Homework Assignment #9
Visualization of Graphs (Summer Term 2016)
This assignment corresponds to the material covered in Lecture 10

Exercise 1 – Adding an edge

Suppose you are given a planar drawing of a graph $G = (V, E)$ with corresponding combinatorial embedding $\mathcal{E}$. Let $u, v \in V$ be vertices such that $e = \{u, v\} \notin E$.

a) Devise a polynomial-time algorithm that adds $e$ to the existing drawing causing as few crossings as possible. *Hint:* Use the dual graph.

4 points

b) Argue why your algorithm is correct.

2 points

c) What is the runtime of your algorithm?

1 point

Exercise 2 – Fixed Linear Crossing Number

Consider the problem $\textsc{Fixed Linear Crossing Number}$ that was introduced in the lecture. A graph $G = (V, E)$ with given vertex numbering $V = \{v_1, v_2, \ldots, v_n\}$ has to be drawn such that $v_i$ has position $(i, 0)$, every edge is a semicircle and the number of crossings is minimal. The only decision when drawing an edge is therefore whether it is drawn above or below the $x$ axis. Given a graph $G$ with numbered vertices and an integer $k$, it is NP-hard to decide whether an admissible drawing with at most $k$ crossings exists.

a) Devise an algorithm that decides for a given graph $G = (V, E)$ with vertex numbering $V = \{v_1, \ldots, v_n\}$ in polynomial, whether an admissible drawing without crossings exists; if a planar drawing exists, the algorithm should return one.

4 points

b) Argue why your algorithm is correct.

3 points
c) What is the runtime of your algorithm?

1 point

d) Show that when restricting the input graphs to matchings (i.e. all nodes have degree 1), Fixed Linear Crossing Number is still NP-hard.

5 extrapoints

Exercise 3

Show that the bound $\text{cr}(G) \geq \frac{1}{16} \cdot \frac{m^3}{n^2} \in \Omega\left(\frac{m^3}{n^2}\right)$ from the lecture is asymptotically tight.

*Hint:* Consider the graph $G = (V,E)$ with $V = \{v_0,v_1,v_2,\ldots,v_{n-1}\}$ and $E = \{\{v_i,v_j\} \mid i < j \leq i + k \mod n\}$ for $0 < k < n/2$. Use a drawing in which $v_0,v_1,\ldots,v_{n-1}$ in this order are in concave position (e.g. on a circle) and the edges are drawn as straight lines.

5 points

This assignment is due at the beginning of the next lecture, that is, on July 7 at 10:15. Please hand in your solutions in the lecture or submit them via WueCampus.

The exercises on this assignment will be discussed in the tutorial session on July 4 at 16:00.