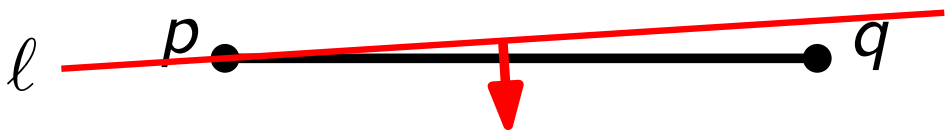

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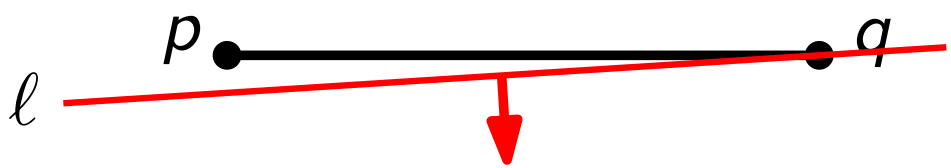

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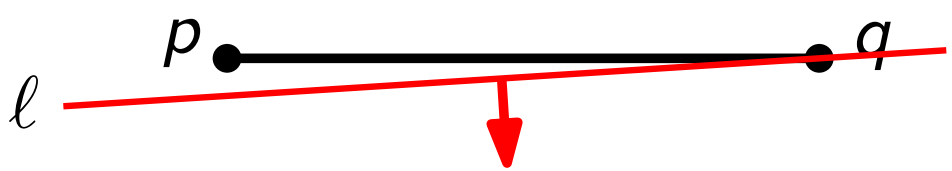

Store event pts in *balanced binary search tree* acc. to $\prec$

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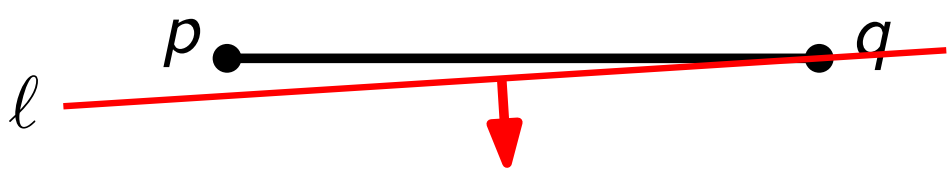

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$\Rightarrow$ nextEvent() and del/insEvent() take $O(\log |\mathcal{Q}|)$ time

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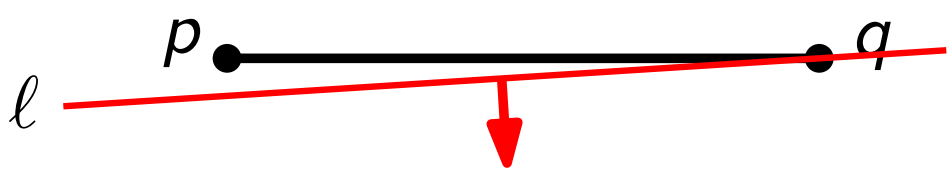

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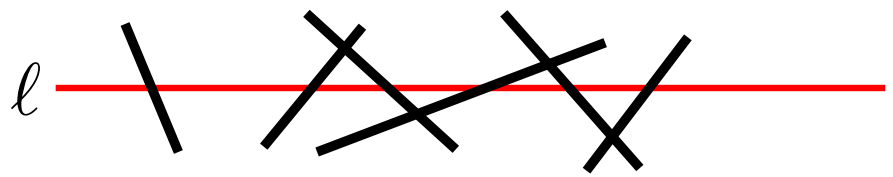

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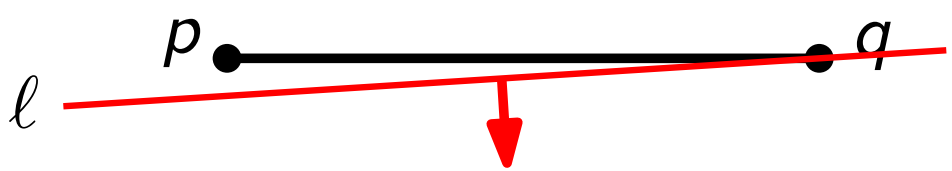
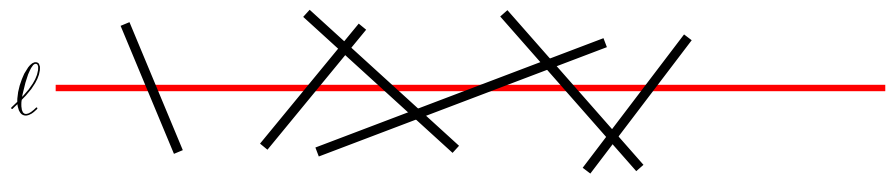
$p \prec q \quad \Leftrightarrow_{\text{def.}} \quad y_p > y_q \quad$ or $\quad (y_p = y_q \text{ and } x_p < x_q)$



Store event pts in *balanced binary search tree* acc. to $\prec$

$\Rightarrow$ nextEvent() and del/insEvent() take $O(\log |\mathcal{Q}|)$ time

2) (sweep-line) status $\mathcal{T}$



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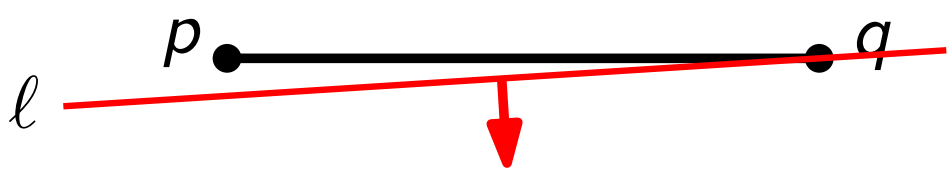

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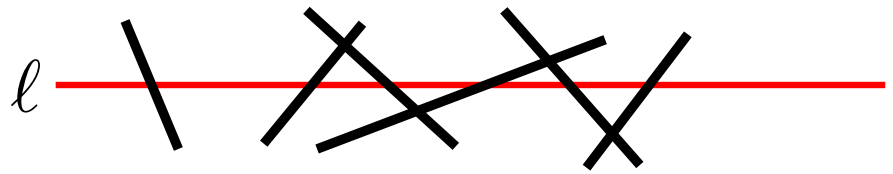

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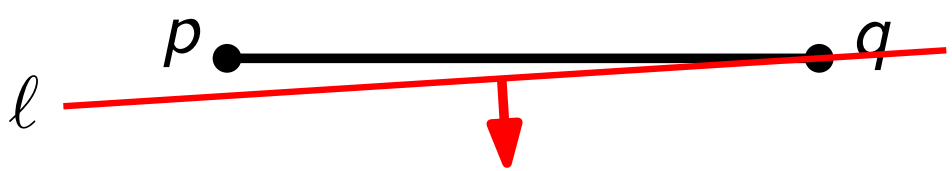

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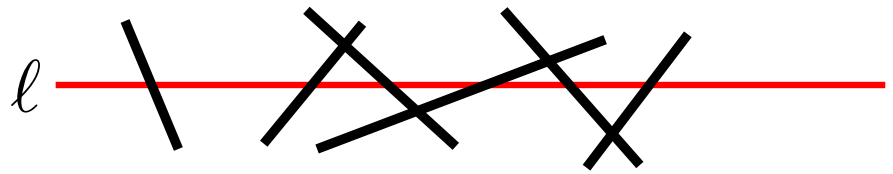

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How? In a balanced binary search tree!

Pseudo-code

findIntersections(S)

Input: set S of n non-overlapping closed line segments

Output: – set I of intersection pts
– for each $p \in I$ every $s \in S$ with $p \in s$

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$\mathcal{Q} \leftarrow \emptyset$; $\mathcal{T} \leftarrow \langle$ vertical lines at $-\infty$ and $+\infty \rangle$ // sentinels
foreach $s \in S$ **do**
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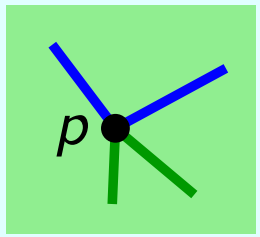

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```
Q ← ∅;  T ← ⟨ vertical lines at −∞ and +∞ ⟩   // sentinels
foreach s ∈ S do
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while Q ≠ ∅ do
    p ← Q.nextEvent()
    Q.deleteEvent(p)
    handleEvent(p)
```

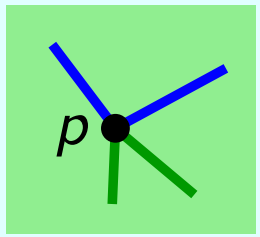

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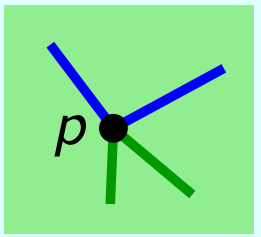

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This subroutine does the real work – how would you implement it?

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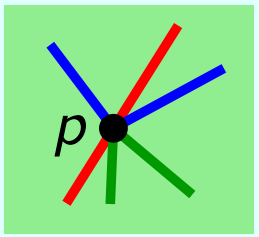

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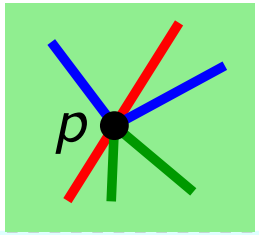

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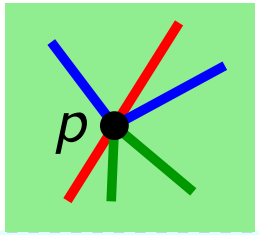
handleEvent(event p)

Handling an Event



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report intersection in p, report segments in $U(p) \cup L(p) \cup C(p)$

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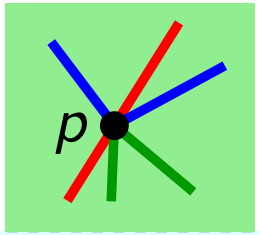

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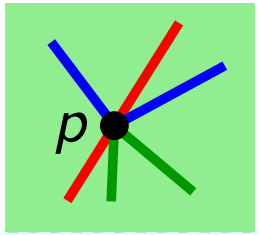

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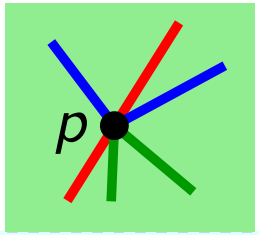

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  $b_{\text{left}}/b_{\text{right}}$ = left/right neighbor of p in $\mathcal{T}$

  findNewEvent(b_{left}, b_{right}, p)

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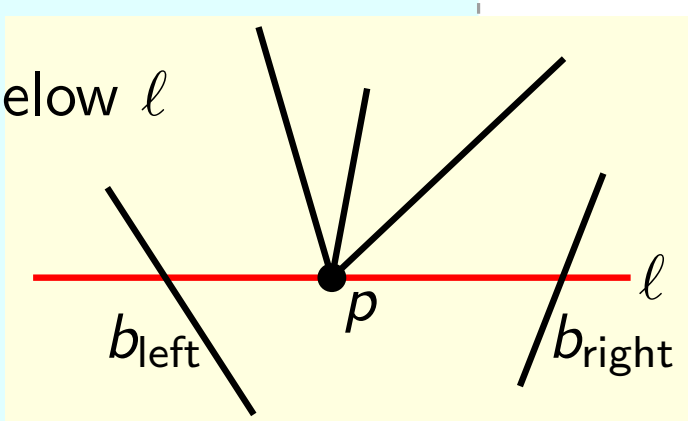

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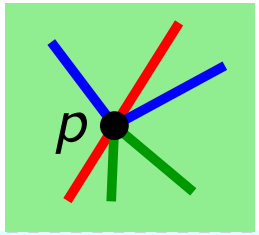

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if $s \cap s' = \emptyset$ **then return**

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if x below ℓ or to the right of p **then**

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  if $x \in$ rel-int(s) **then** $C(x) \leftarrow C(x) \cup \{s\}$

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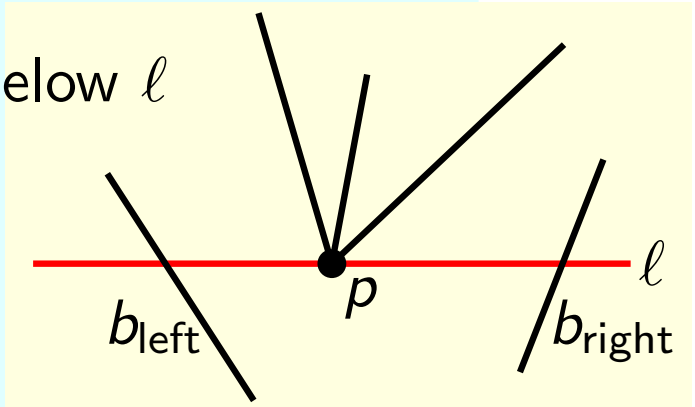

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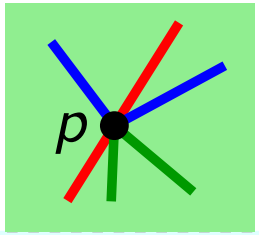

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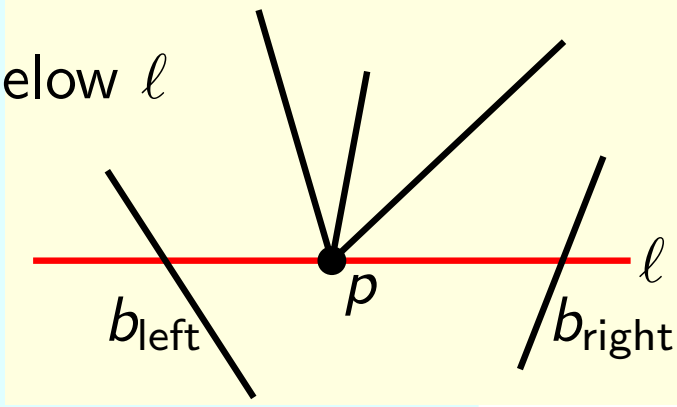

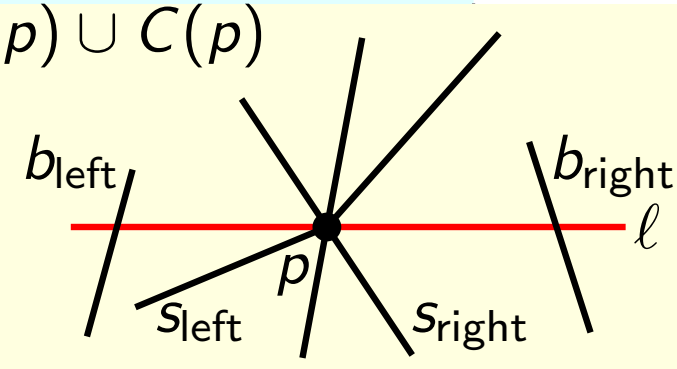

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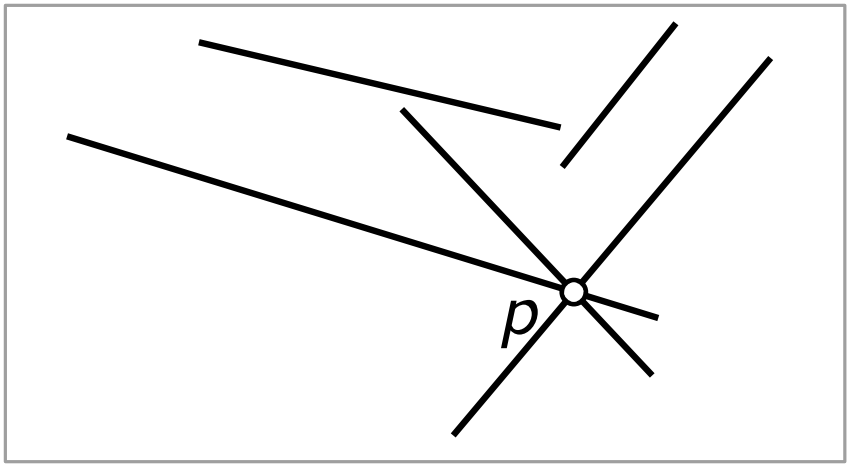

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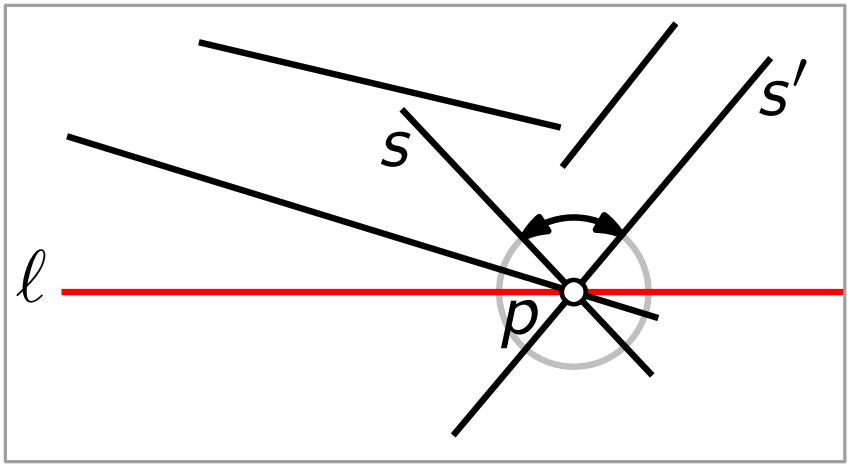

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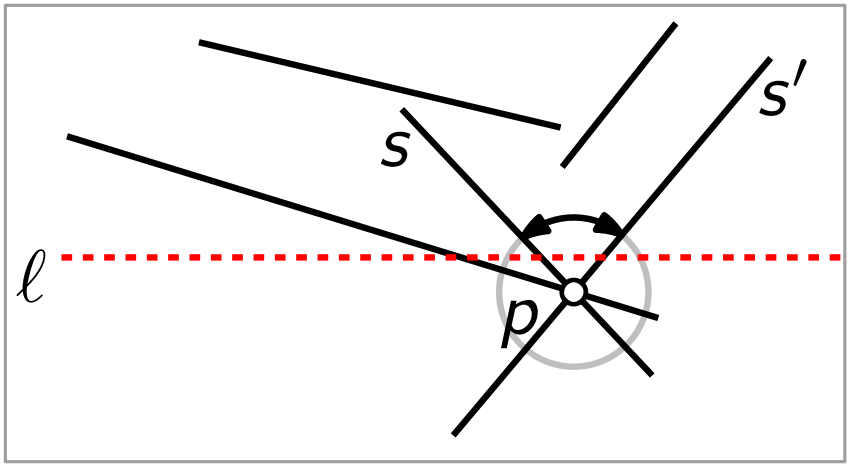

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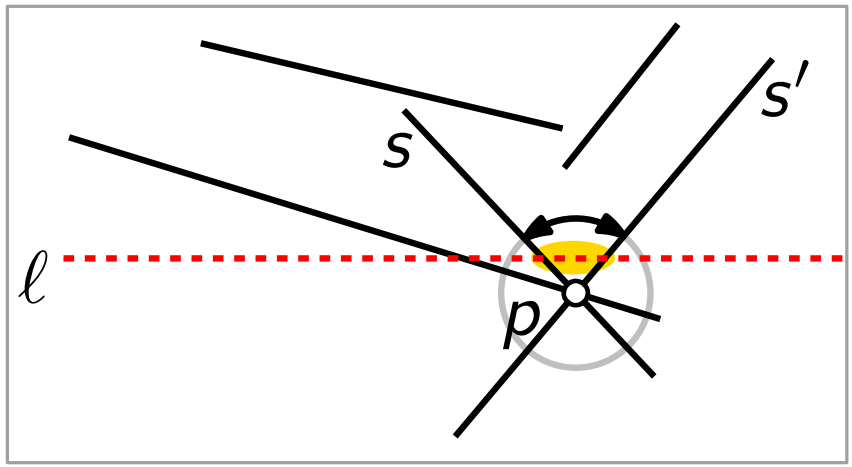

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Then s, s' were neighbors in the left-to-right order on ℓ (in $\mathcal{T}$).

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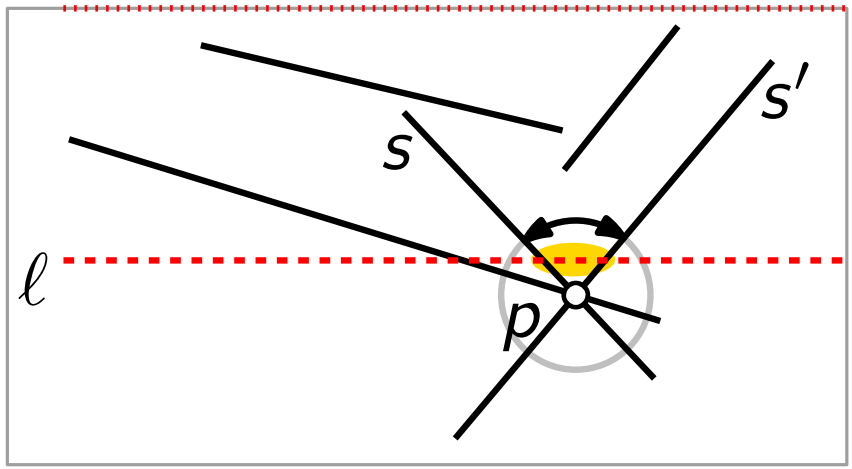

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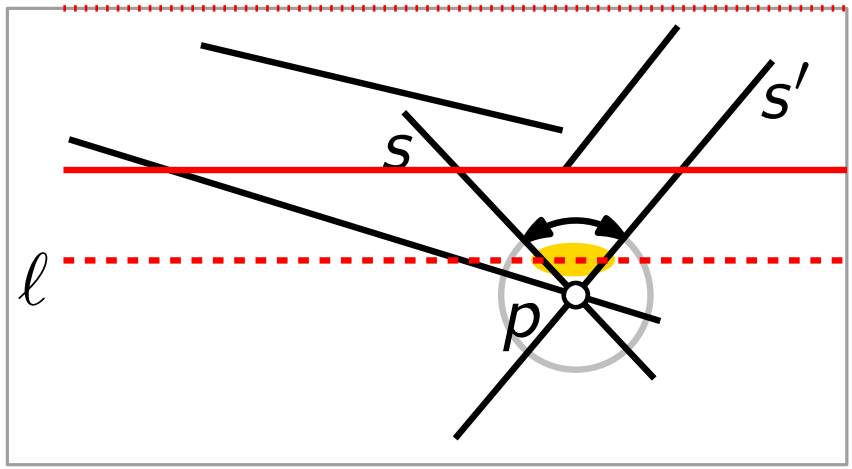

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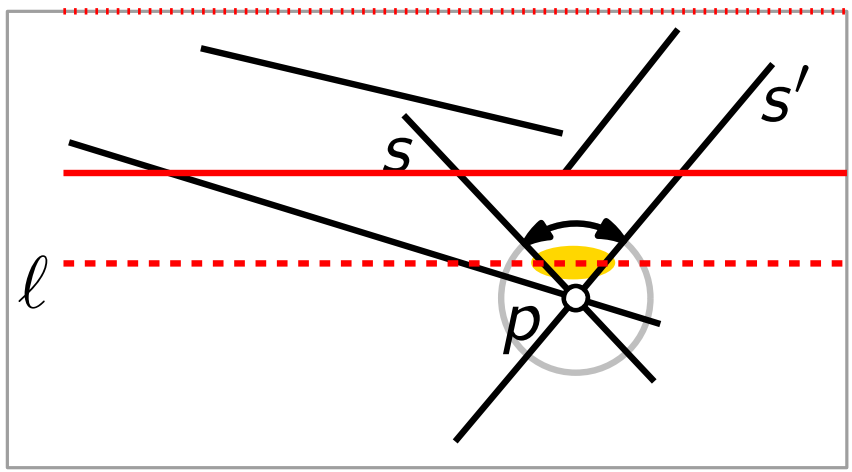

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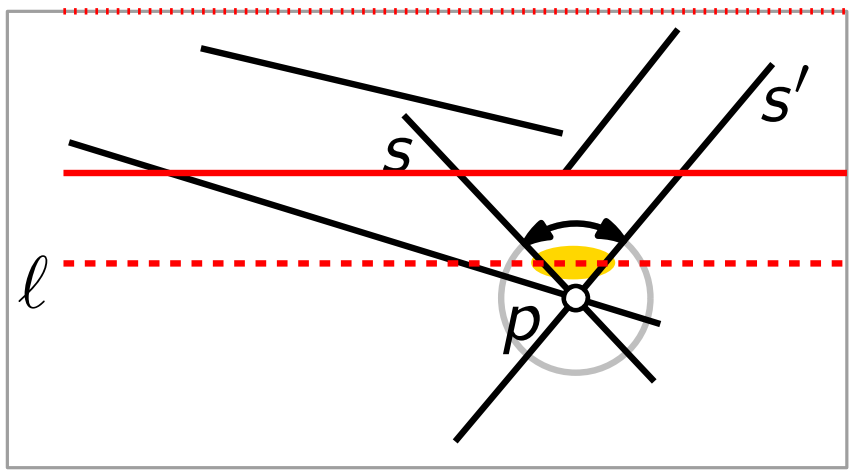

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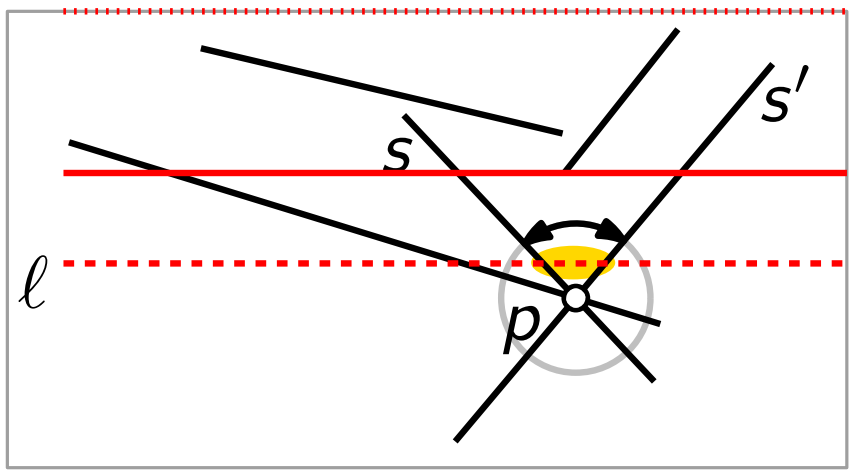

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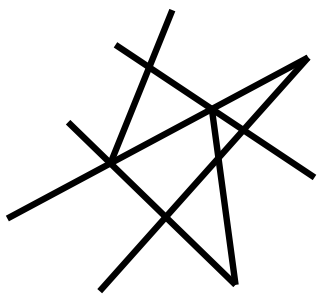

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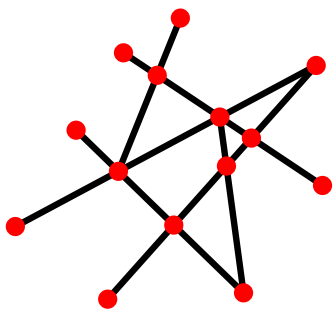

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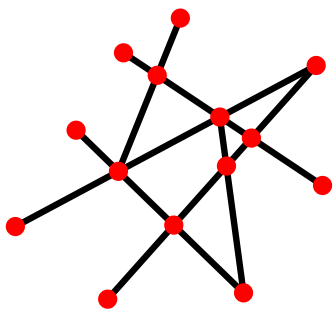

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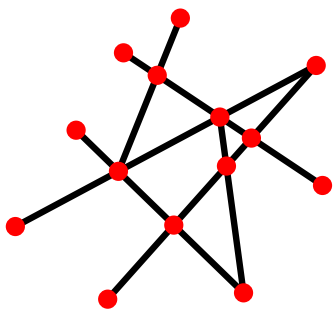

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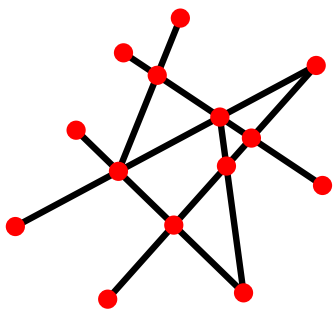

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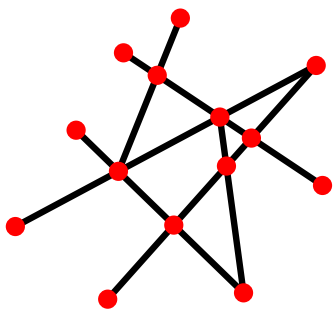

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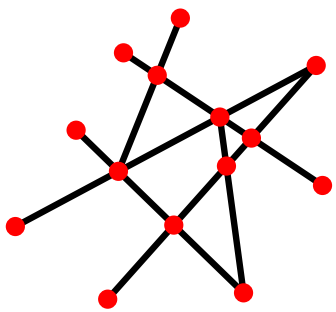

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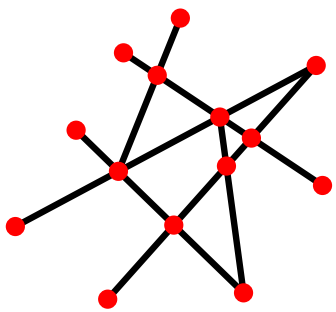

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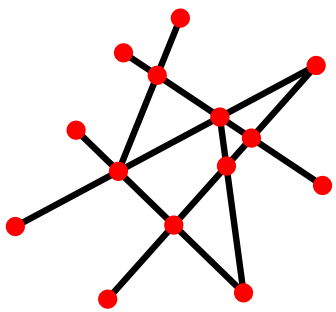

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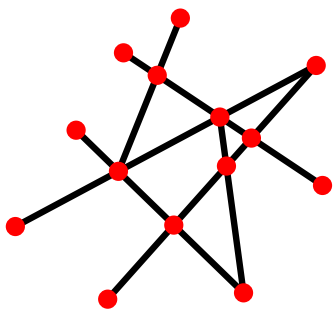

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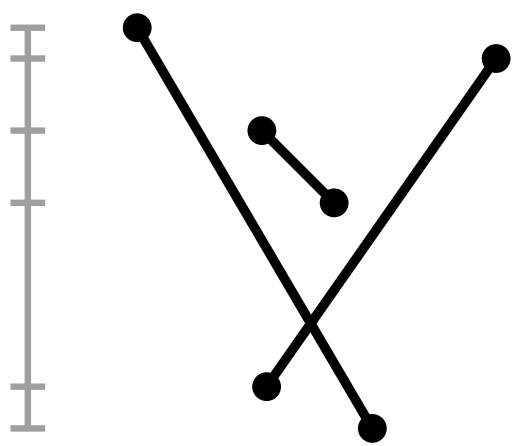

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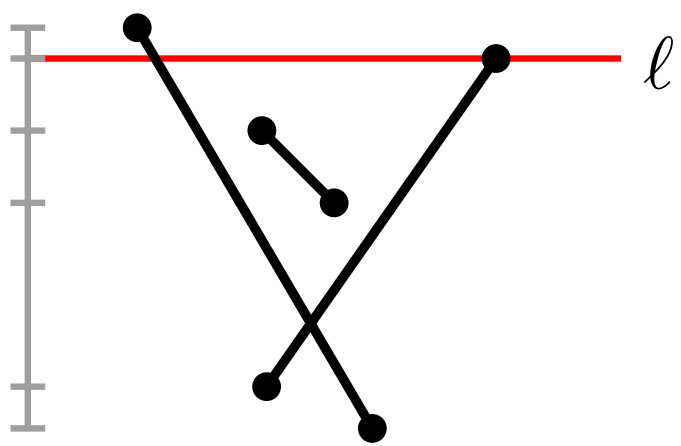

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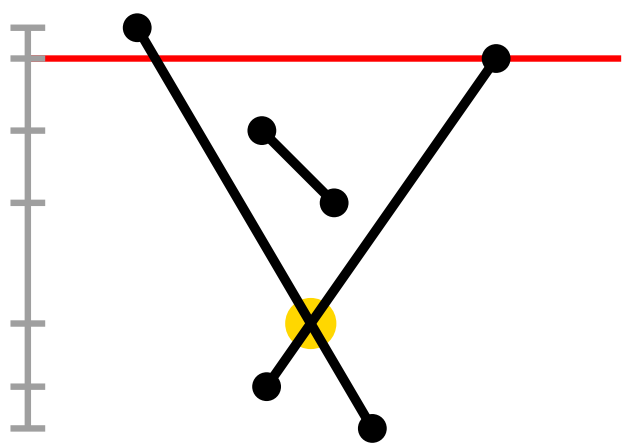

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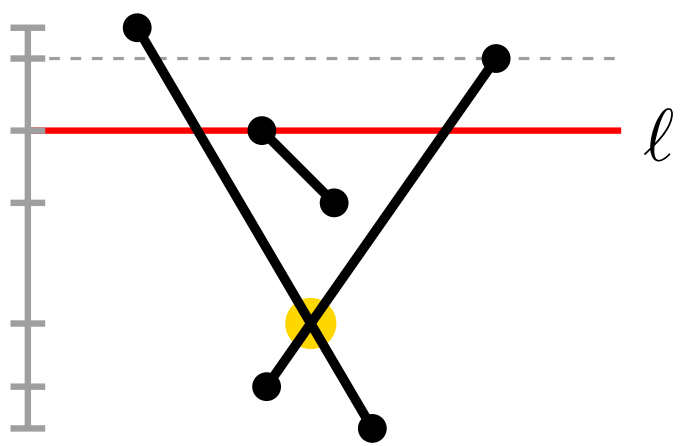

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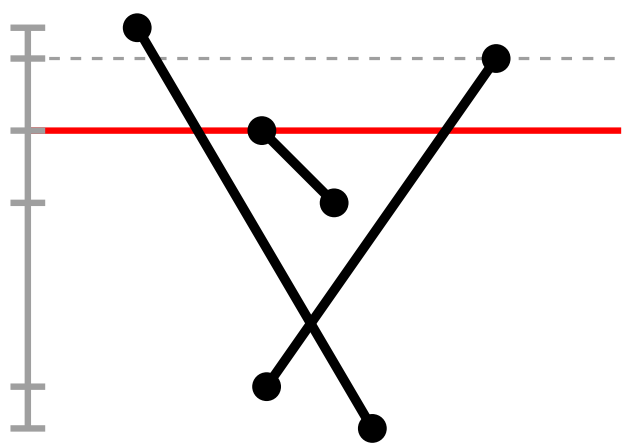

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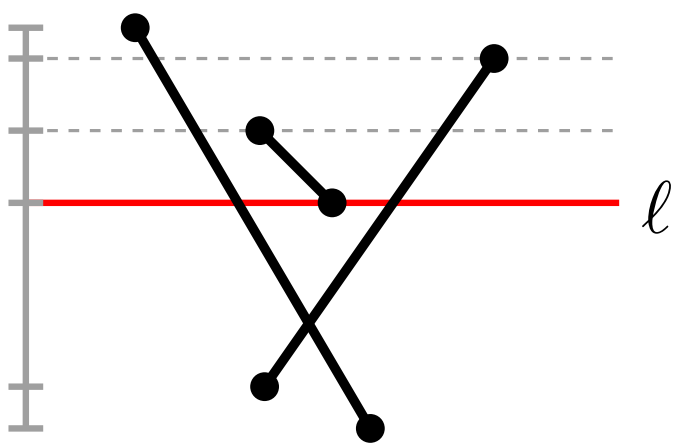

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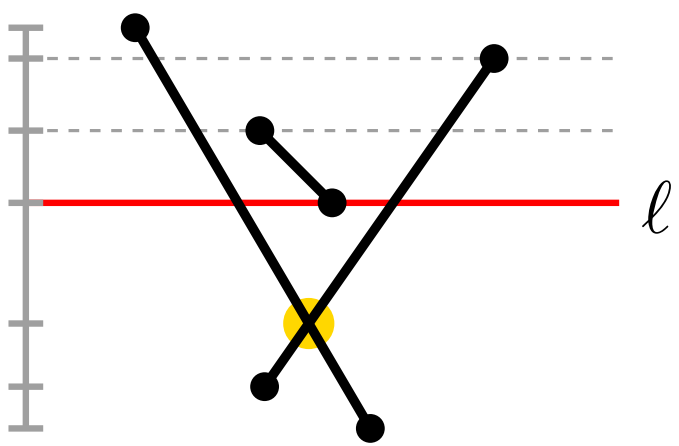

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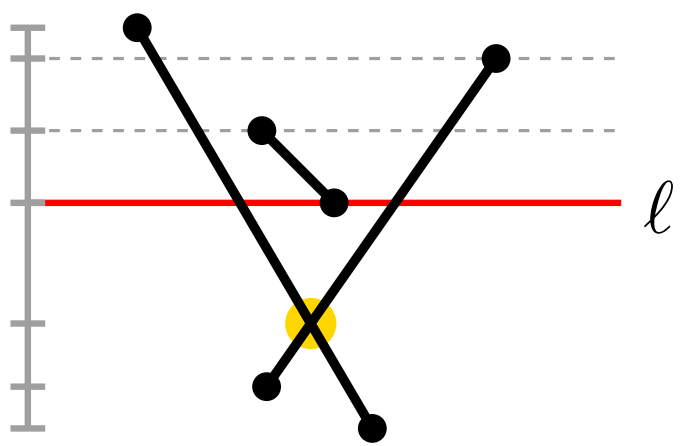


– insert $s \cap s'$ into $\mathcal{Q}$
– remove $s \cap s'$ from $\mathcal{Q}$
– re-insert $s \cap s'$ into $\mathcal{Q}$

$\Rightarrow$ need just $O(n)$ space,

Today's Main Result

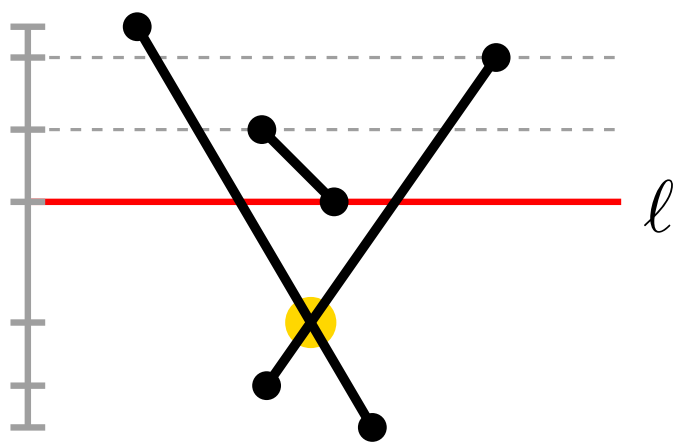
Theorem: We can report all I intersection points among n non-overlapping line segments in the plane and report the segments involved in the intersections in $O((n+I)\log n)$ time and $O(n)$ space.

Sure? The event-point queue $\mathcal{Q}$ contains

- all segment end pts below the sweep line
- all intersection pts below the sweep line

$\Rightarrow$ (worst-case) space consumption $\in \Theta(n+I)$:-(

Can we do better?



– insert $s \cap s'$ into $\mathcal{Q}$
– remove $s \cap s'$ from $\mathcal{Q}$
– re-insert $s \cap s'$ into $\mathcal{Q}$

$\Rightarrow$ need just $O(n)$ space, (asymptotic) running time doesn't change $\square$