

Homework Assignment #11

Computational Geometry (Winter Term 2014/15)

Exercise 1

Draw the Minkowski sum $P_1 \oplus P_2$ for the case where

- a) both P_1 and P_2 are unit discs (radius 1); [1 point]
- b) both P_1 and P_2 are unit squares (side length 1); [1 point]
- c) P_1 is a unit disc and P_2 is a unit square; [1 point]
- d) P_1 is a unit square and P_2 is a triangle with vertices $(0, 0)$, $(1, 0)$ and $(0, 1)$. [1 point]

Exercise 2

Let P_1 and P_2 be two sets of points in the plane. Prove that the “shape” of the Minkowski sum $P_1 \oplus P_2$ is invariant under translations of P_1 and P_2 . More precisely: Let P'_1 and P'_2 be two sets of points resulting from translating P_1 and P_2 , respectively. Prove that the Minkowski sum $P'_1 \oplus P'_2$ results from translating $P_1 \oplus P_2$. [3 points]

Exercise 3

Let P_1 and P_2 be two convex polygons with vertex sets S_1 and S_2 , respectively. Further, let $\text{CH}(S)$ be the convex hull of a set of points S .

- a) Prove that $\text{CH}(S_1 \oplus S_2) \subseteq P_1 \oplus P_2$. [3 points]
- b) Does even $\text{CH}(S_1 \oplus S_2) = P_1 \oplus P_2$ hold? [4 points]

Exercise 4

In the lecture we gave an $O(n)$ bound on the complexity of the union of a set \mathcal{P} of polygonal pseudodiscs with n vertices in total. We are interested in the precise bound.

- a) Assume that the union boundary contains m original vertices of the polygons in \mathcal{P} . Show that the complexity of the union boundary is at most $2n - m$. Use this to prove an upper bound of $2n - 3$ on the complexity of the union boundary. **[3 points]**
- b) Prove a lower bound of $2n - 6$ by constructing an example that has this complexity. **[3 points]**

This assignment is due at the beginning of the next lecture, that is, on January 7 at 10:15. Solutions will be discussed in the tutorial on Friday, January 9, 14:00–15:30 in room SE I.