

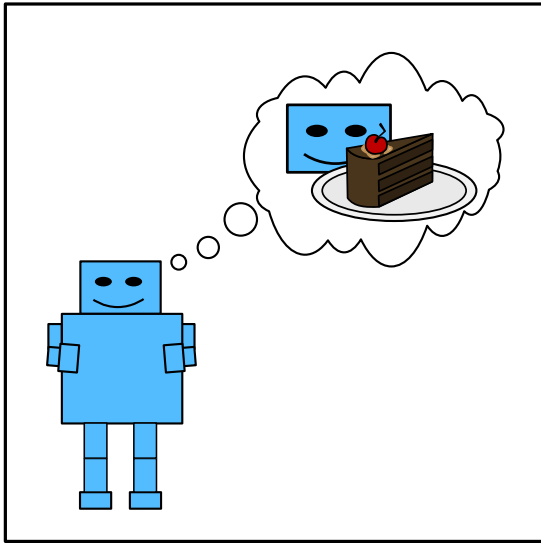
Computational Geometry

Winter semester 2014/15

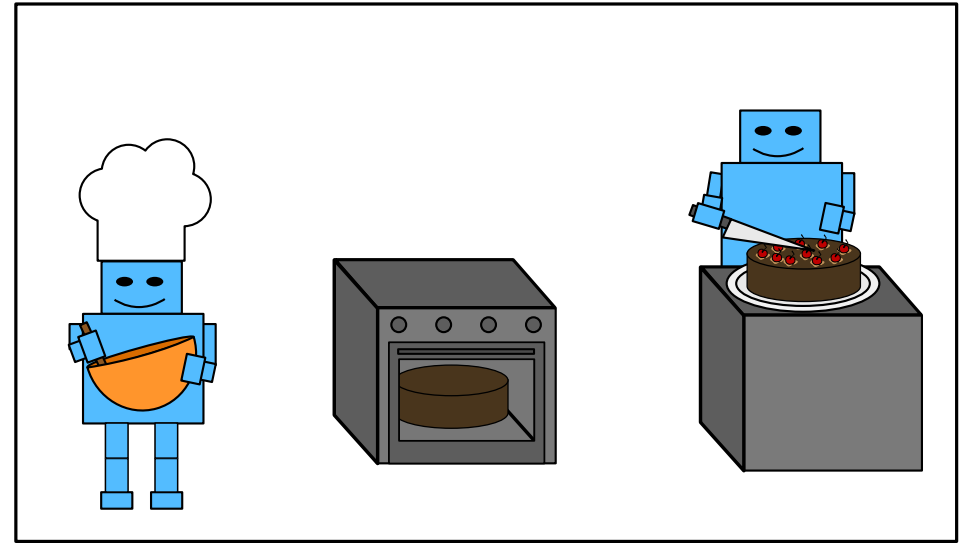
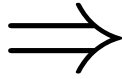
Motion Planning

Lecture #10

Planning

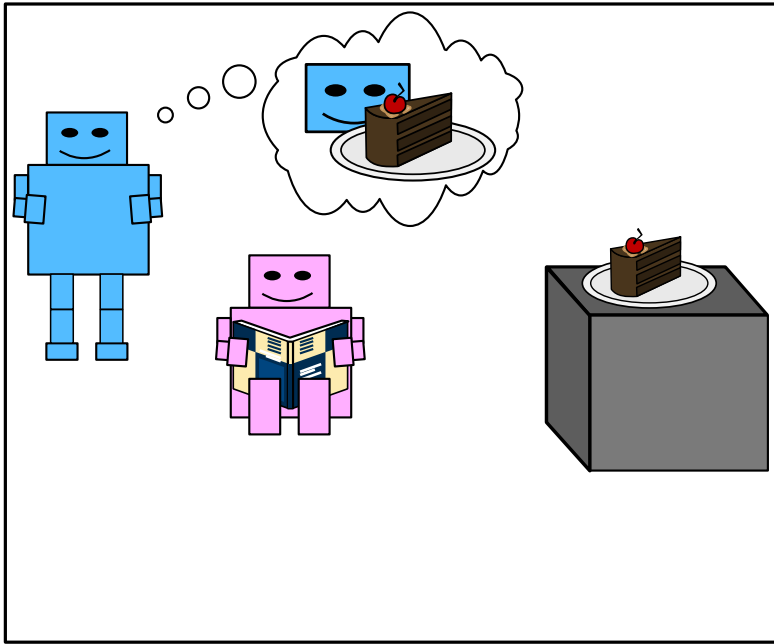


current situation,
desired situation

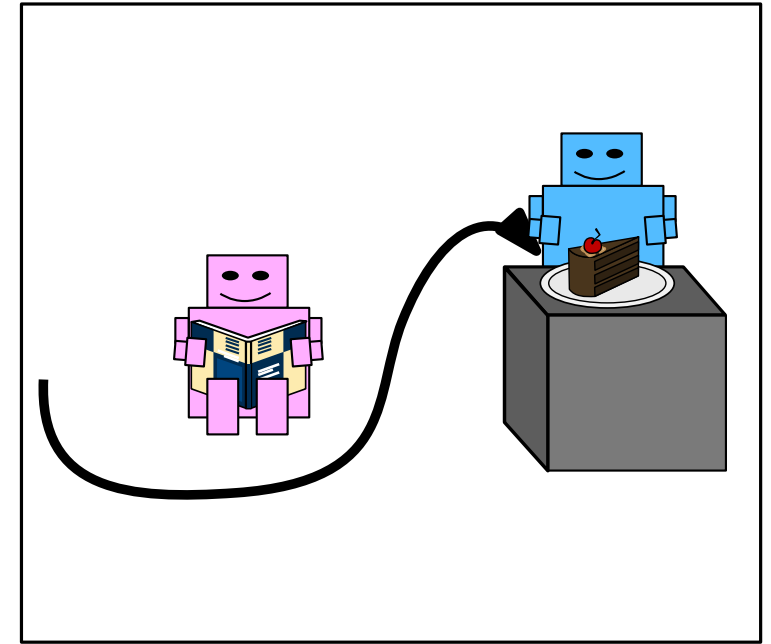
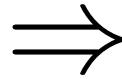


sequence of steps to reach
the one from the other

Path Planning



current location,
desired location



path to reach the one
from the other

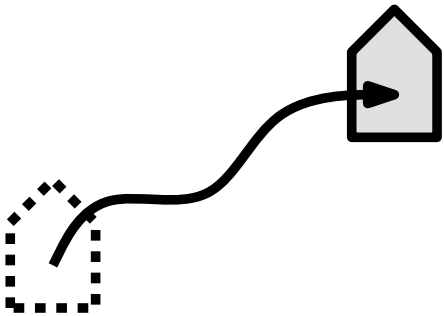
A First Result

Theorem: We can preprocess a set of polygonal obstacles with a total of n edges in $O(n \log n)$ expected time such that, given a start and a goal position, we can find a collision-free path for a **point robot** in $O(n)$ time if it exists.

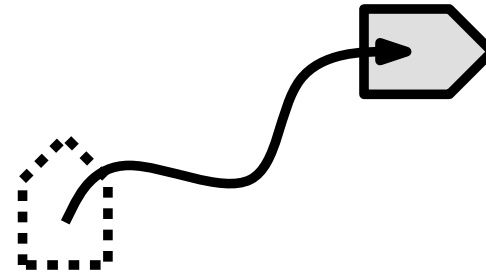
What about, say, *polygonal* robots?

Degrees of Freedom

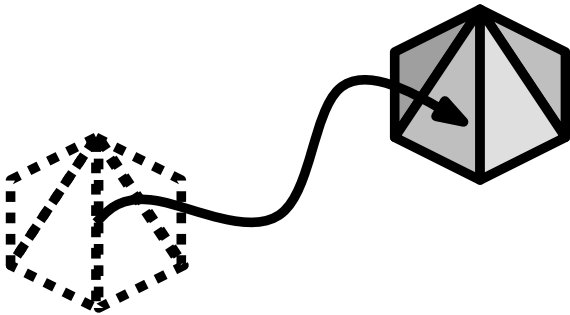
Every robot has some number d of *degrees of freedom*, meaning that its *configuration* with respect to the world can be specified by d parameters.



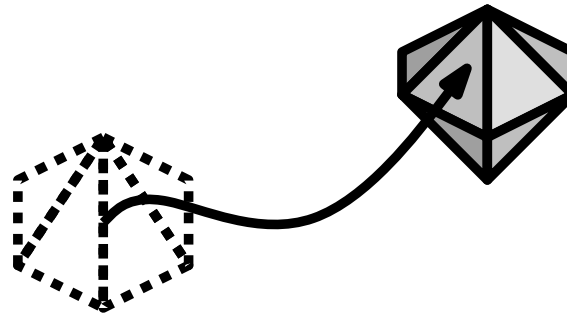
2D translating robot



2D translating, rotating robot

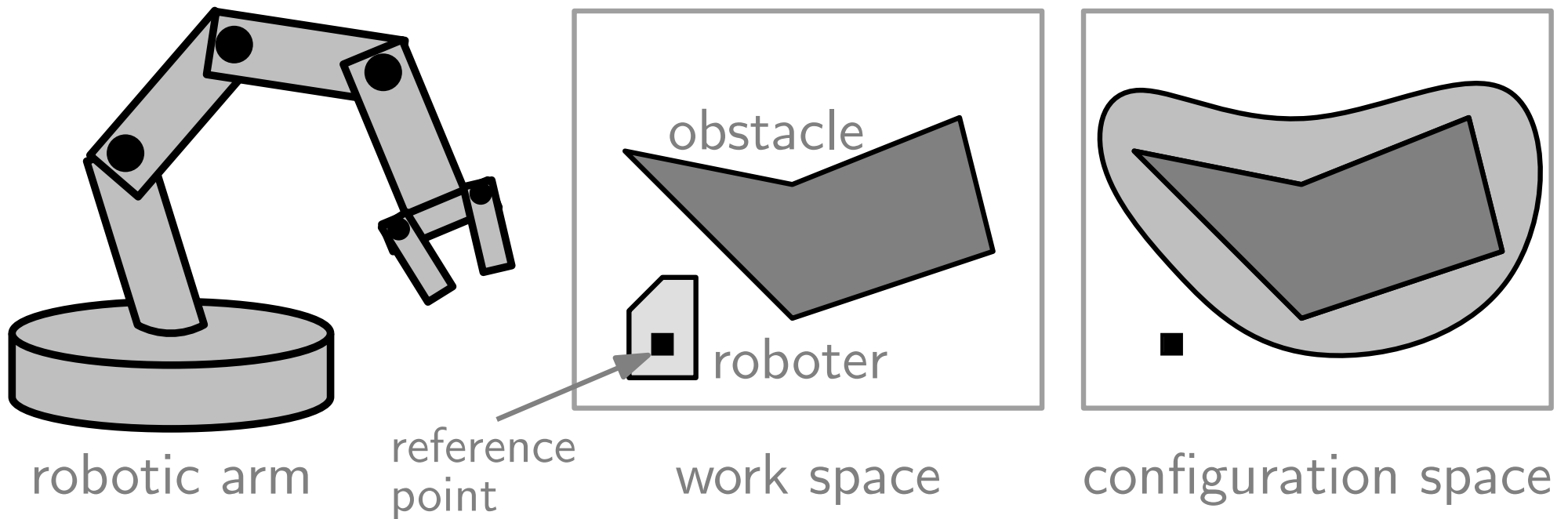


3D translating robot



3D translating, rotating robot

Configuration Space



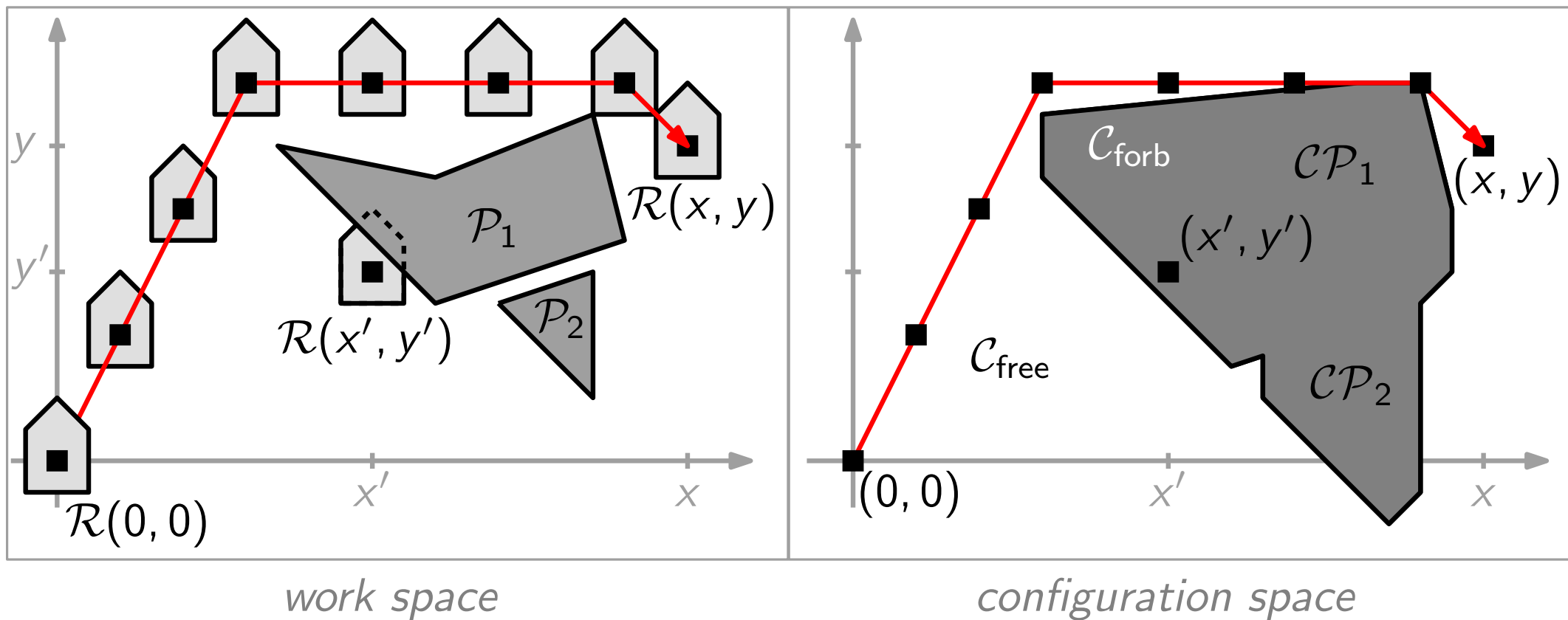
The *configuration space* is the d -dimensional space of all possible (i.e., obstacle avoiding) parameter value combinations.

Path for a *point* through configuration space



path for the *robot* in the original space.

Example: Translating 2D Polygonal Robots



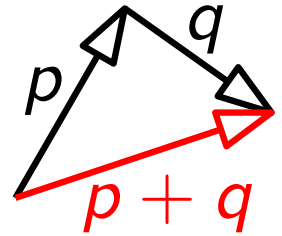
- Compute $\mathcal{CP}_i = \{(x, y) : \mathcal{R}(x, y) \cap \mathcal{P}_i \neq \emptyset\}$ for each \mathcal{P}_i .
- Compute their union $\mathcal{C}_{\text{forb}} = \bigcup_i \mathcal{CP}_i$.
- Find a path for a point in the complement $\mathcal{C}_{\text{free}}$ of $\mathcal{C}_{\text{forb}}$.
 \Rightarrow collision-free path for the robot in work space

Some Linear Algebra

Vector sums

Algebra: $(p_x, p_y) + (q_x, q_y) = (p_x + q_x, p_y + q_y)$

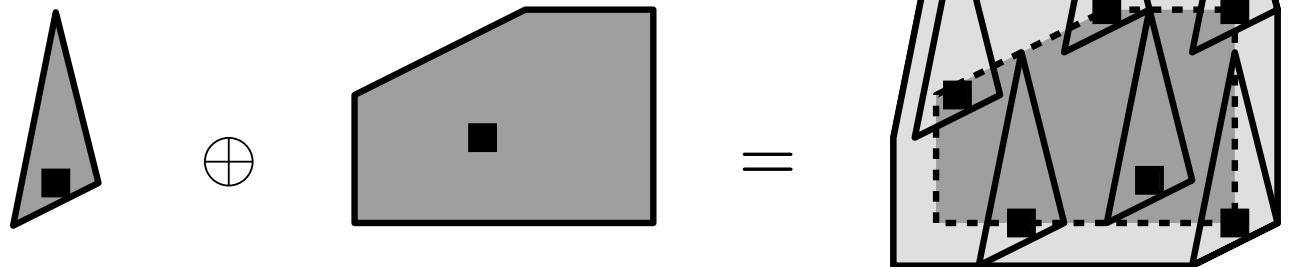
Geometry: place vectors head to tail



Minkowski sums

Algebra: $S_1 \oplus S_2 = \{p + q \mid p \in S_1, q \in S_2\}$

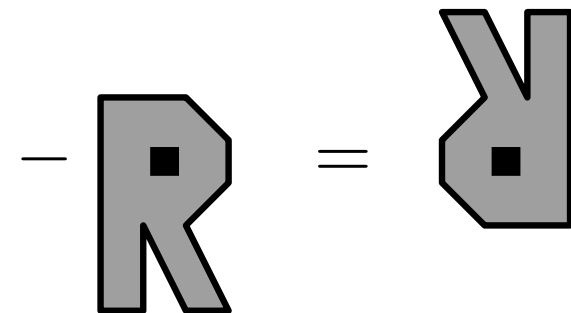
Geometry: place copy of one shape at every point of the other



Inversion

Algebra: $-S = \{-p \mid p \in S\}$

Geometry: rotate 180° (point-mirror)
around reference point



Characterizing \mathcal{CP}

Recall that $\mathcal{CP} = \{(x, y) : \mathcal{R}(x, y) \cap \mathcal{P} \neq \emptyset\}$ for an obstacle \mathcal{P} .

In other words: $\mathcal{R}(x, y)$ intersects $\mathcal{P} \iff (x, y) \in \mathcal{CP}$.

Theorem. $\mathcal{CP} = \mathcal{P} \oplus (-\mathcal{R}(0, 0))$

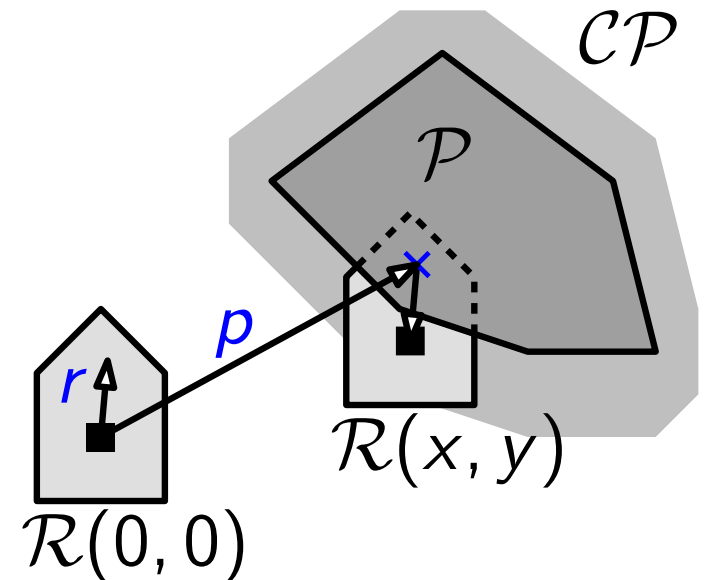
Proof. Show: $\mathcal{R}(x, y)$ intersects $\mathcal{P} \iff (x, y) \in \mathcal{P} \oplus (-\mathcal{R}(0, 0))$.

“ \Rightarrow ” Suppose $\mathcal{R}(x, y)$ intersects \mathcal{P} .

Let $q \in \mathcal{R}(x, y) \cap \mathcal{P}$. Then...

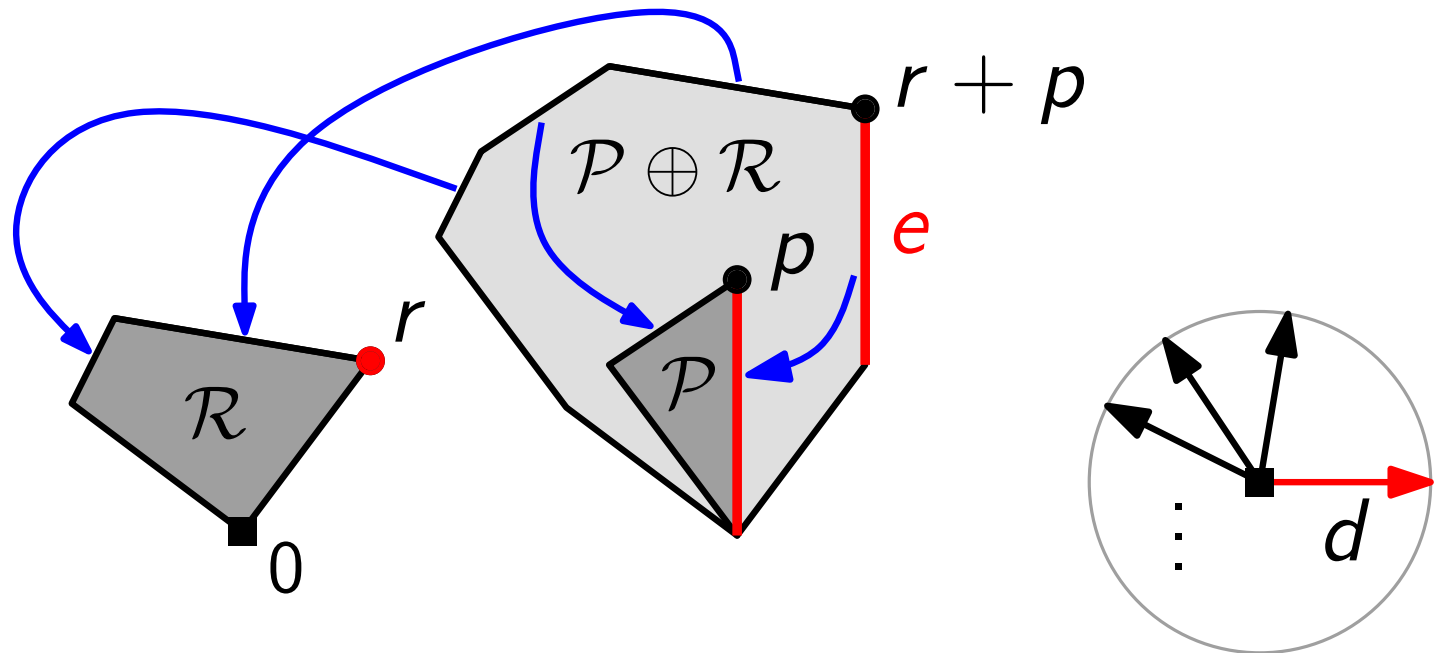
“ \Leftarrow ” Let $(x, y) \in \mathcal{P} \oplus (-\mathcal{R}(0, 0))$.

Then there are points
 $r \in \mathcal{R}(0, 0)$ and $p \in \mathcal{P}$
 such that ...



Minkowski Sums: Complexity

Theorem: If \mathcal{P} and \mathcal{R} are convex polygons with n and m edges, respectively, then $\mathcal{P} \oplus \mathcal{R}$ is a convex polygon with at most $n + m$ edges.



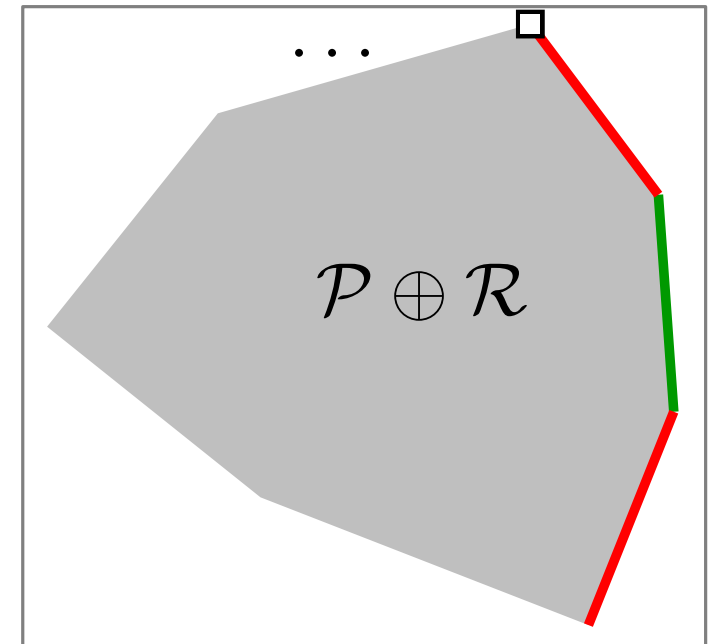
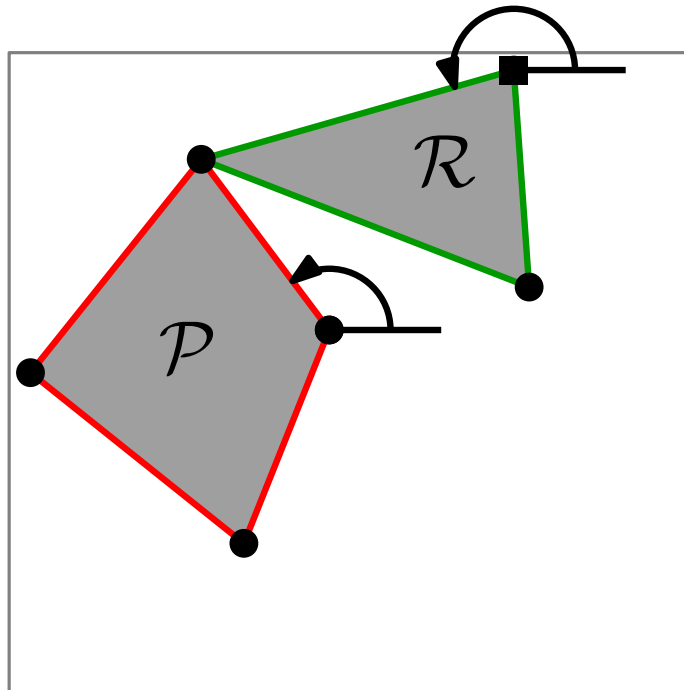
Minkowski Sums: Computation

Task: How would you compute $\mathcal{P} \oplus \mathcal{R}$ given \mathcal{P} and \mathcal{R} ?

Idea: $\mathcal{P} \oplus \mathcal{R} = \text{CH}(\underbrace{\{p + r \mid p \in \mathcal{P}, r \in \mathcal{R}\}}_{\text{Minkowski sum of vertices}})$ (Proof?)

Problem: complexity $\in \Theta(|\mathcal{P}| \cdot |\mathcal{R}|)$:-)

Theorem. The Minkowski sum of two convex polygons \mathcal{P} and \mathcal{R} can be computed in $O(|\mathcal{P}| + |\mathcal{R}|)$ time.



Einschub: Vorlesungsumfrage

Nutzen Sie die Vorlesungsbefragung – zu *konstruktiver* Kritik!

Von besseren Vorlesungen profitieren

- Sie (wenn Sie die nächste Veranstaltung bei mir hören)
- Ihre Nachfolger (im nächsten Wintersemester)
- ich (Gute Lehre macht Spaß! Gute Klausurergebnisse auch!)

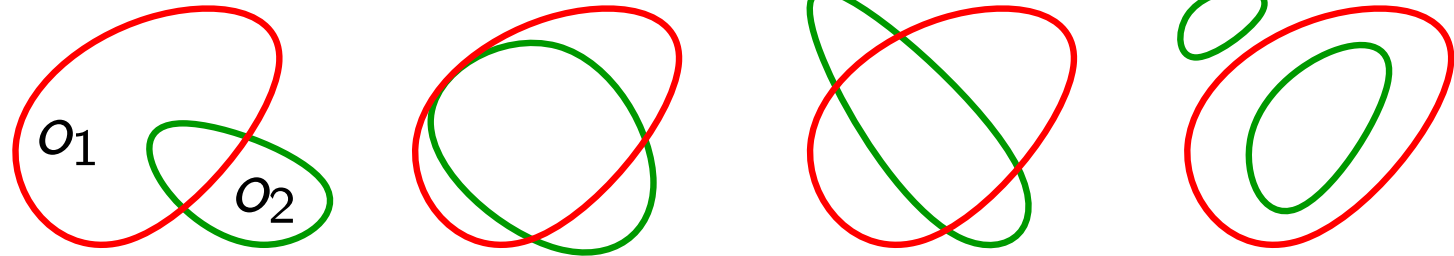


JETZT!

Vielen Dank!

*Suchen Sie in Ihrem stud-mail-Konto nach einer Email von EvaSys!
Folgen Sie dem Link und geben Sie die TAN ein.*

Pseudodisks



Definition:

A pair of planar objects o_1 and o_2 is a pair of pseudodisks if:

- $\partial o_1 \cap \text{int}(o_2)$ is connected, and
- $\partial o_2 \cap \text{int}(o_1)$ is connected.

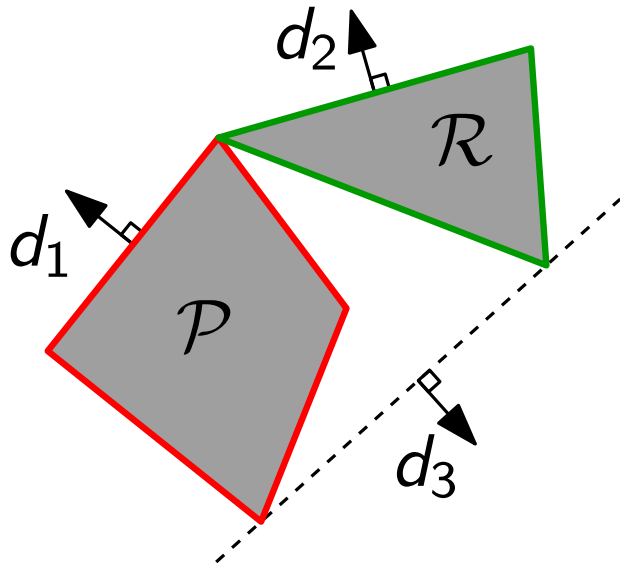
$p \in \partial o_1 \cap \partial o_2$ is a *boundary crossing* if ∂o_1 crosses at p from the interior to the exterior of o_2 .

Observation:

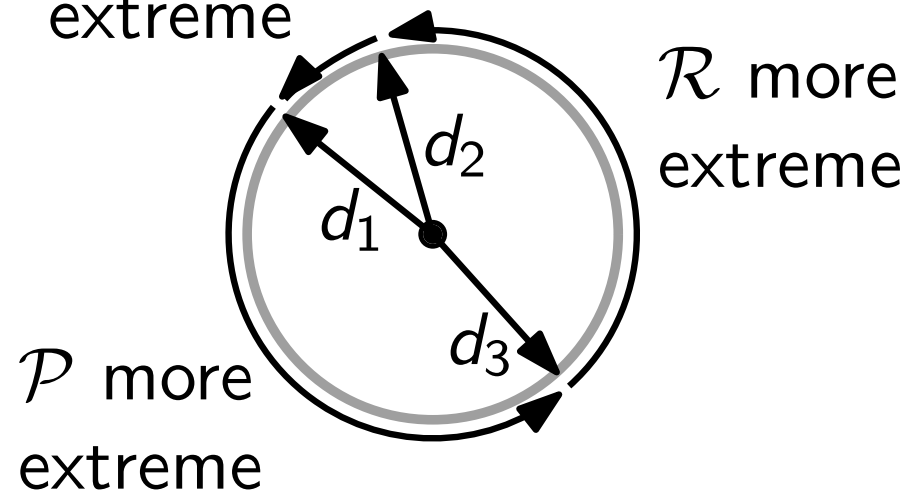
A pair of polygonal pseudodisks defines at most two boundary crossings.

Extreme directions

Observation: Let \mathcal{P} , \mathcal{R} be interior-disjoint convex polygons. Let d_1 and d_2 be directions in which \mathcal{P} is more extreme than \mathcal{R} . Then \mathcal{P} is more extreme than \mathcal{R} either in $[d_1, d_2]$ or in $[d_2, d_1]$.



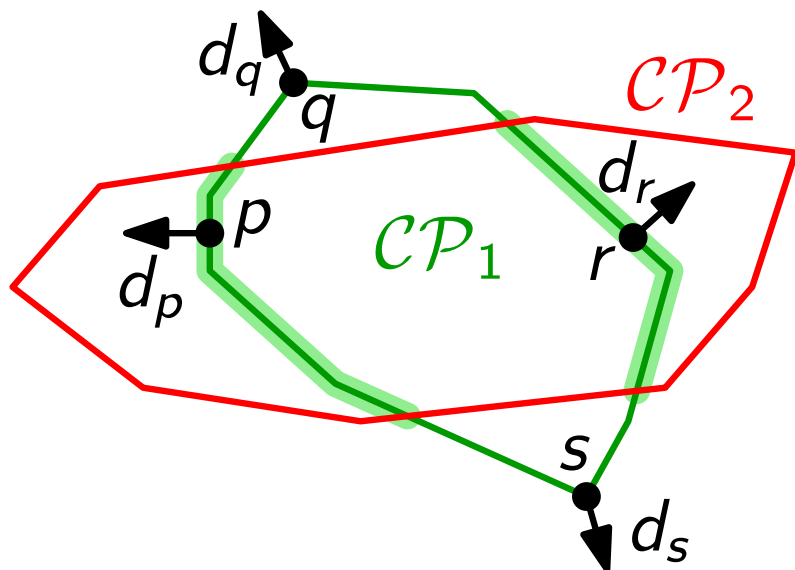
\mathcal{P} and \mathcal{R}
equally
extreme



Polygonal Pseudodisks

Theorem: If \mathcal{P}_1 and \mathcal{P}_2 are convex polygons with disjoint interiors, and \mathcal{R} is another convex polygon, then $\underbrace{\mathcal{P}_1 \oplus \mathcal{R}}_{\mathcal{CP}_1}$ and $\underbrace{\mathcal{P}_2 \oplus \mathcal{R}}_{\mathcal{CP}_2}$ is a pair of pseudodisks.

Proof. It suffices to show: $\partial\mathcal{CP}_1 \cap \text{int } \mathcal{CP}_2$ is connected.
Suppose $\partial\mathcal{CP}_1 \cap \text{int } \mathcal{CP}_2$ is not connected...



⚡ to previous observation!

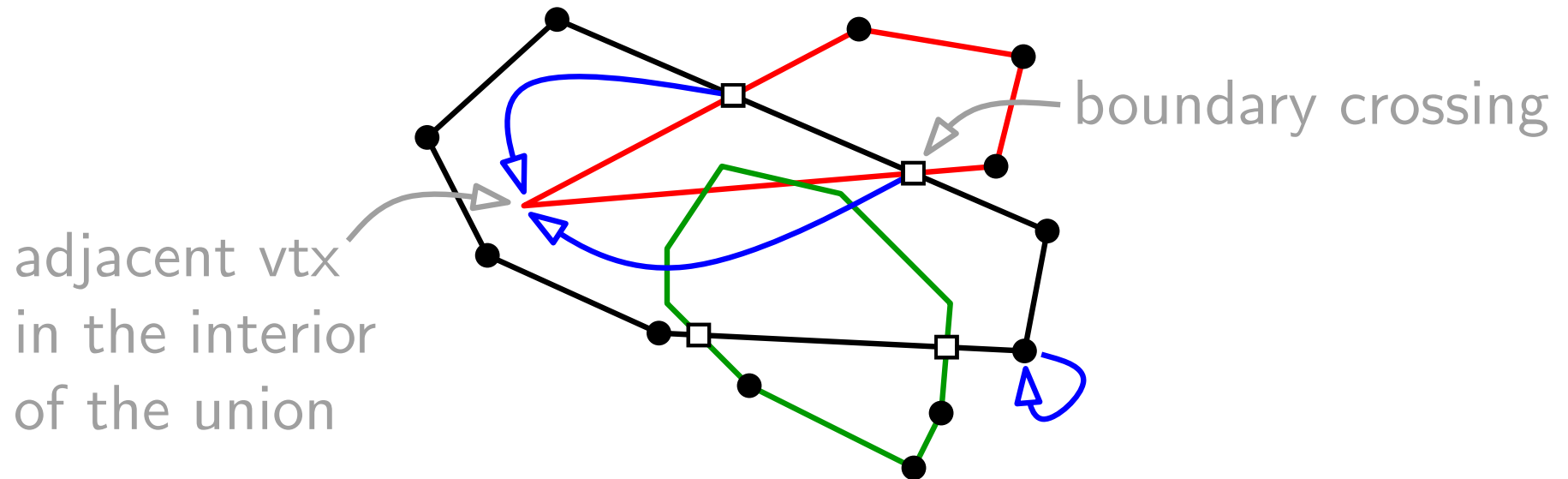
since

- d_q and d_s are also extreme for \mathcal{P}_1 and
- d_p and d_r are also extreme for \mathcal{P}_2 .

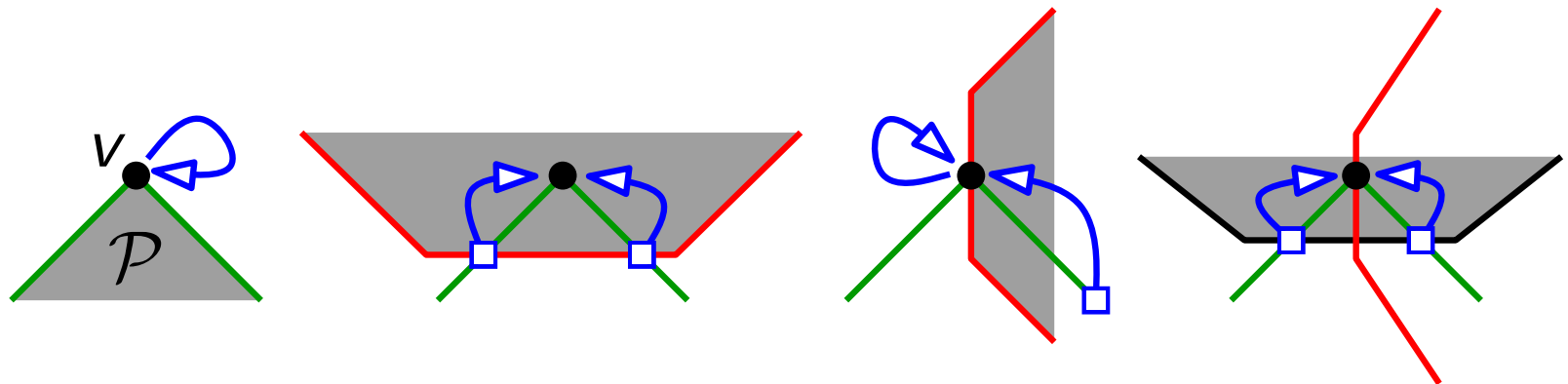
(and \mathcal{P}_1 and \mathcal{P}_2 are convex and interior-disjoint).

Union Complexity

Theorem: A collection S of convex polygonal pseudodiscs with n vtx in total has a union with $\leq 2n$ vtx.



Proof. Charge every vtx of the union to a polygon vtx s.t. every polygon vtx is charged at most twice.



Summary and Main Result

Theorem: Let \mathcal{R} be a constant-complexity convex robot, translating among a set S of disjoint polygonal obstacles with n edges in total.

We can preprocess S in $O(n \log^2 n)$ time such that, given any start and goal position, we can compute in $O(n)$ time a collision-free path for \mathcal{R} if it exists.

Proof.

- Triangulate the obstacles if they're not convex.
- Compute \mathcal{CP}_i for every convex obstacle \mathcal{P}_i .
- Compute their union $\mathcal{C}_{\text{forb}} = \bigcup_i \mathcal{CP}_i$
using divide and conquer (merge by sweeping)
[Argue carefully about the number of intersection pts!]
- Find a path for a point in the complement $\mathcal{C}_{\text{free}}$.