Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems

Institute for Informatics

Computational Geometry

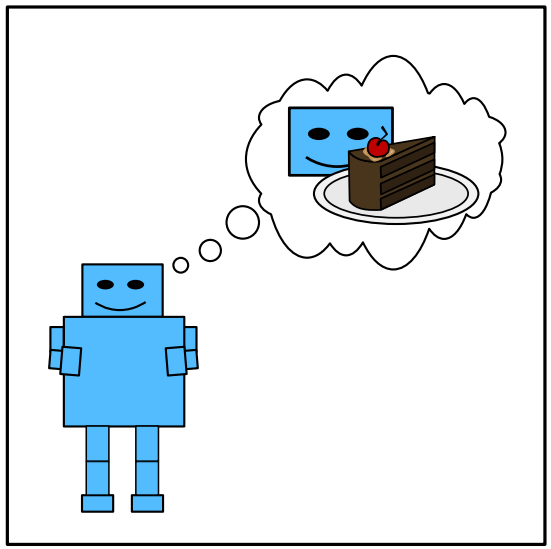
Winter semester 2014/15

Motion Planning

Lecture #10

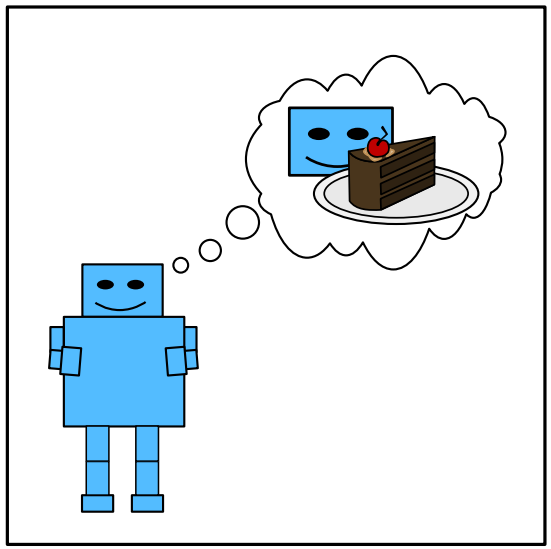
Prof. Dr. Alexander Wolff *Chair for Computer Science I*

Planning



current situation,
desired situation

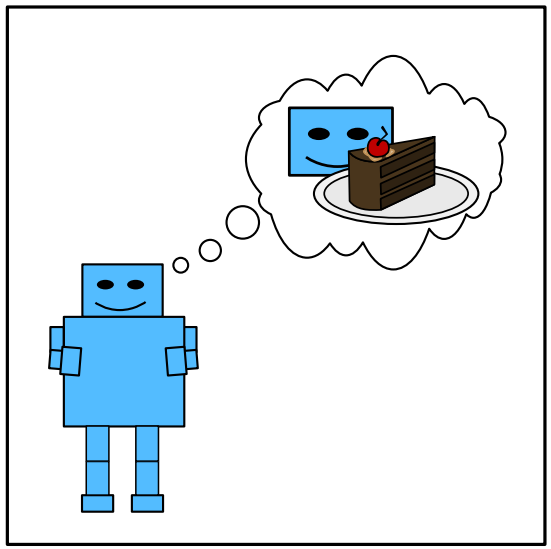
Planning



current situation,
desired situation

$\Rightarrow$

Planning



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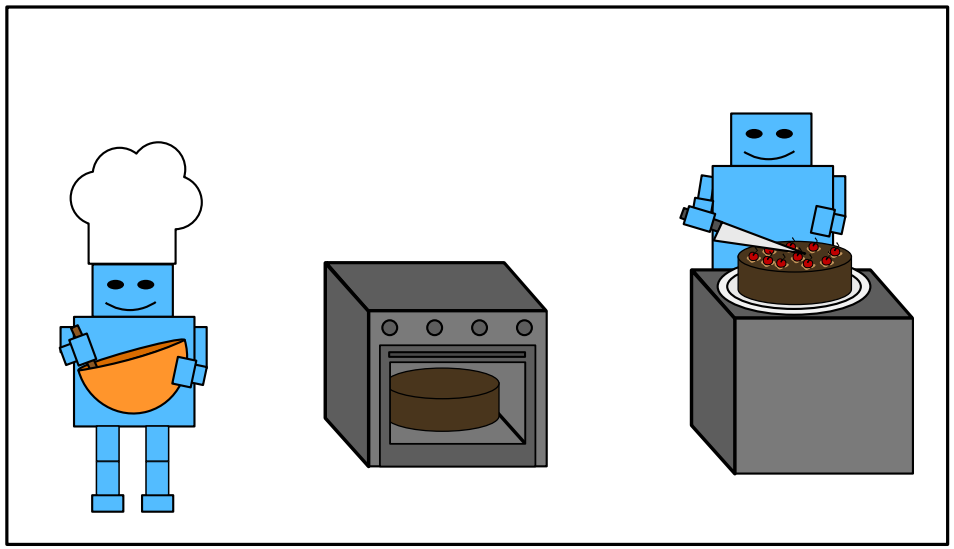
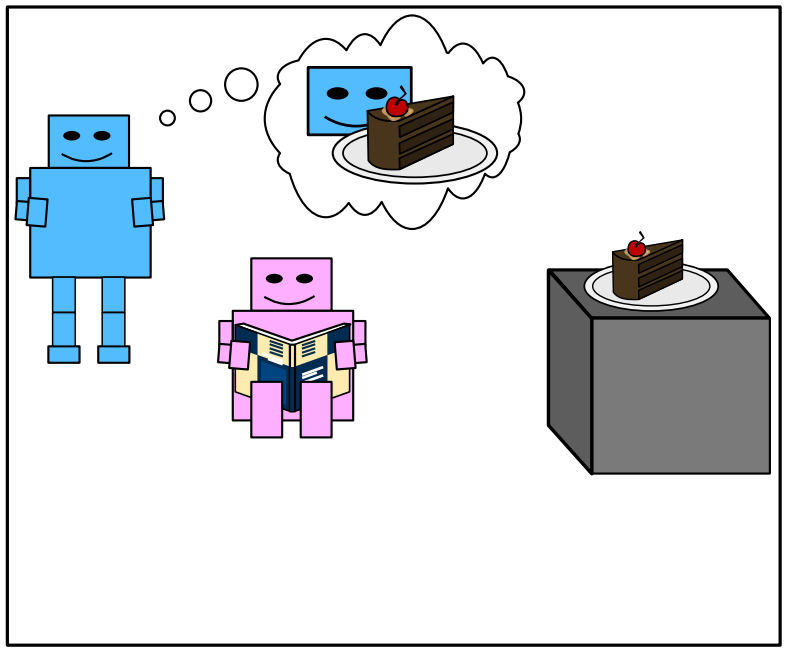


sequence of steps to reach
the one from the other

Path Planning



current location,
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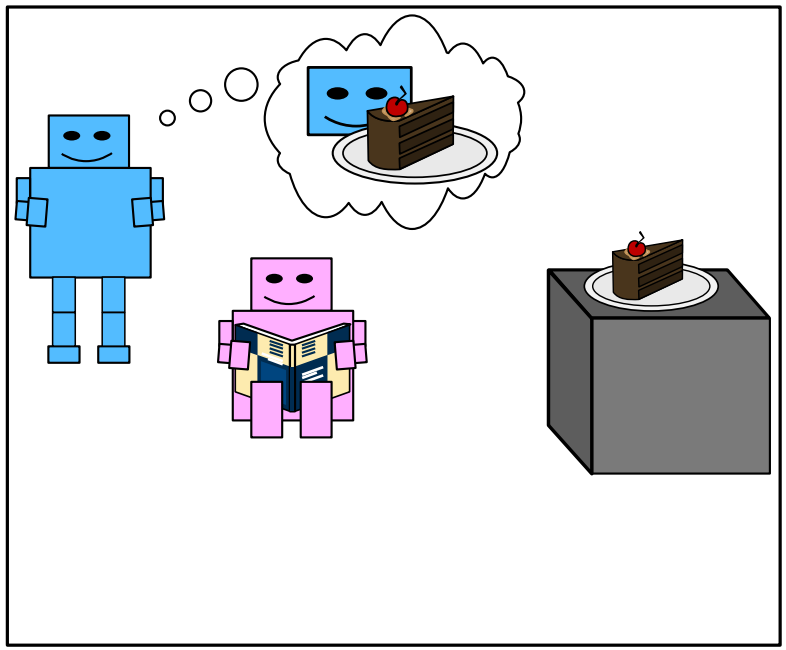
Path Planning



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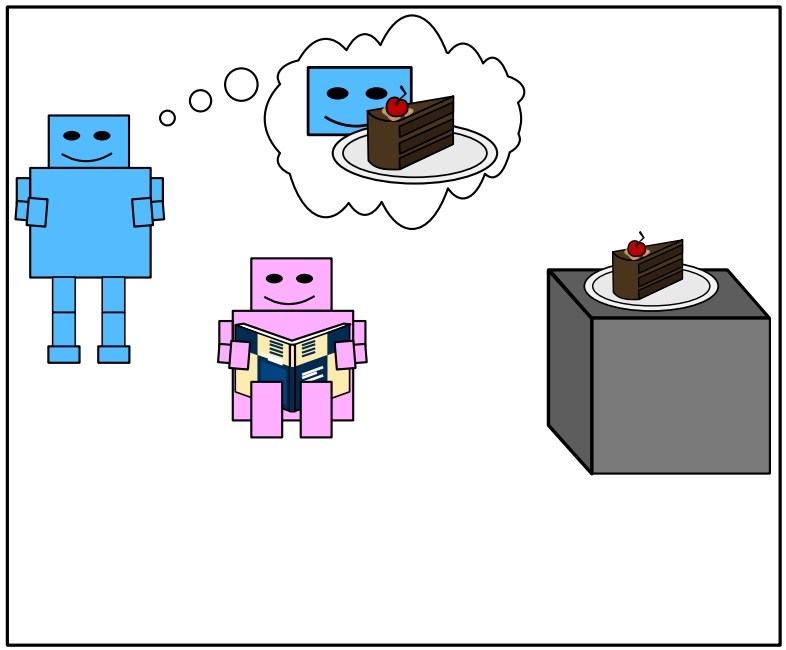
Path Planning



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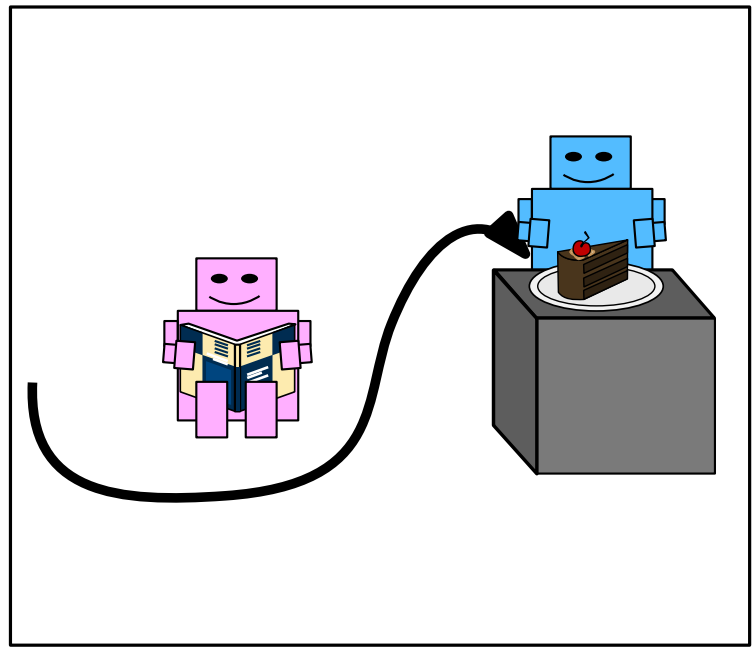
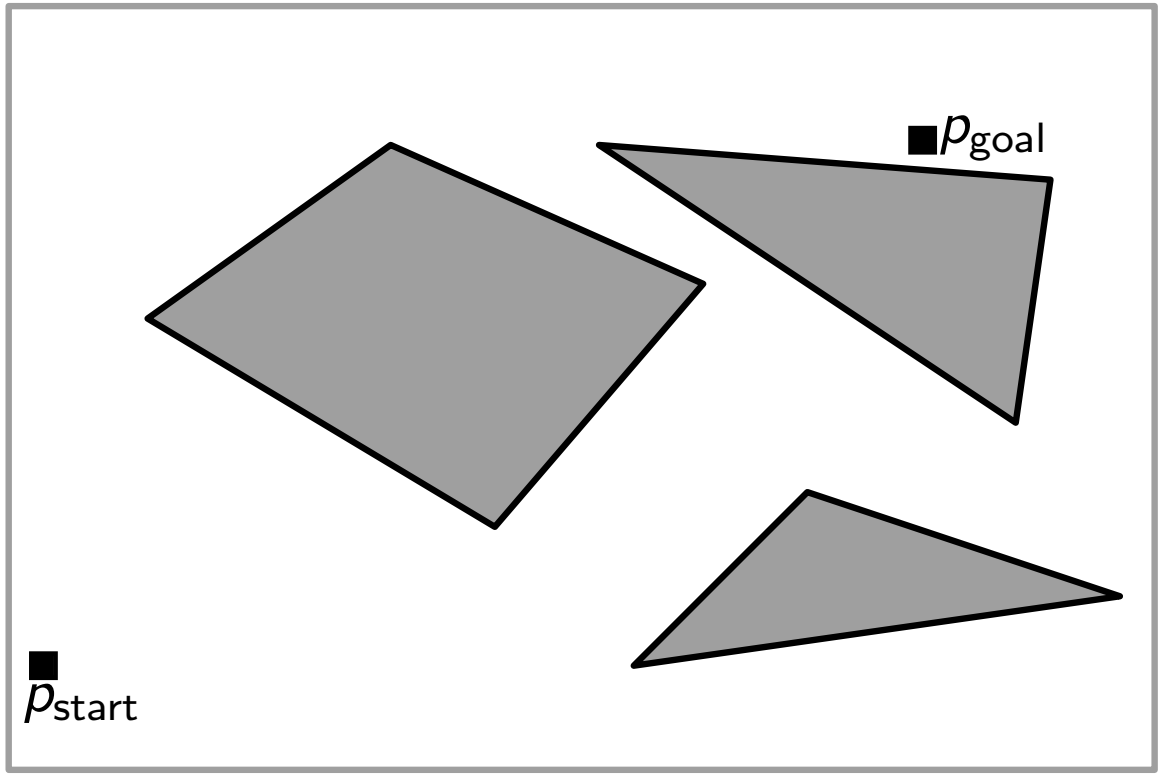


path to reach the one
from the other

Point-Shaped Robots



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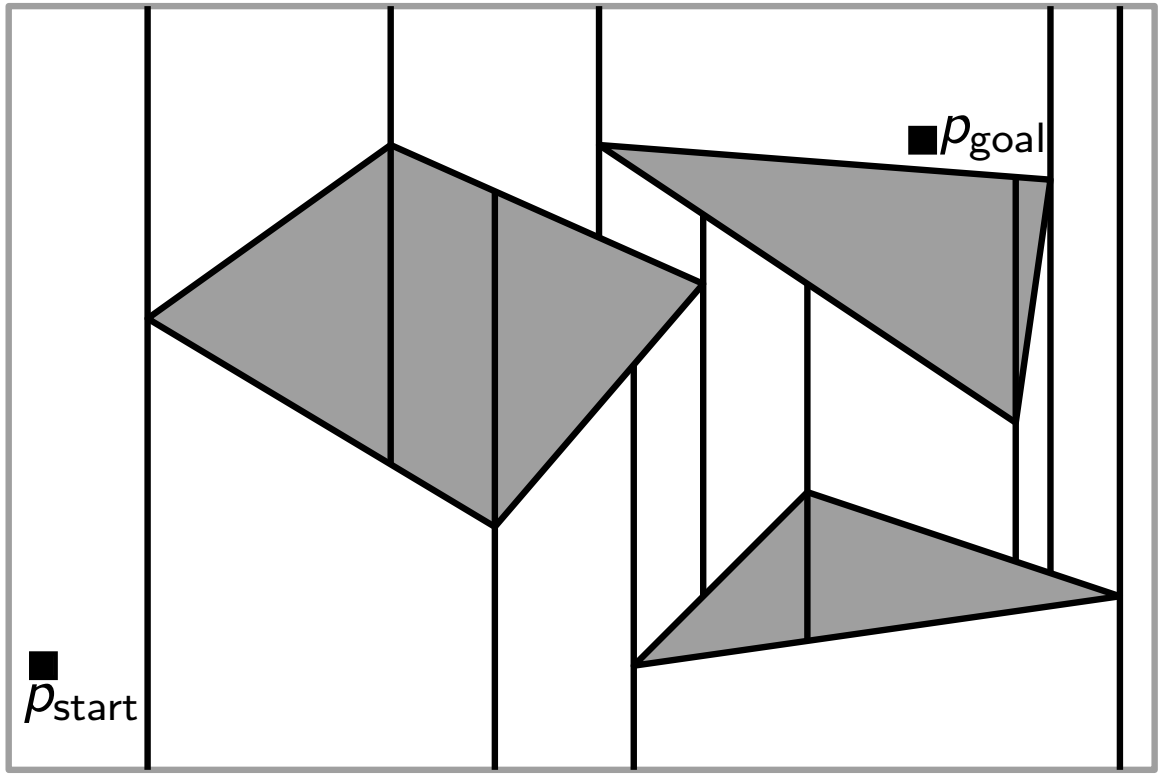
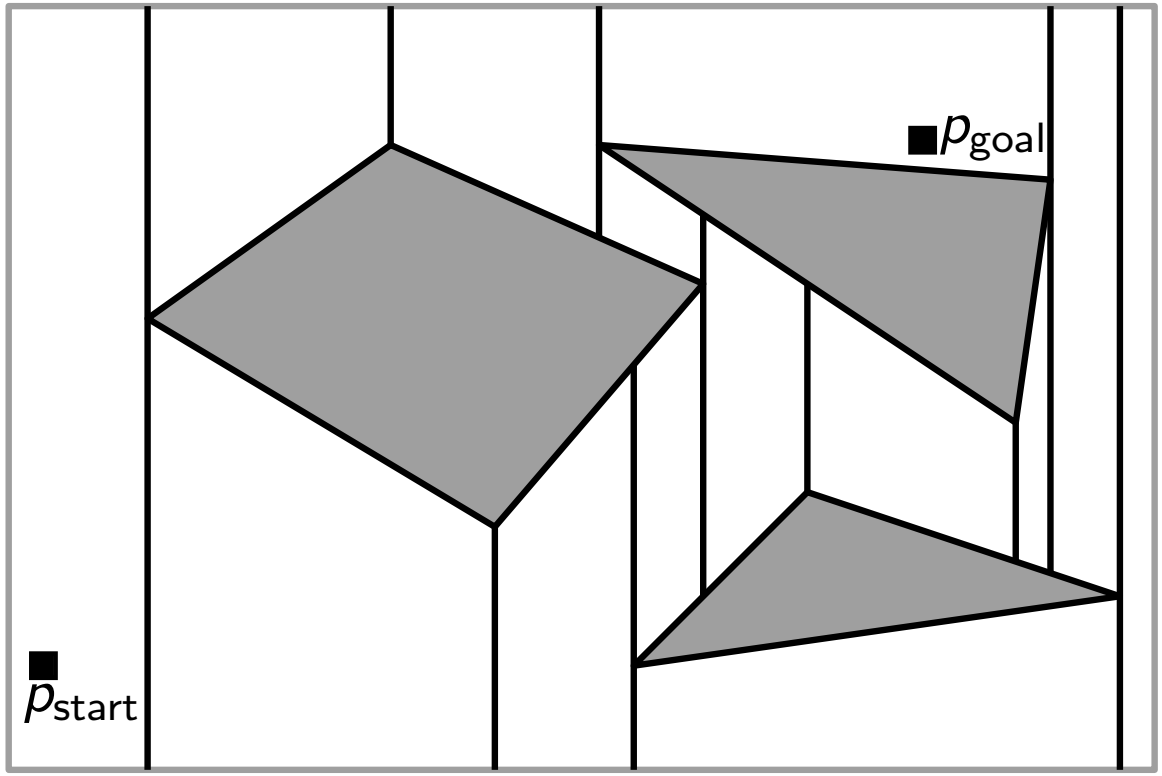


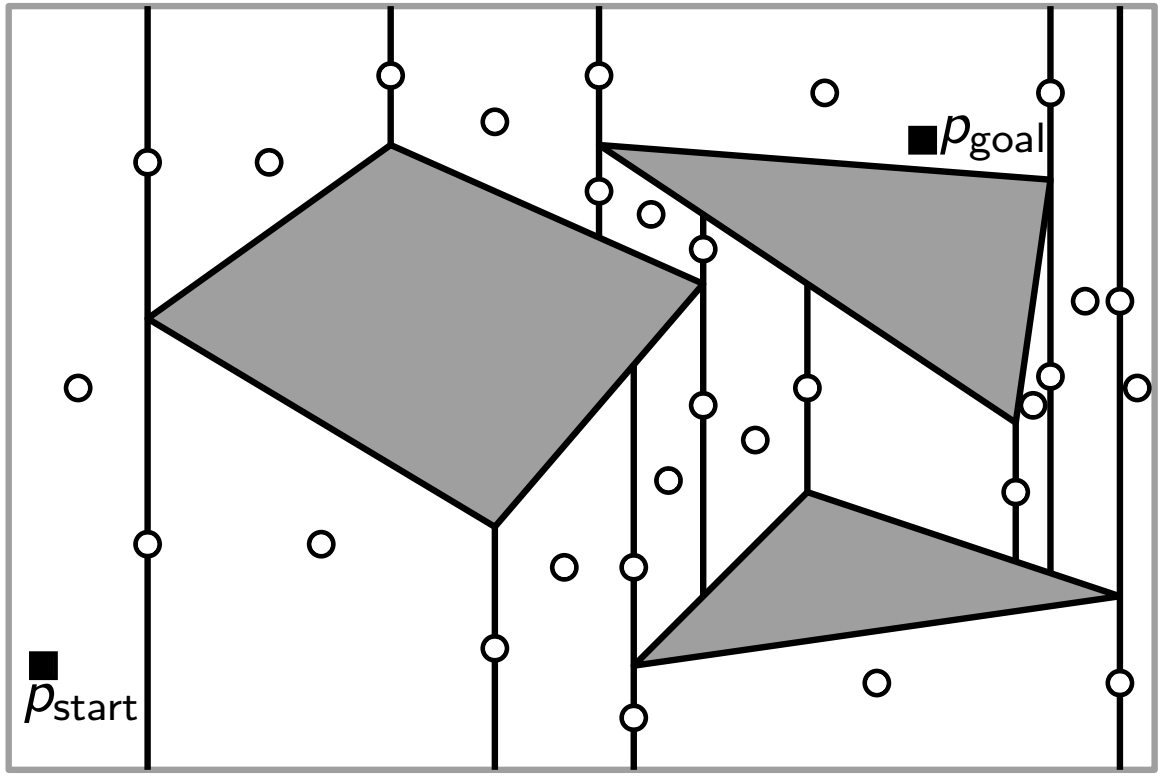
- Create trapezoidal map of obstacle edges.

Point-Shaped Robots



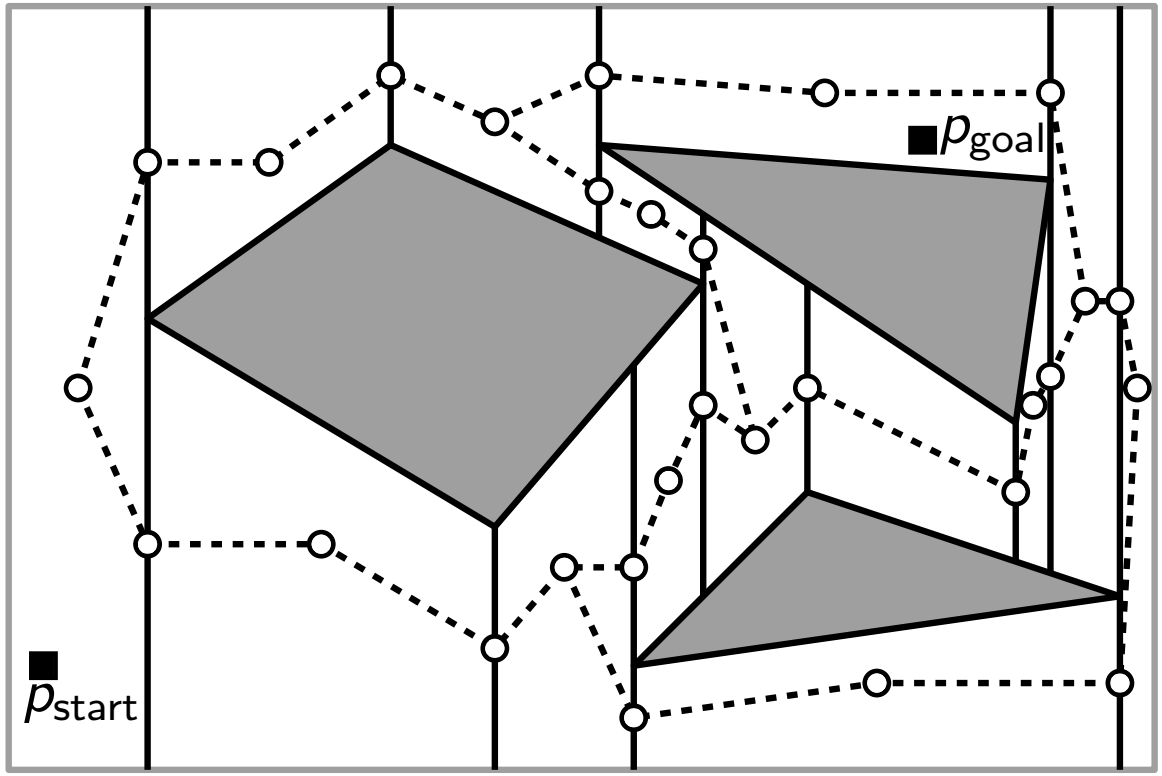
- Create trapezoidal map of obstacle edges.
- Remove vertical extensions inside obstacles.

Point-Shaped Robots



- Create trapezoidal map of obstacle edges.
- Remove vertical extensions inside obstacles.
- Vertices at centers of trapezoids and vertical ext.

Point-Shaped Robots



- Create trapezoidal map of obstacle edges.
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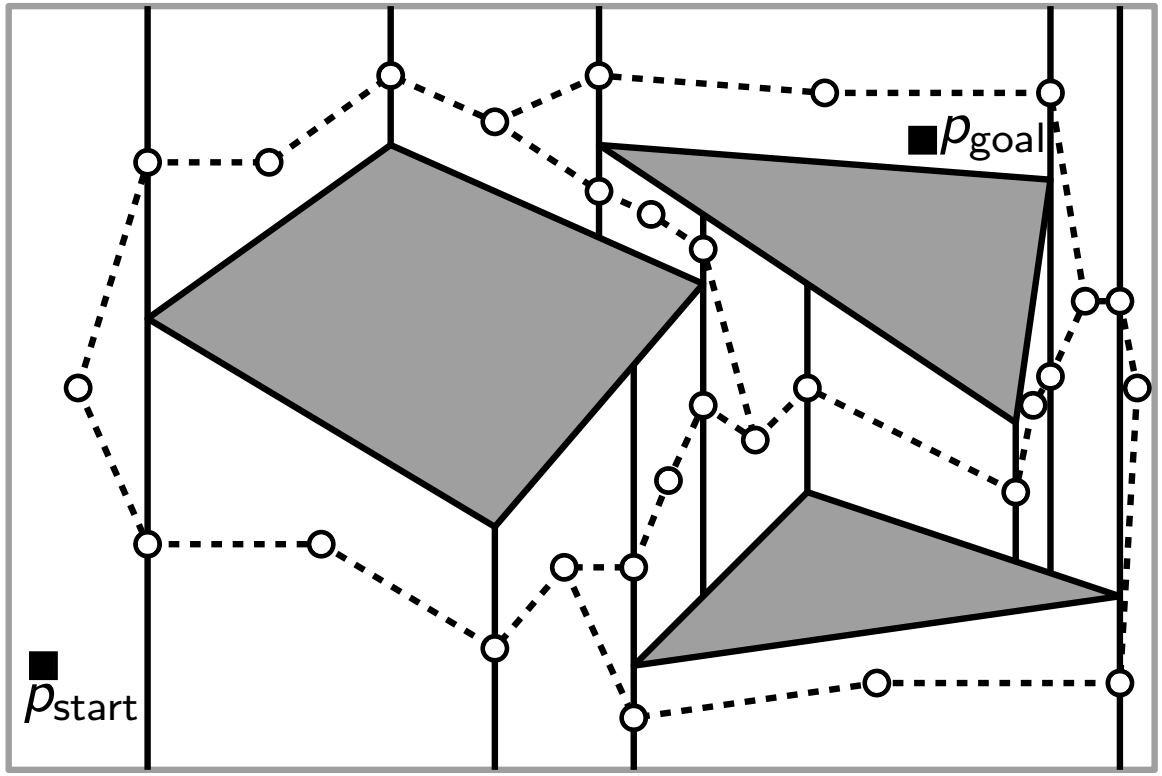

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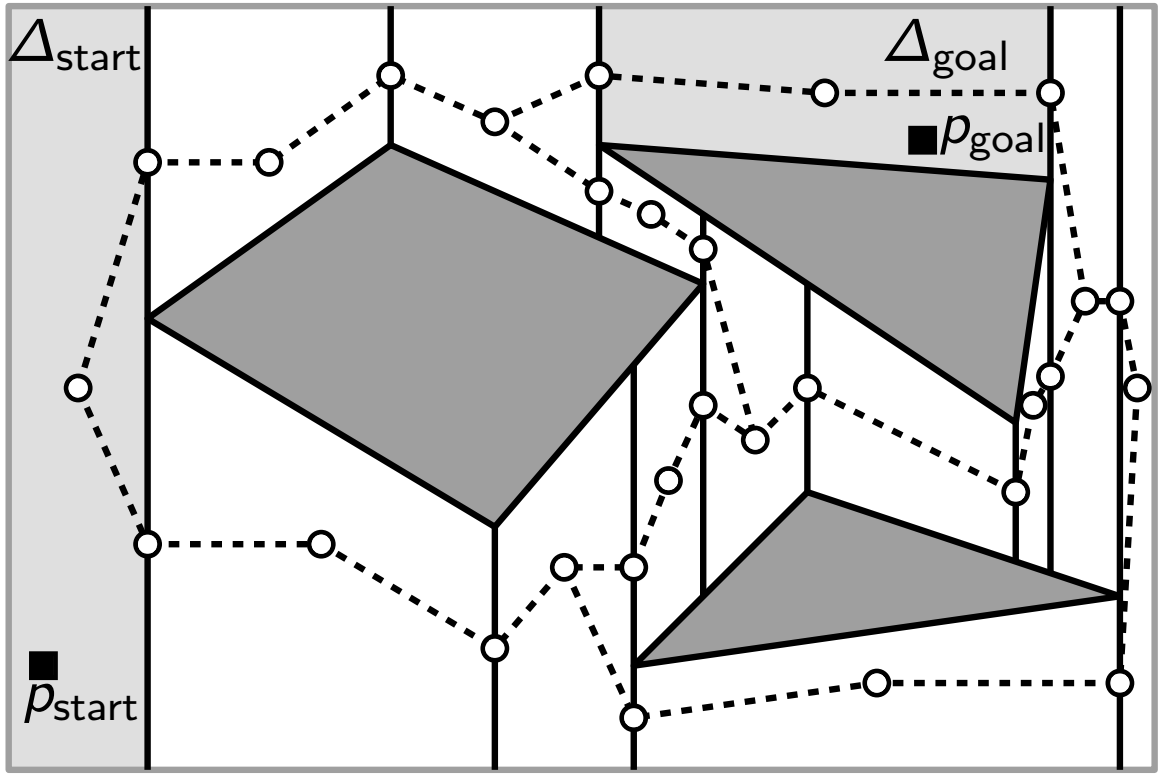
- Locate p_{start}, p_{goal} in map

Point-Shaped Robots



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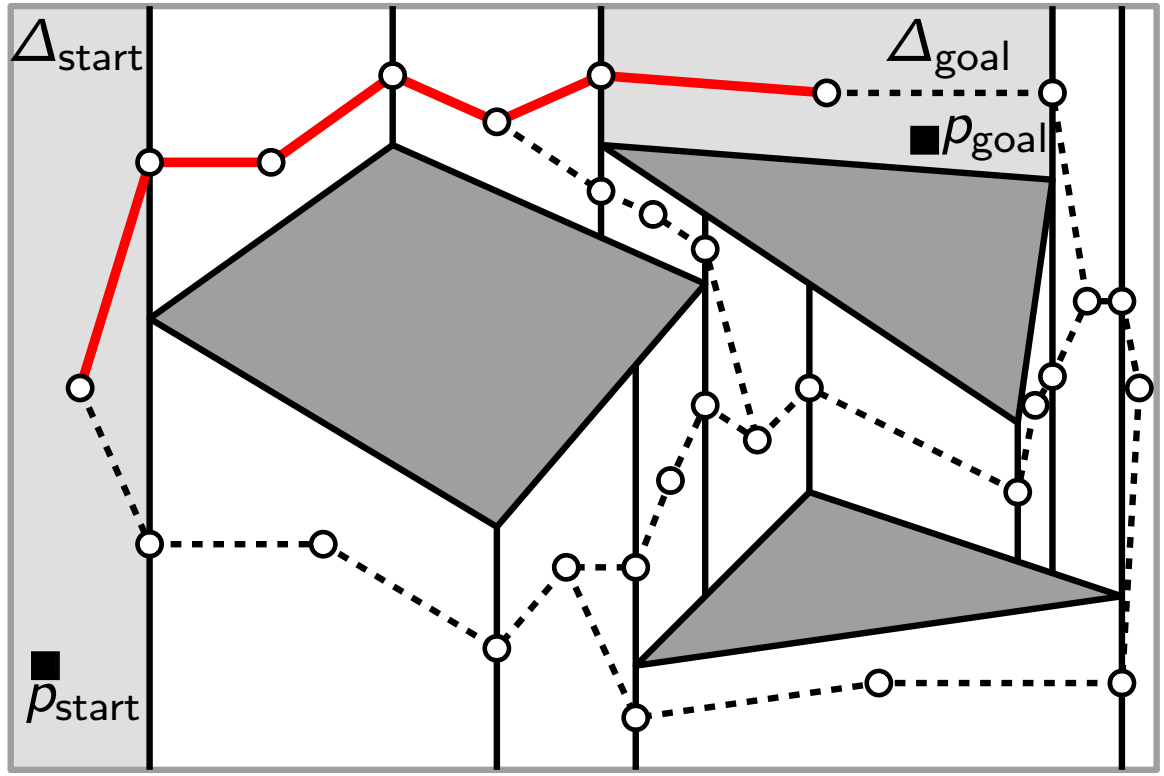

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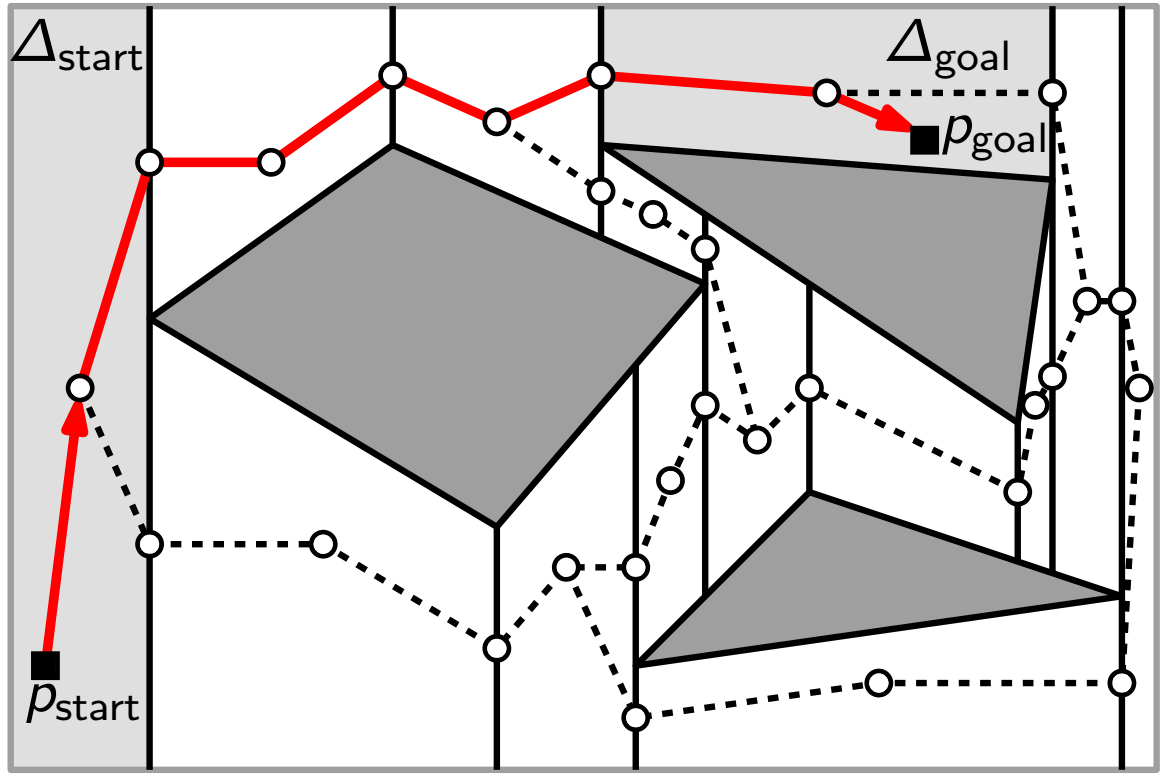
- Do breadth-first search in the *roadmap* to find a path π from Δ_{start} to Δ_{goal}.

Point-Shaped Robots



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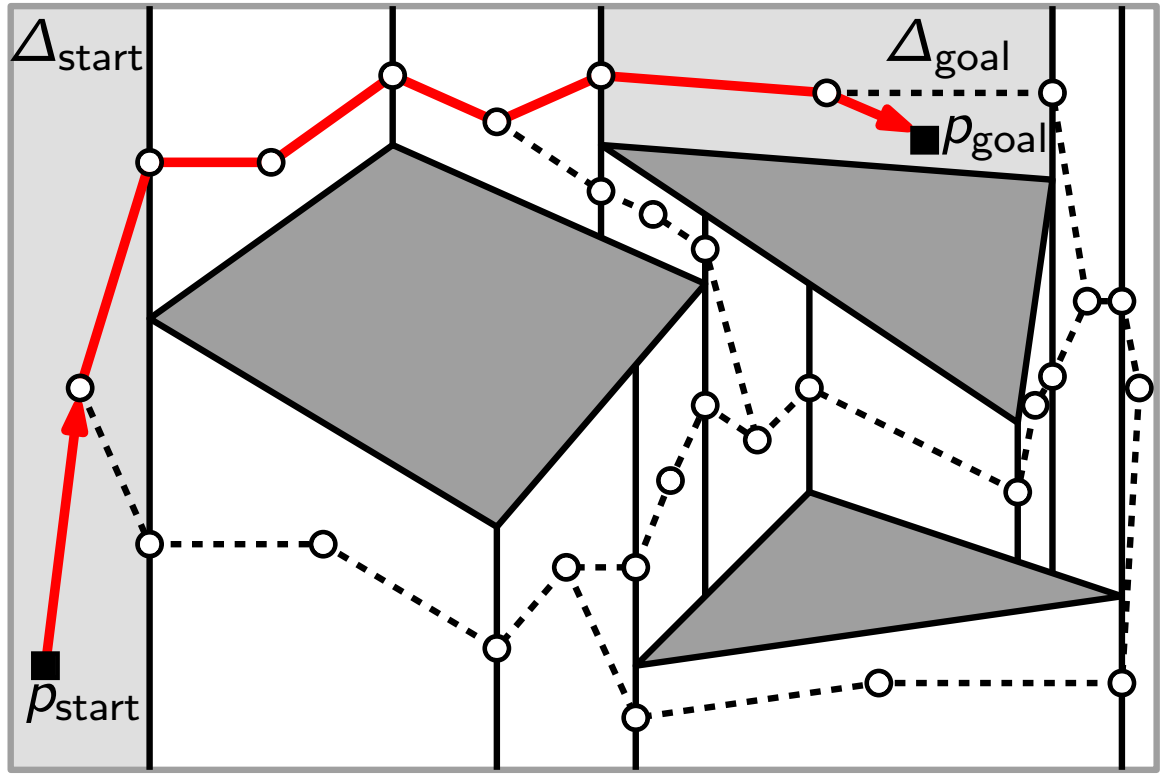

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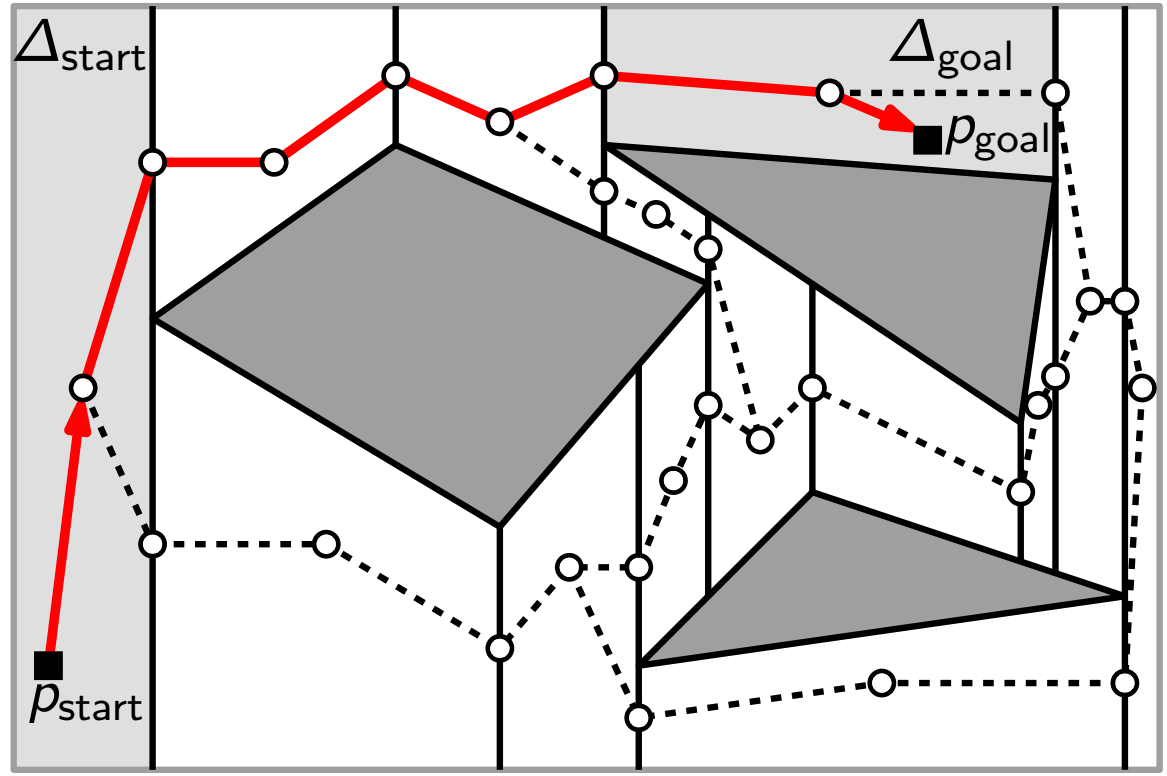

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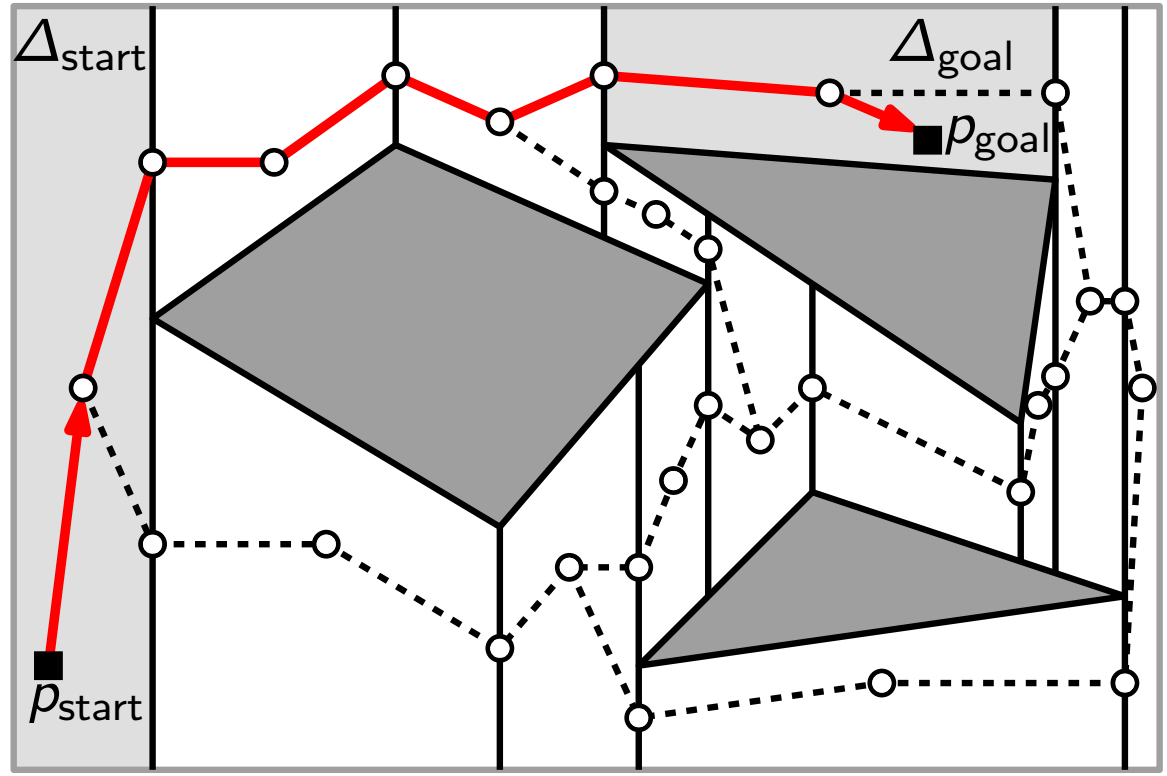

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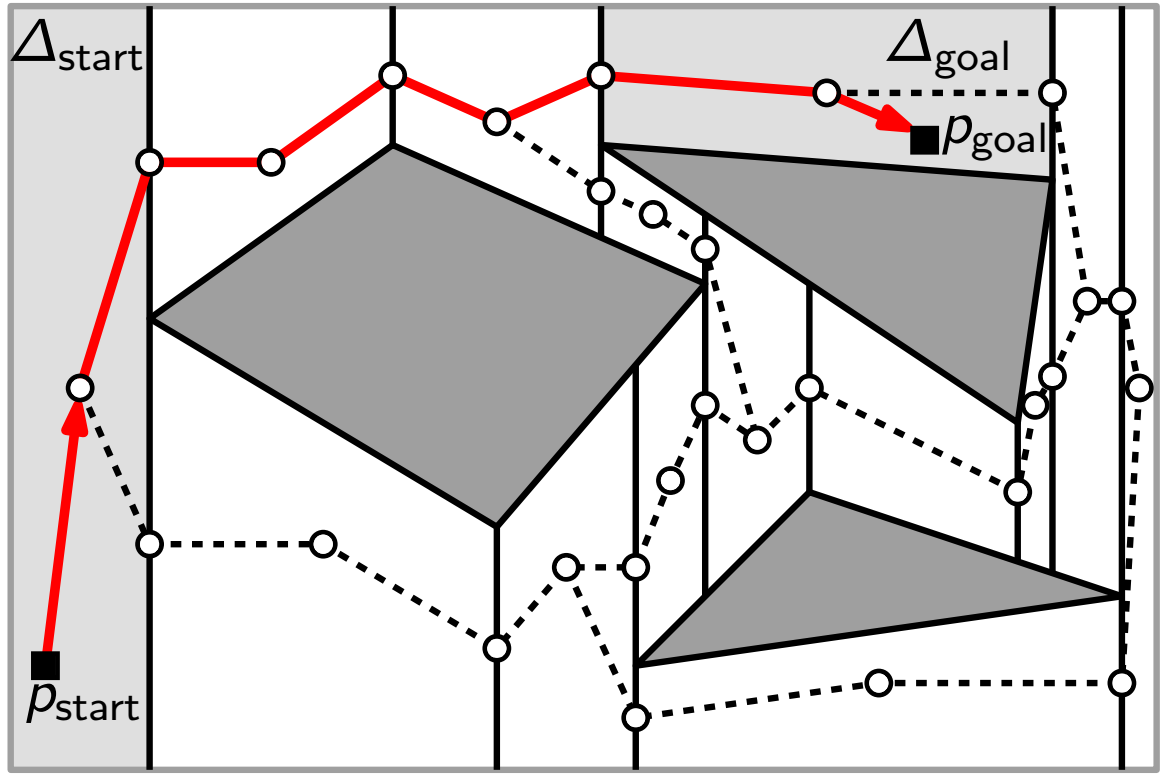

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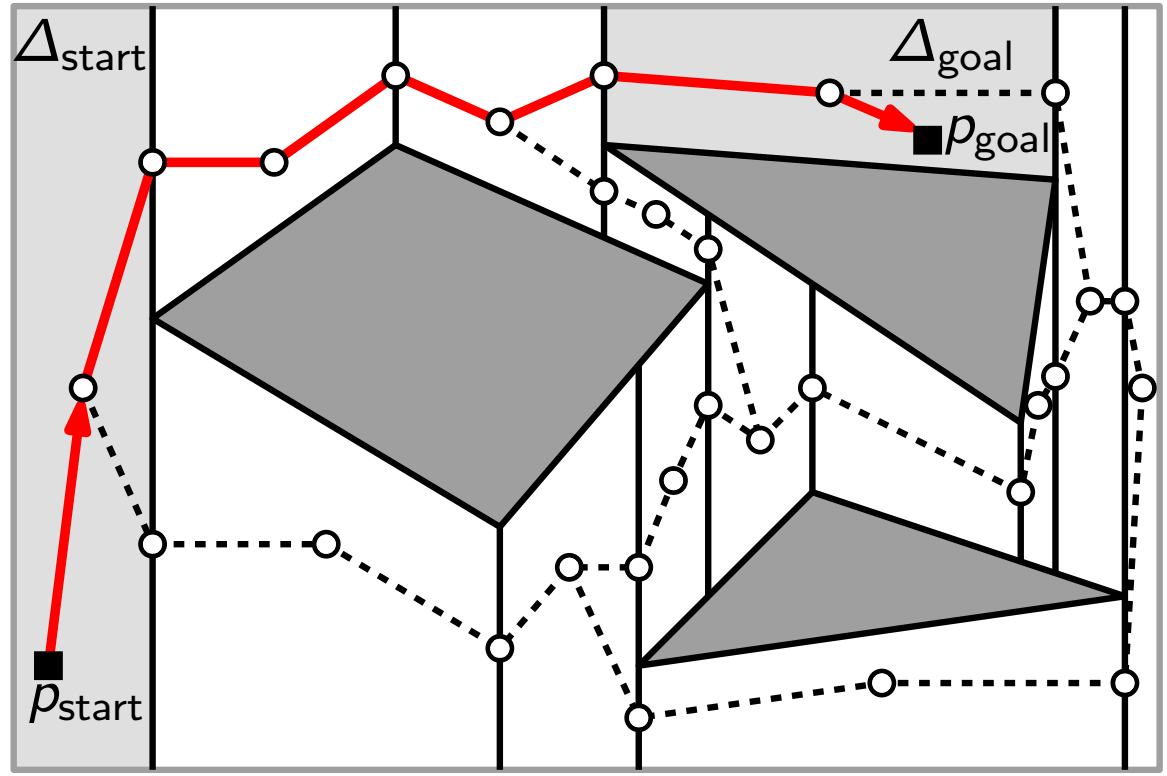

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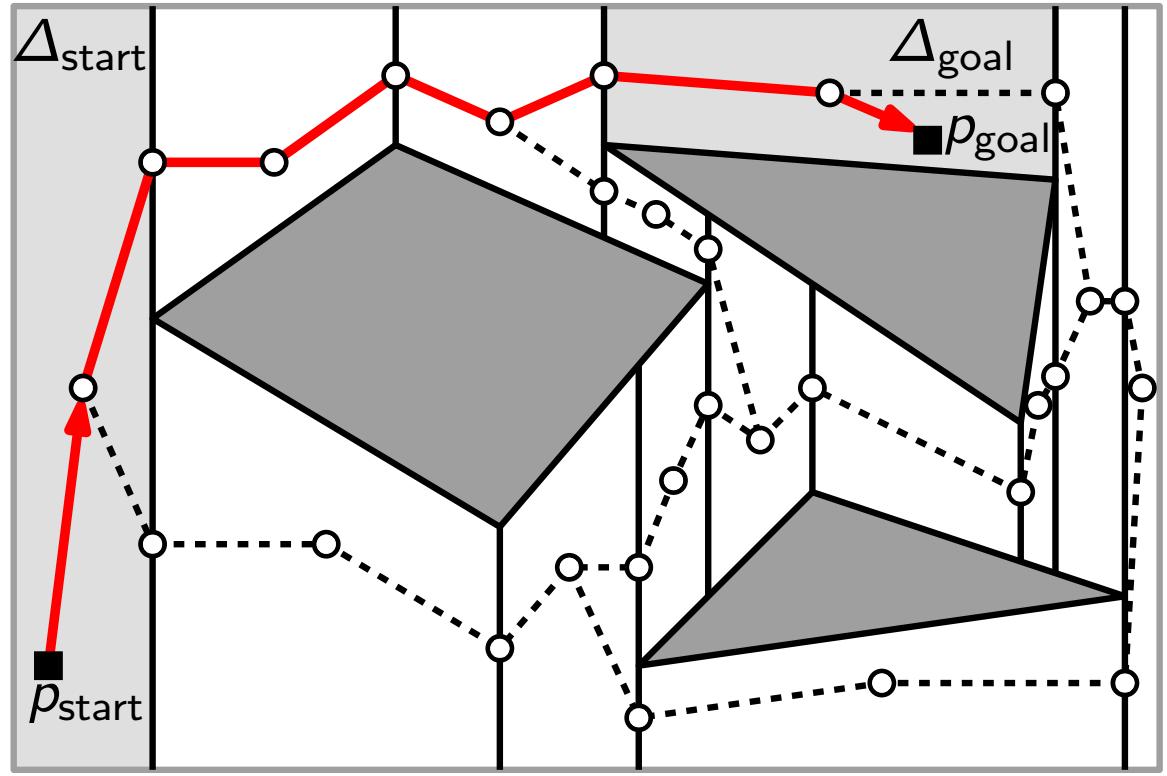

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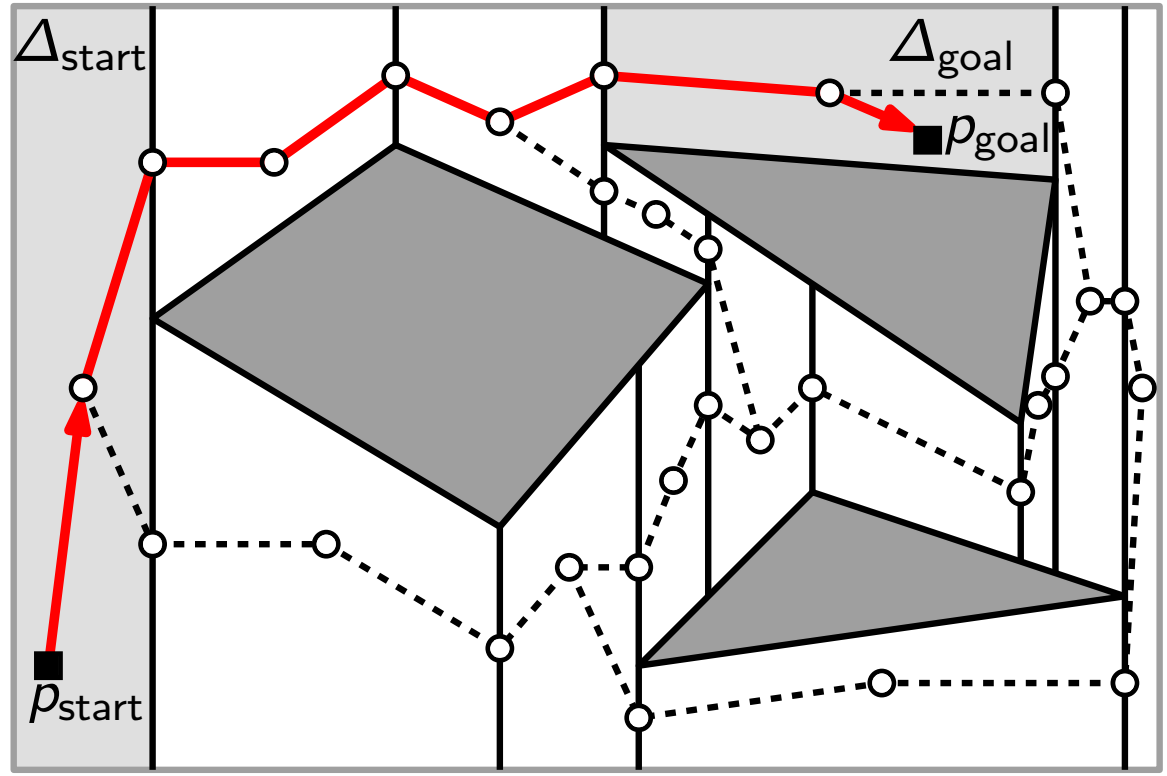

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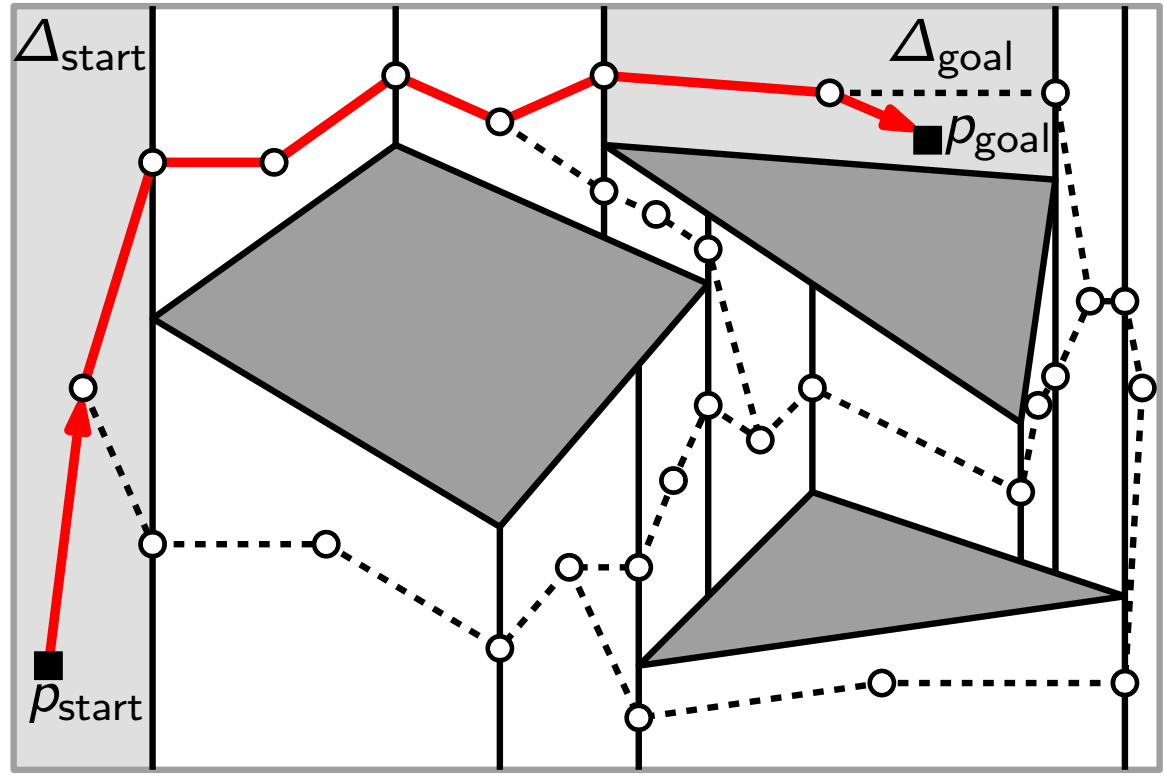

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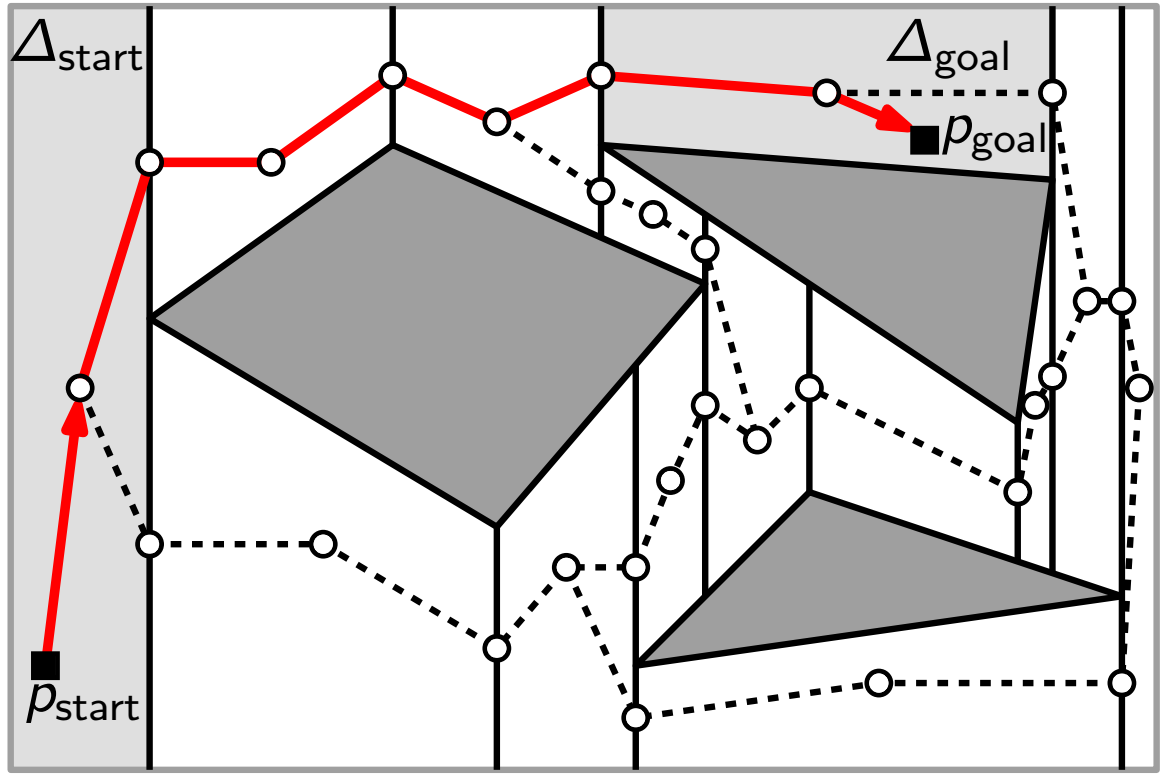

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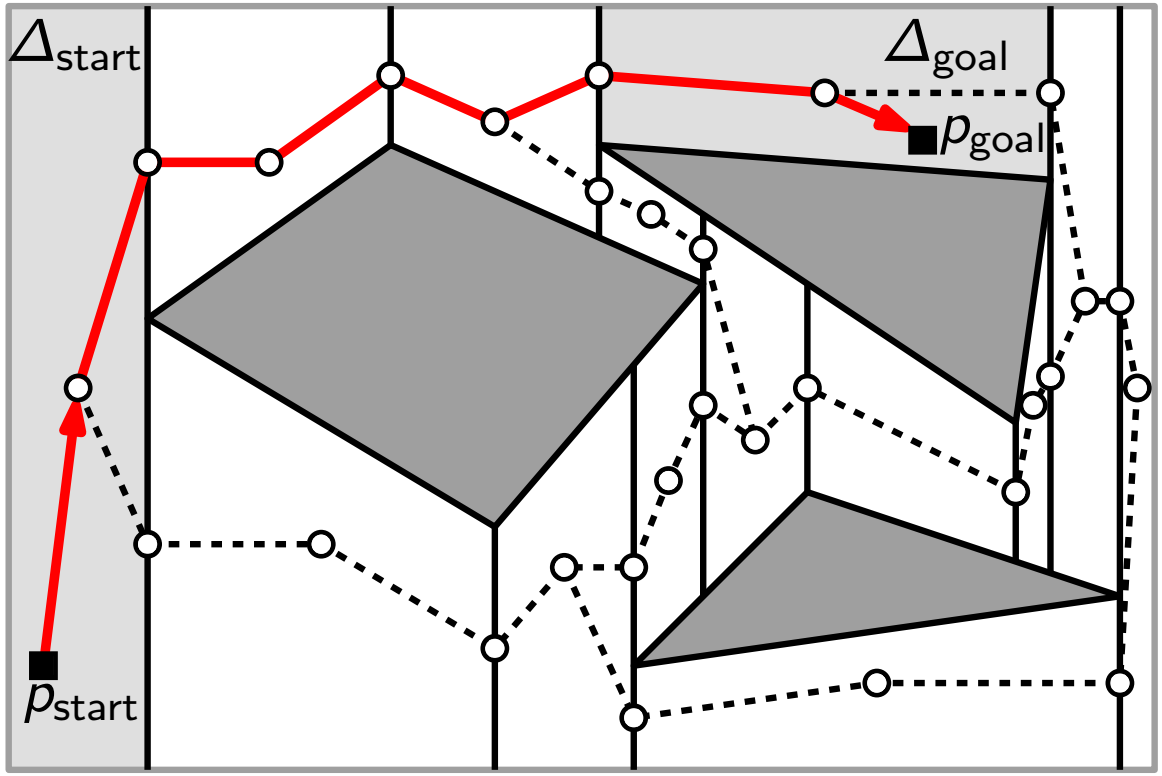

preprocessing	Create trapezoidal map of obstacle edges.	$O(n \log n)$
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A First Result

Theorem: We can preprocess a set of polygonal obstacles with a total of n edges in $O(n \log n)$ expected time such that, given a start and a goal position, we can find a collision-free path for a point robot in $O(n)$ time if it exists.

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What about, say, *polygonal* robots?

Degrees of Freedom

Every robot has some number d of *degrees of freedom*, meaning that its *configuration* with respect to the world can be specified by d parameters.

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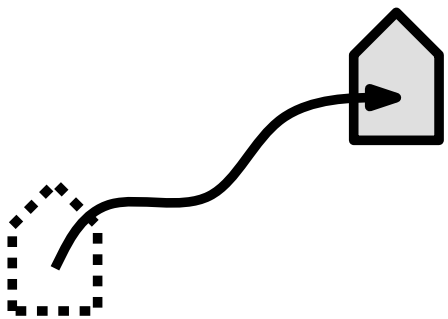

2D translating robot

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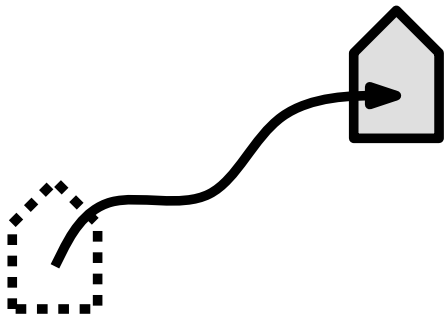
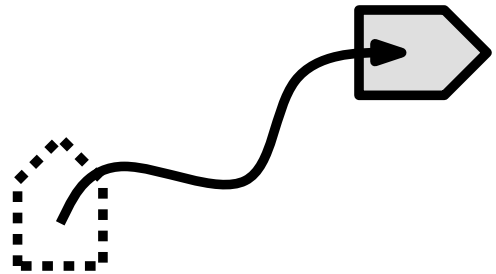


2D translating robot



2D translating, rotating robot

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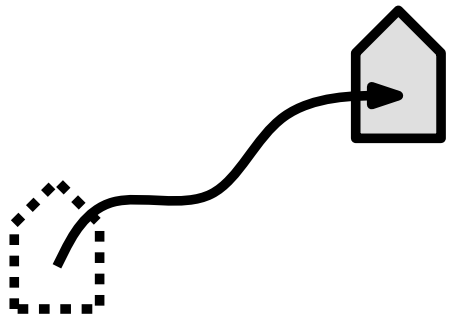
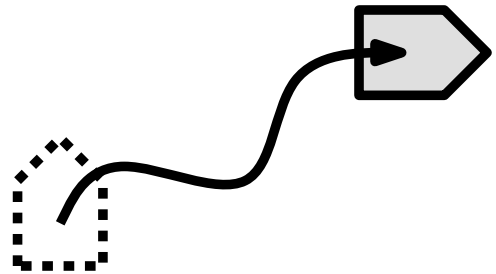
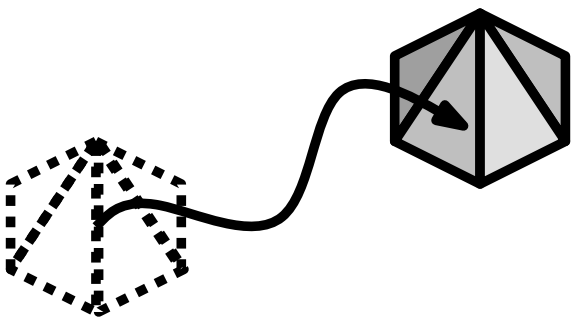


2D translating robot



2D translating, rotating robot



3D translating robot

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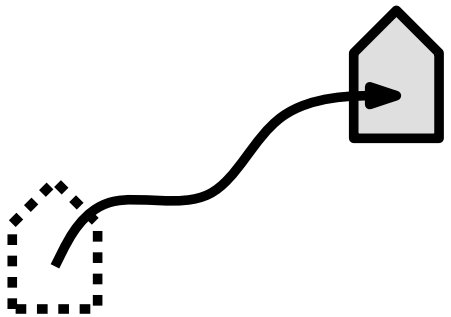
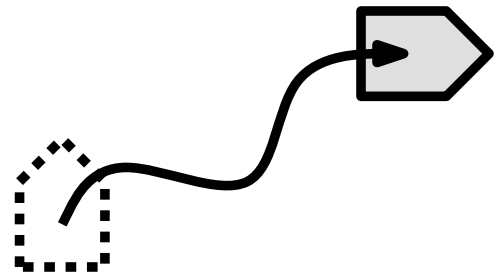
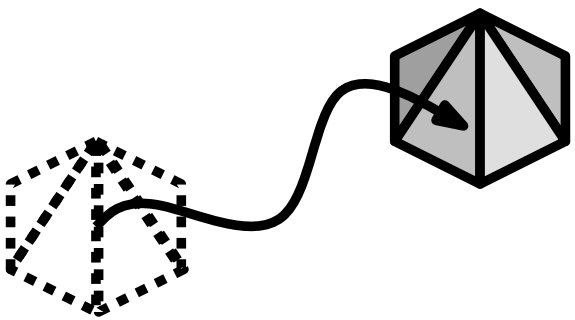


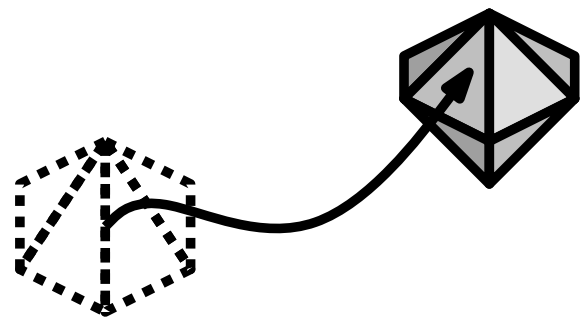
2D translating robot



2D translating, rotating robot

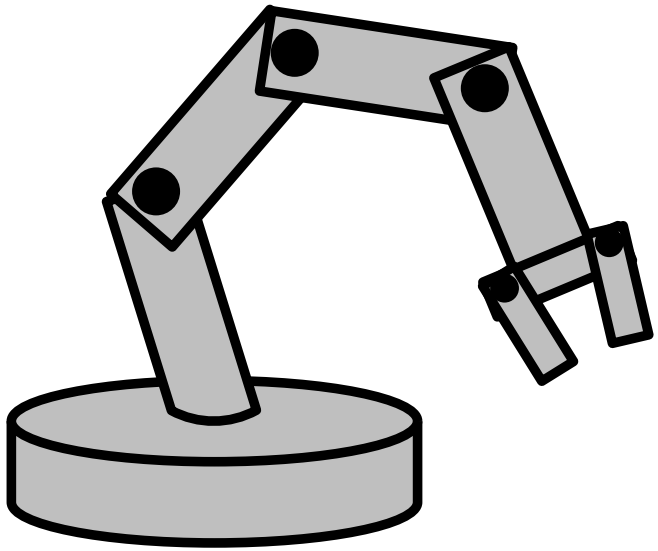


3D translating robot



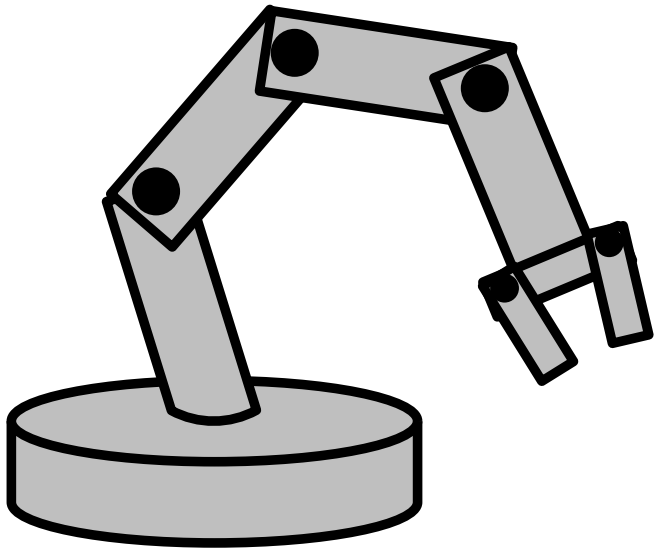
3D translating, rotating robot

Configuration Space



robotic arm

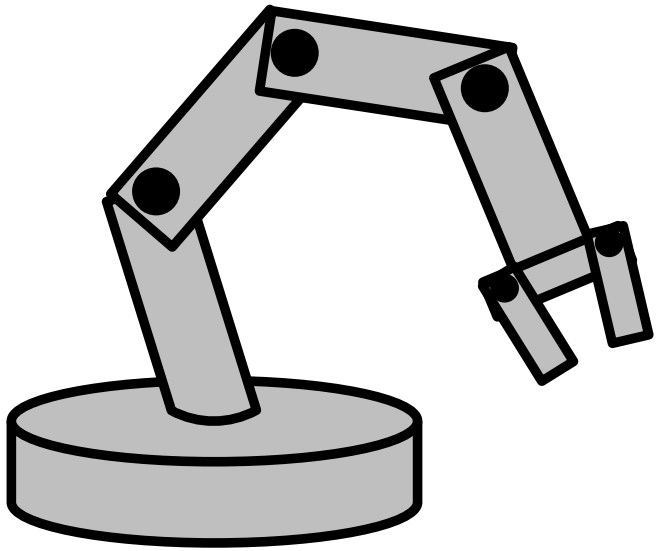
Configuration Space



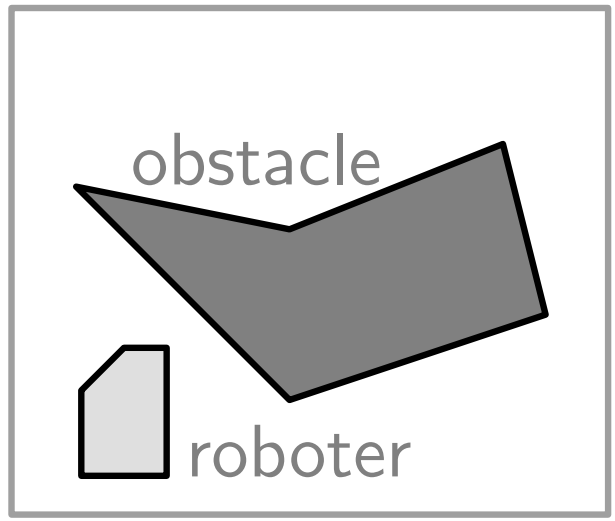
robotic arm

The *configuration space* is the d-dimensional space of all possible (i.e., obstacle avoiding) parameter value combinations.

Configuration Space



robotic arm



work space

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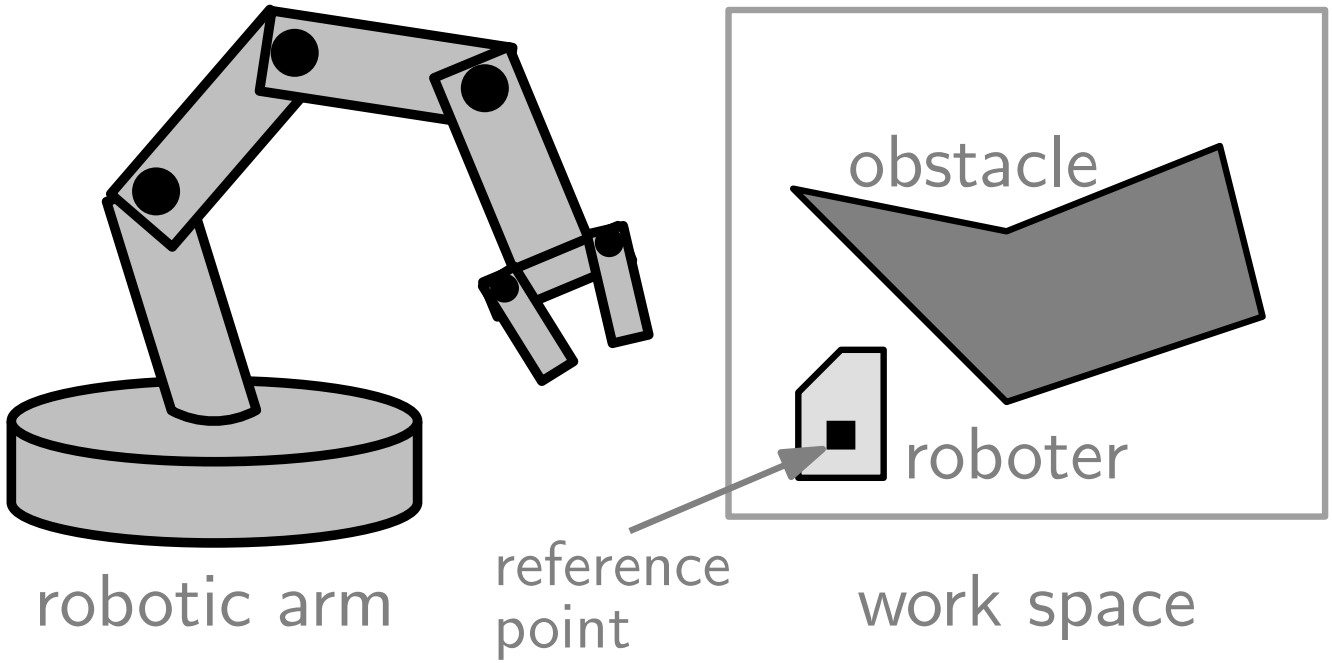

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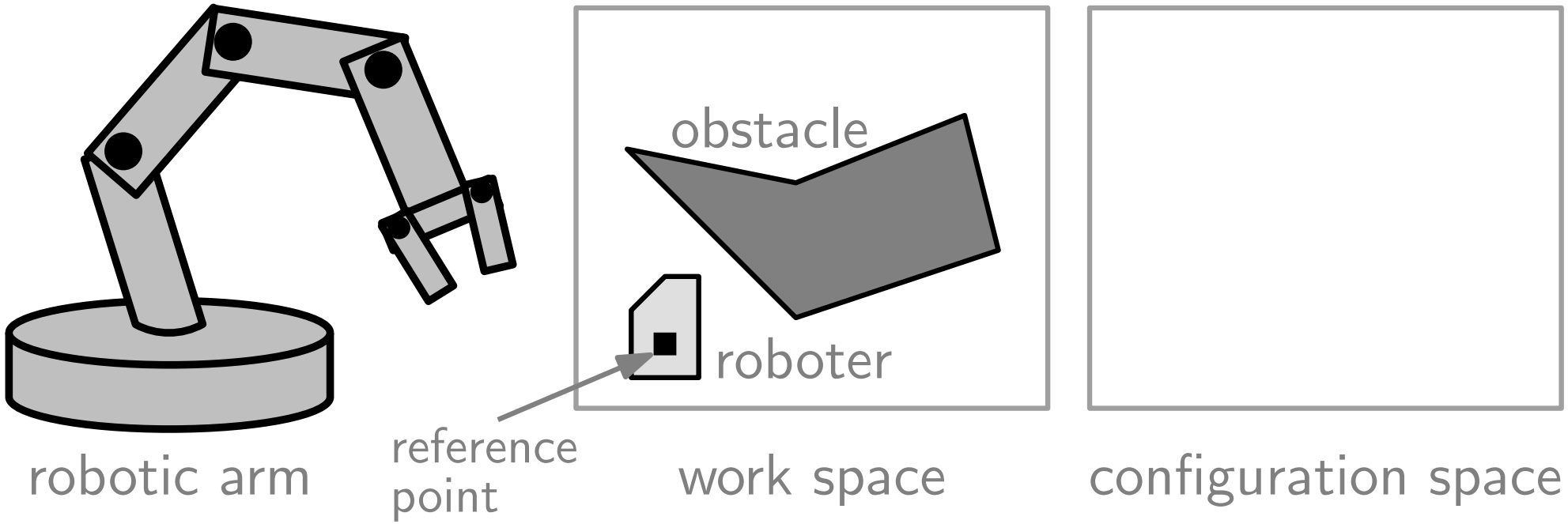

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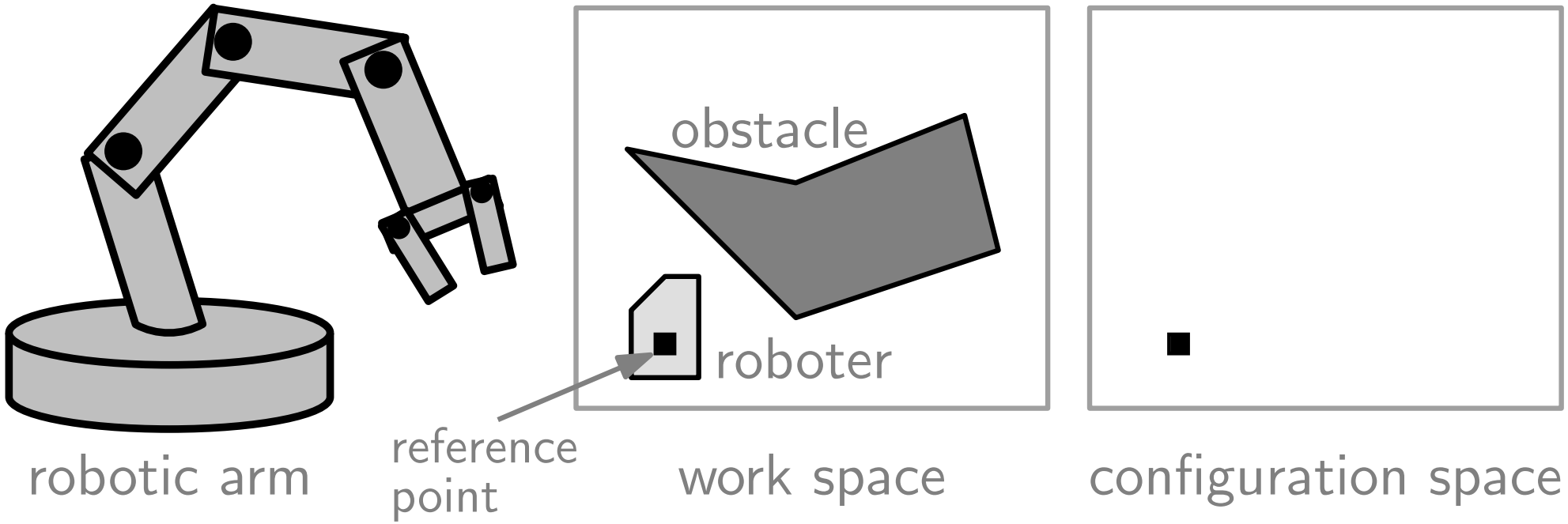

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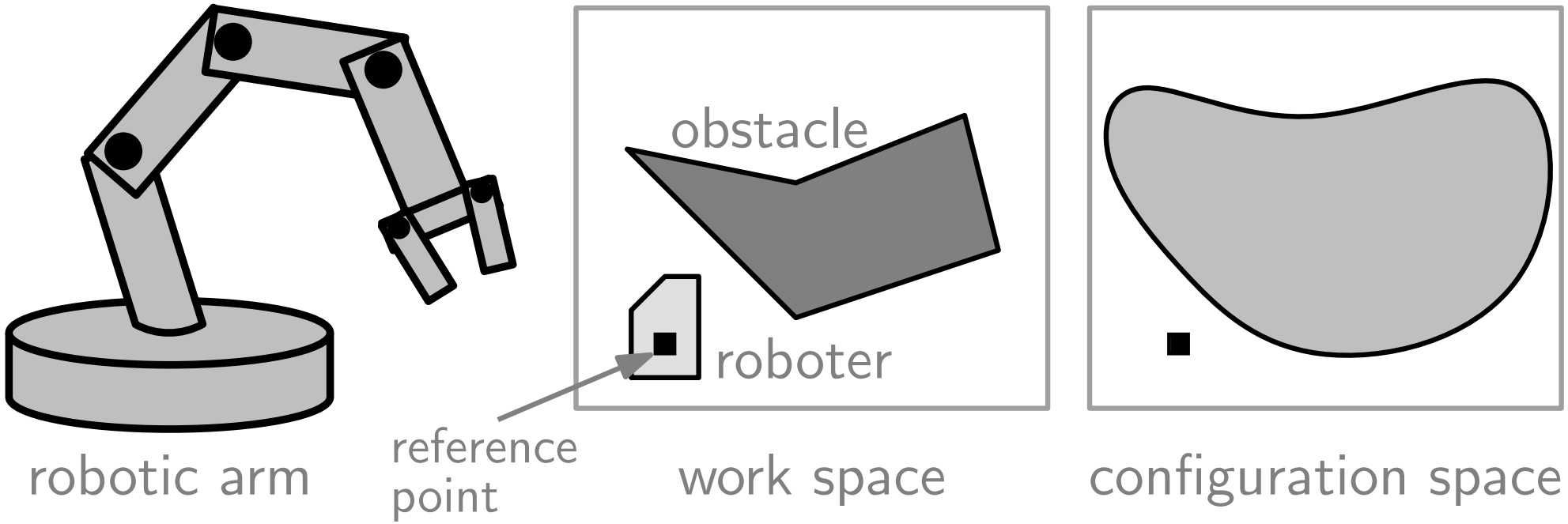

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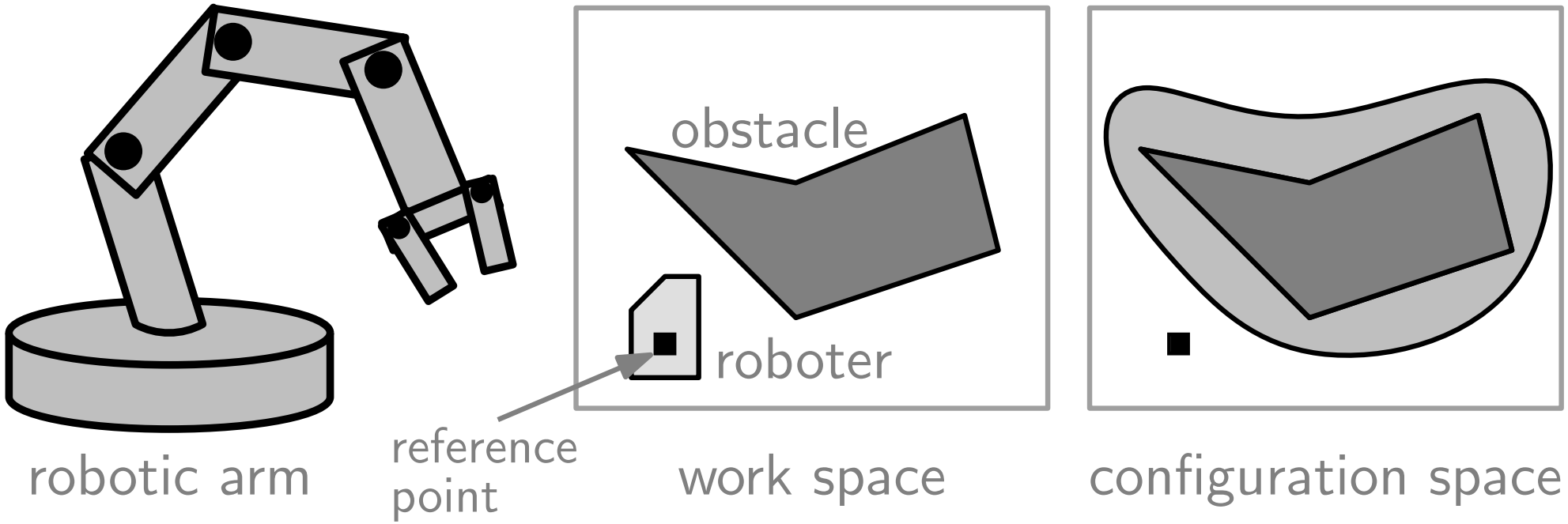

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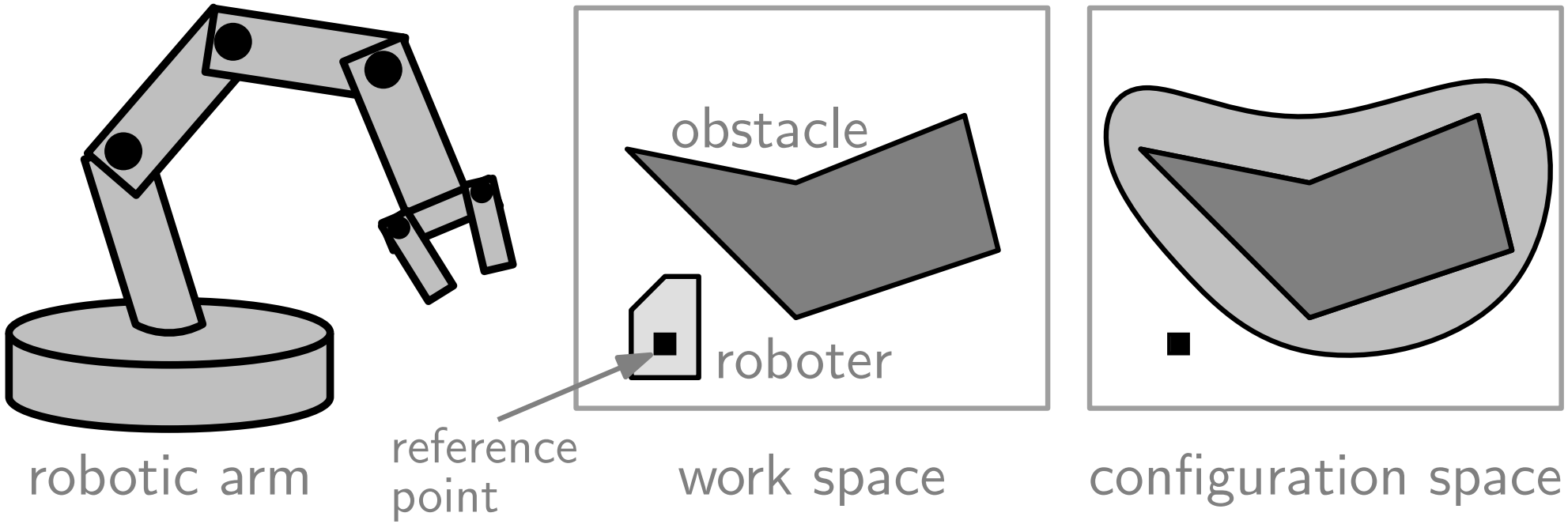

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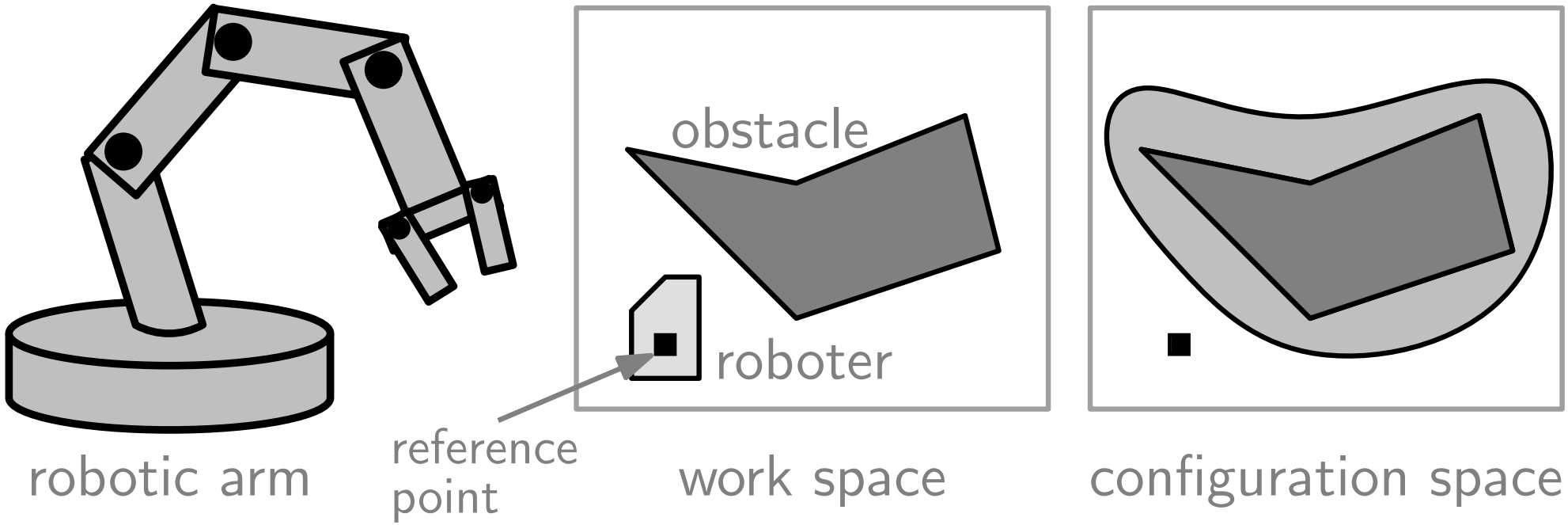
Path for a *point* through configuration space

Configuration Space



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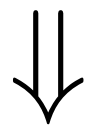
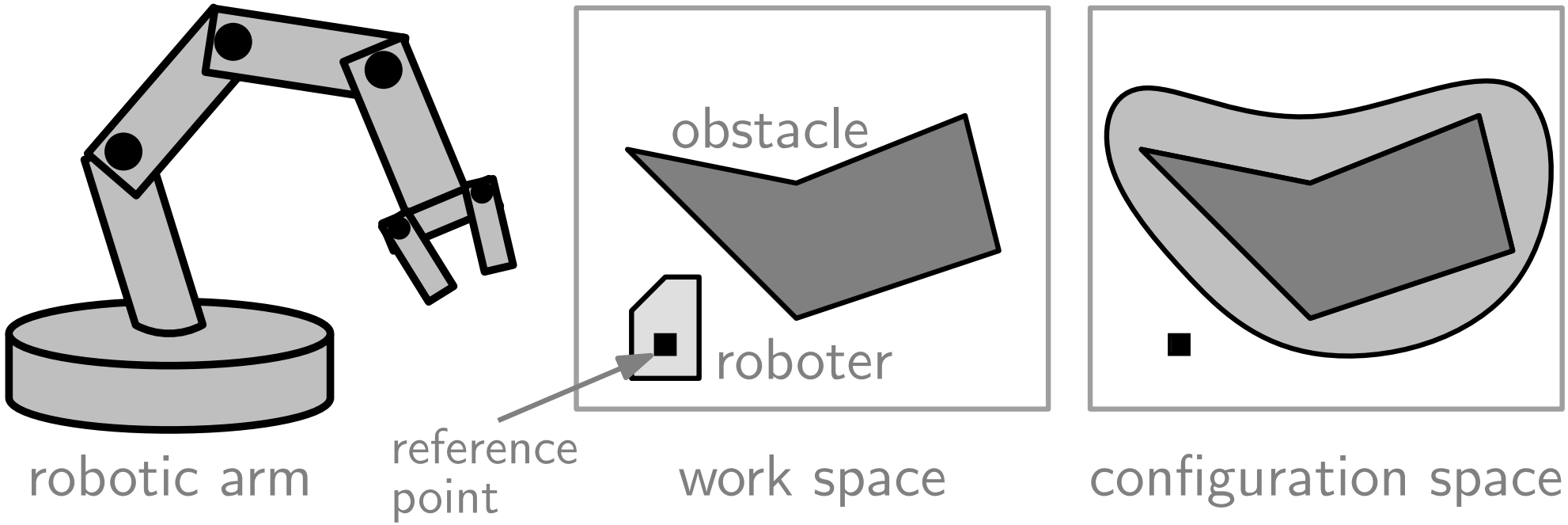


Configuration Space



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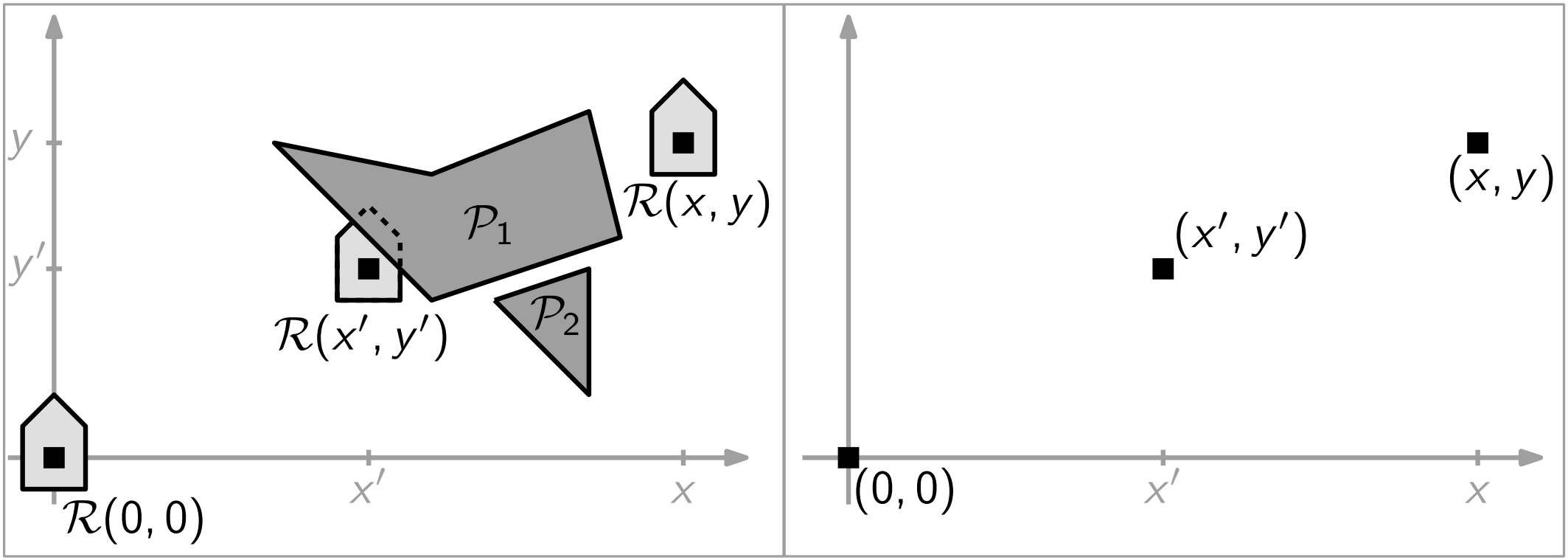
Path for a *point* through configuration space

$\Downarrow$

path for the *robot* in the original space.

Example: Translating 2D Polygonal Robots



work space *configuration space*

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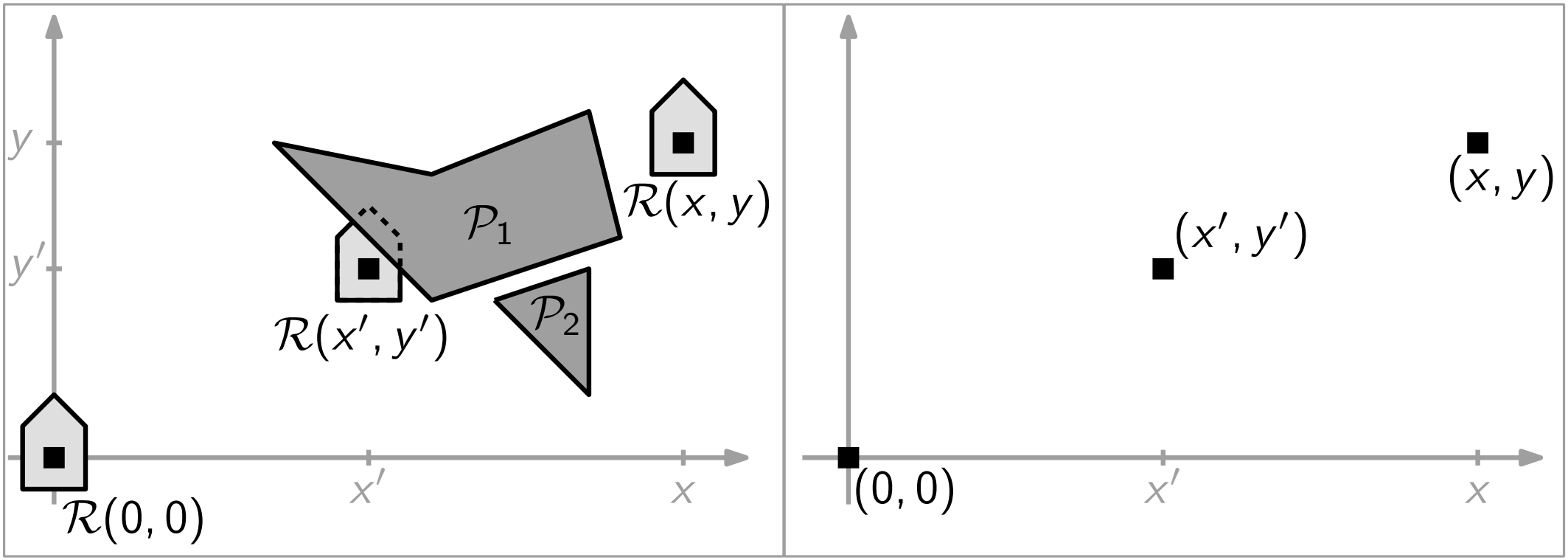

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- Compute $\mathcal{CP}_i = \{(x, y) : \mathcal{R}(x, y) \cap \mathcal{P}_i \neq \emptyset\}$ for each $\mathcal{P}_i$.

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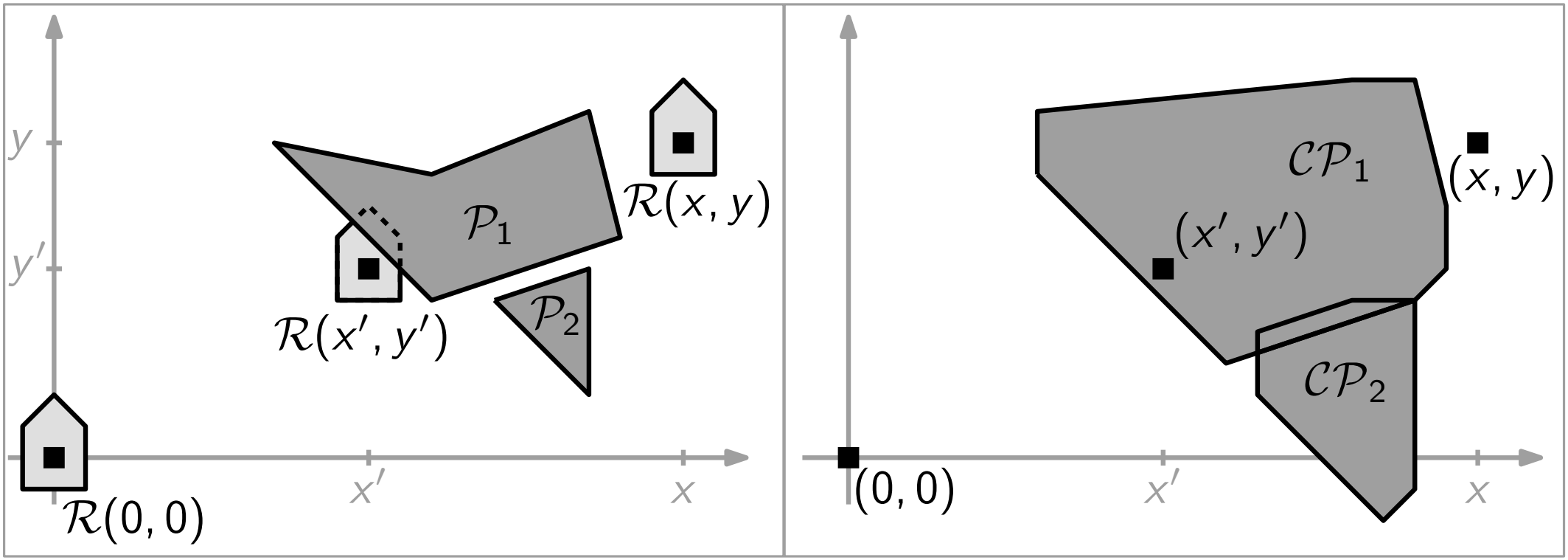

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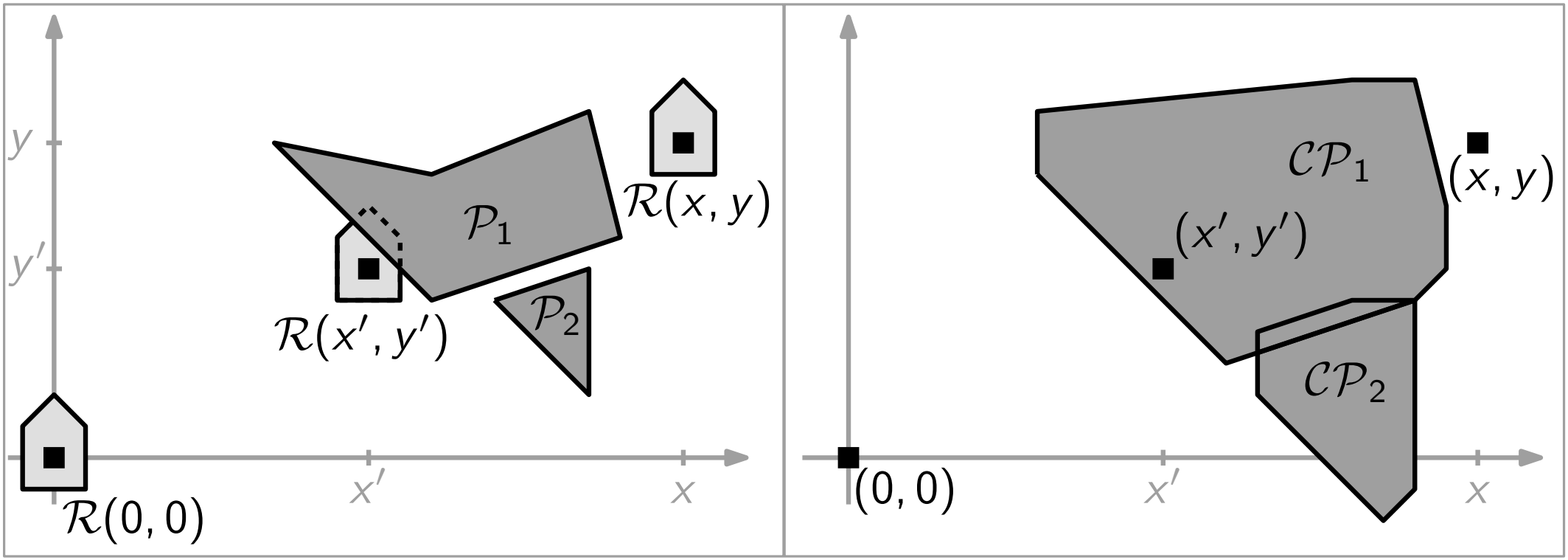

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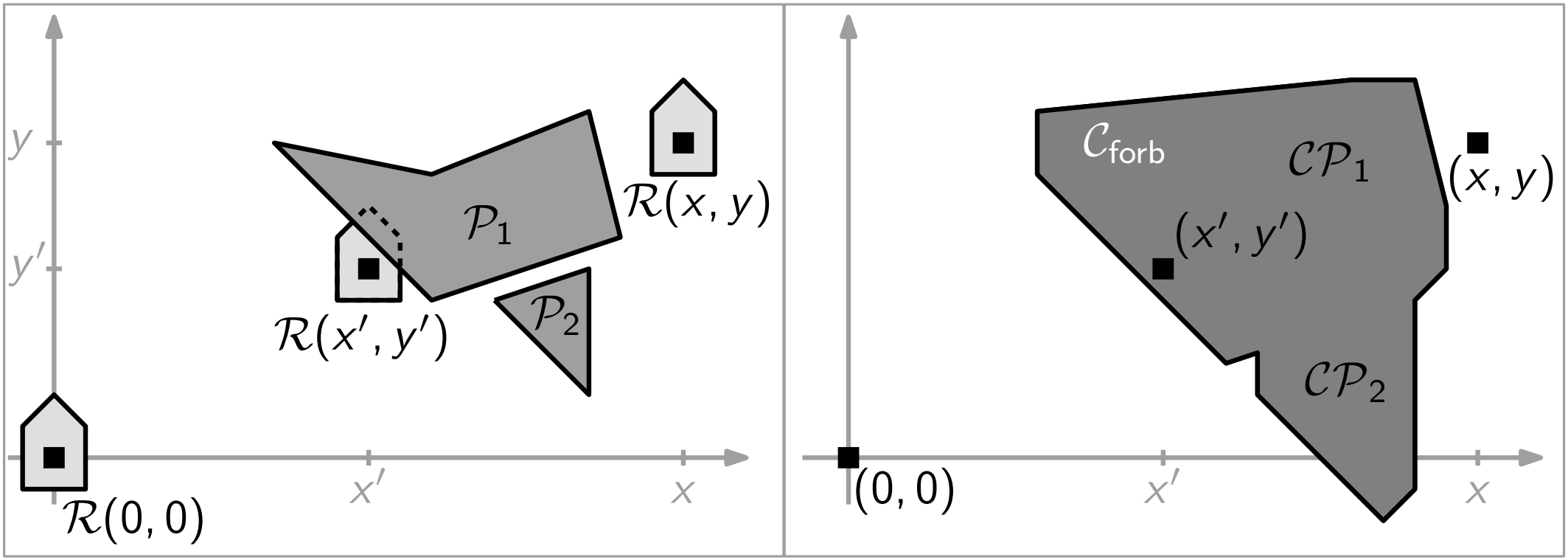

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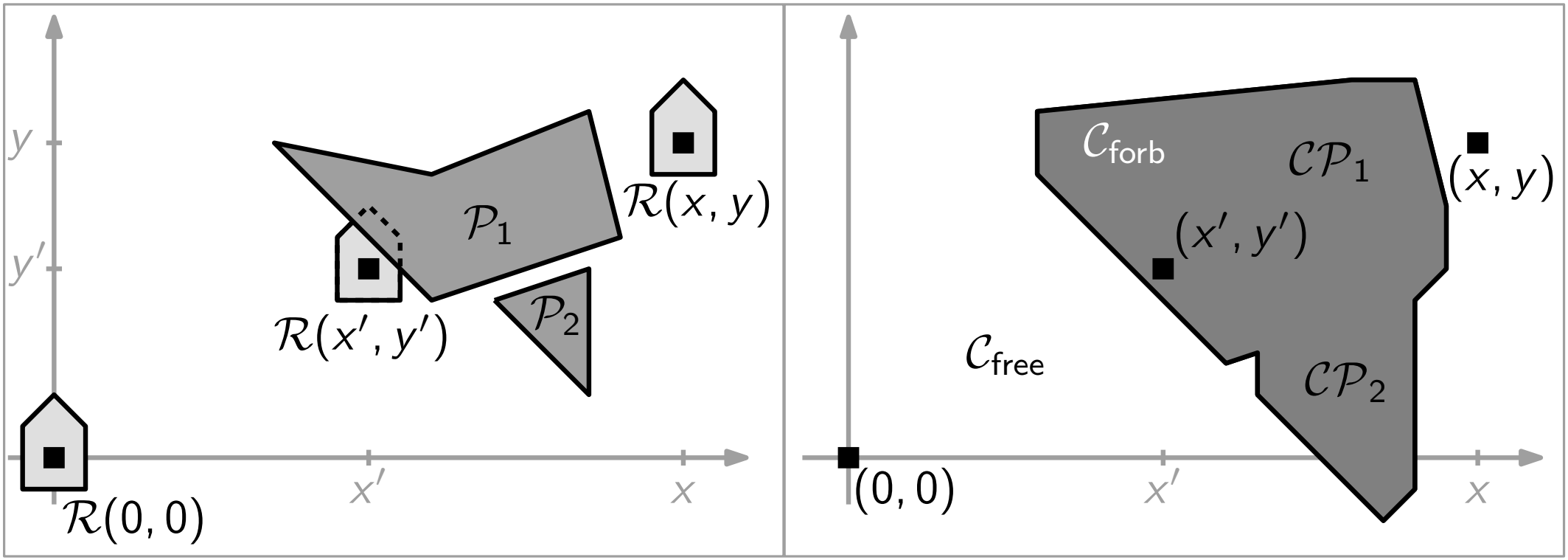

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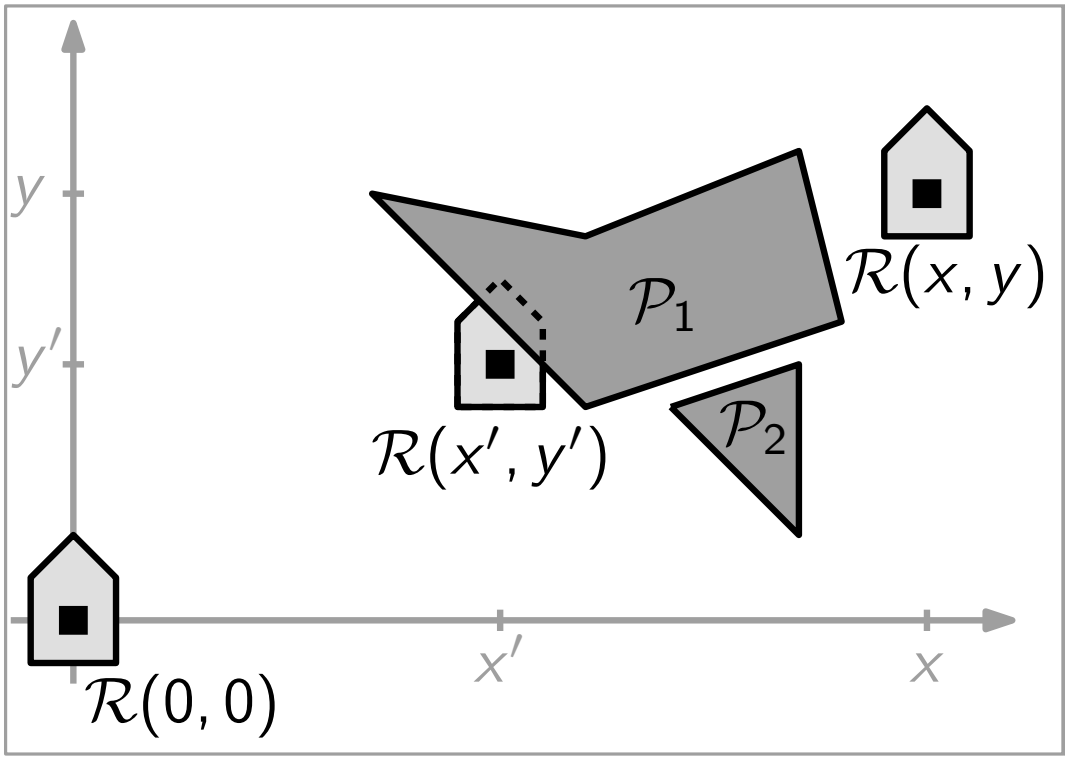
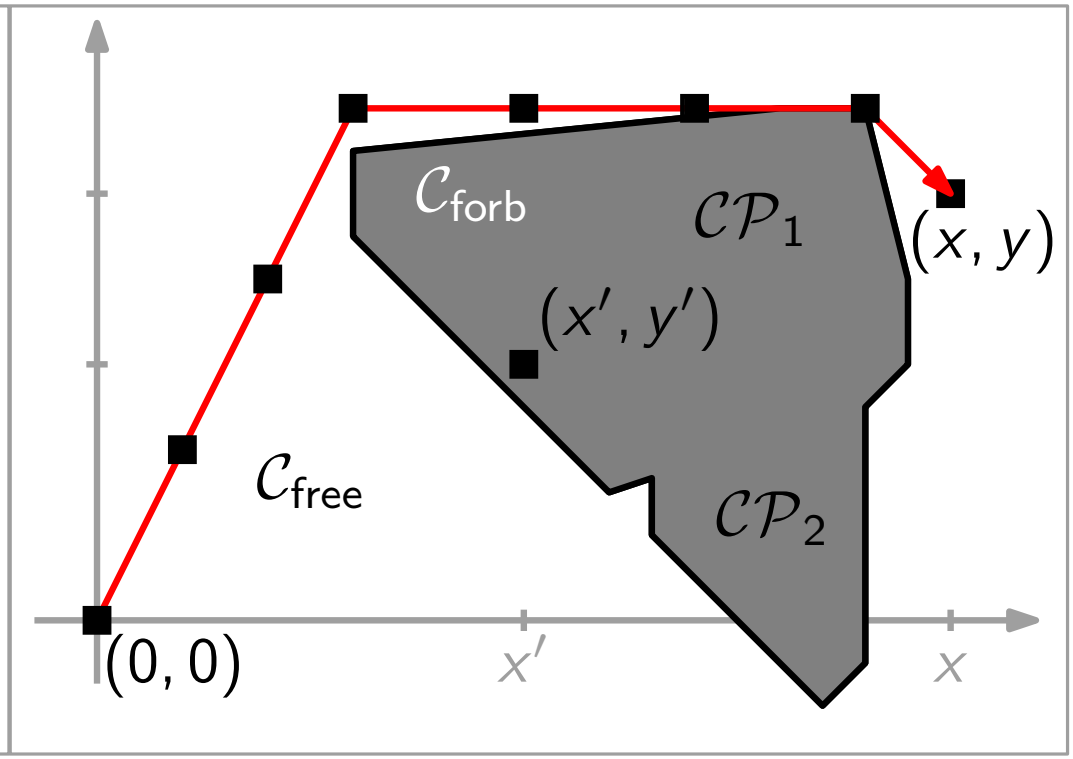


work space



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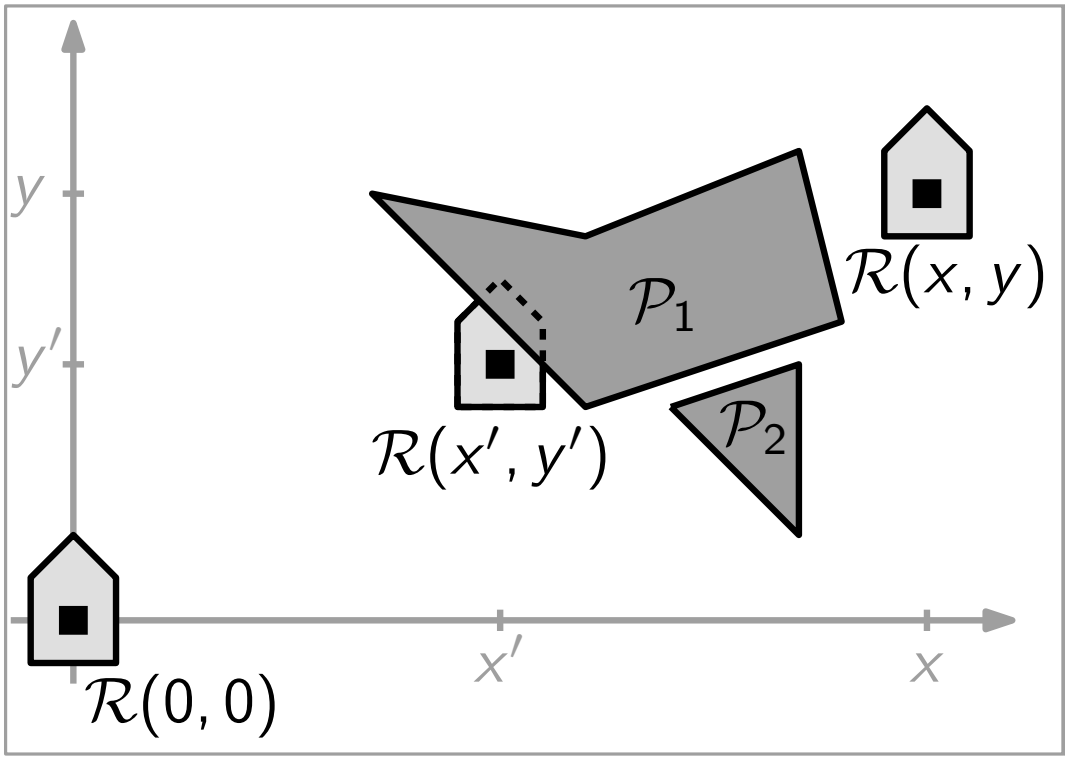

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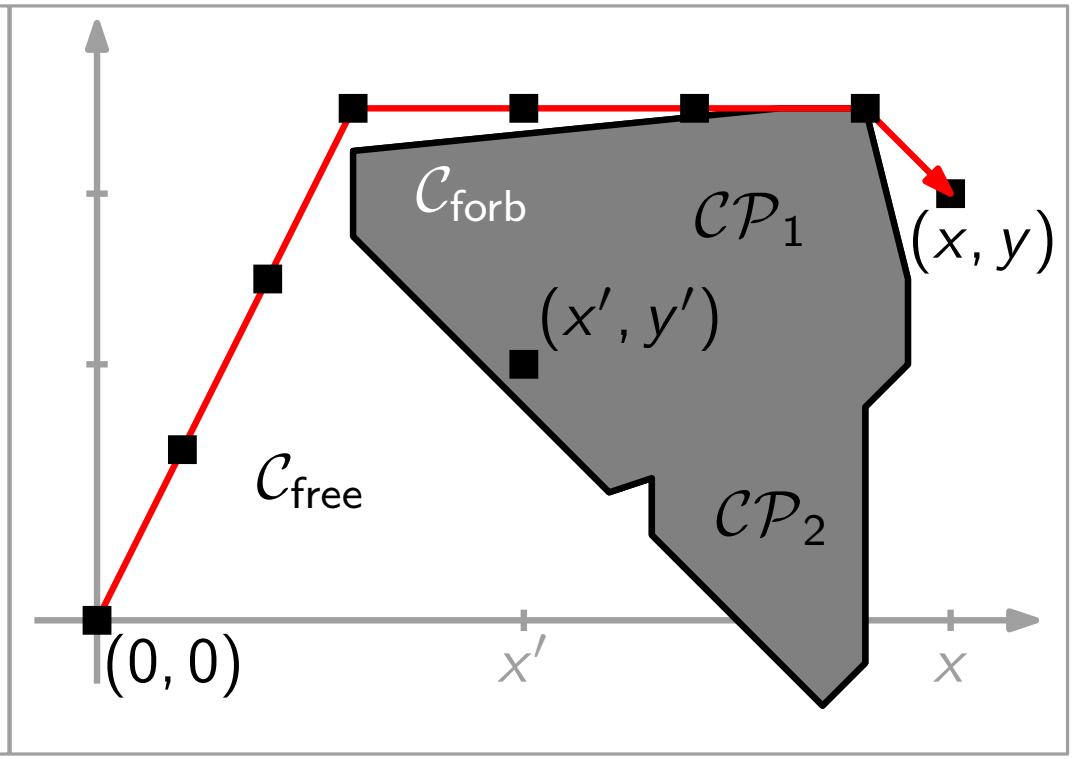

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  $\Rightarrow$ collision-free path for the robot in work space

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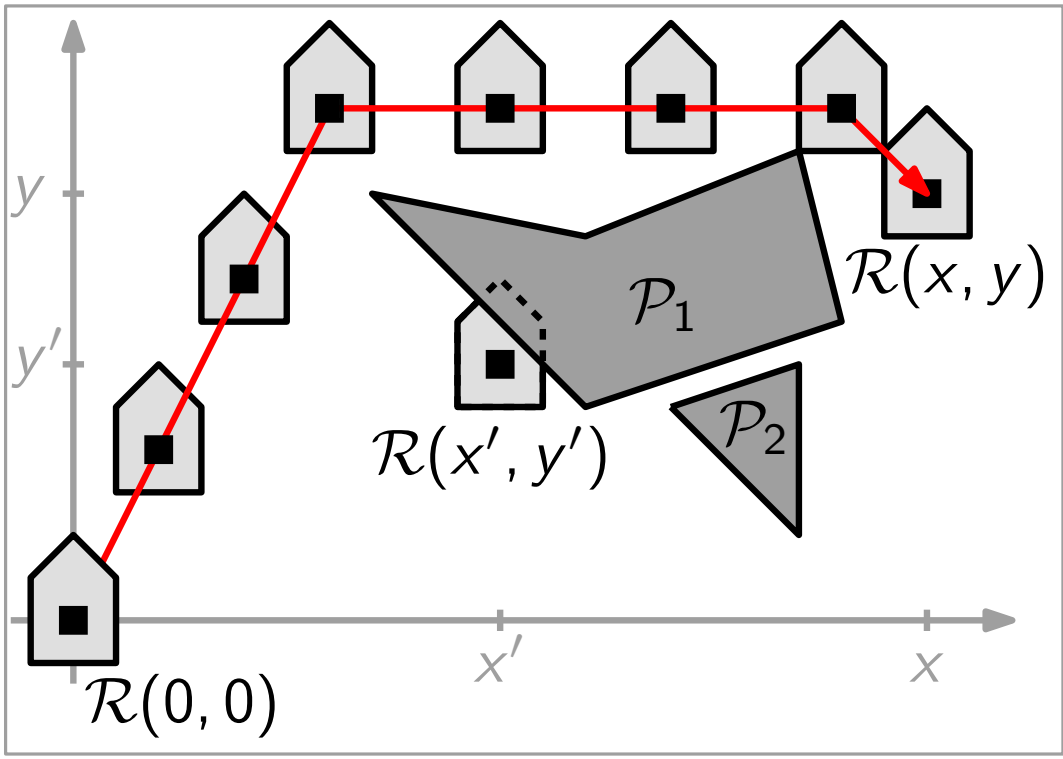

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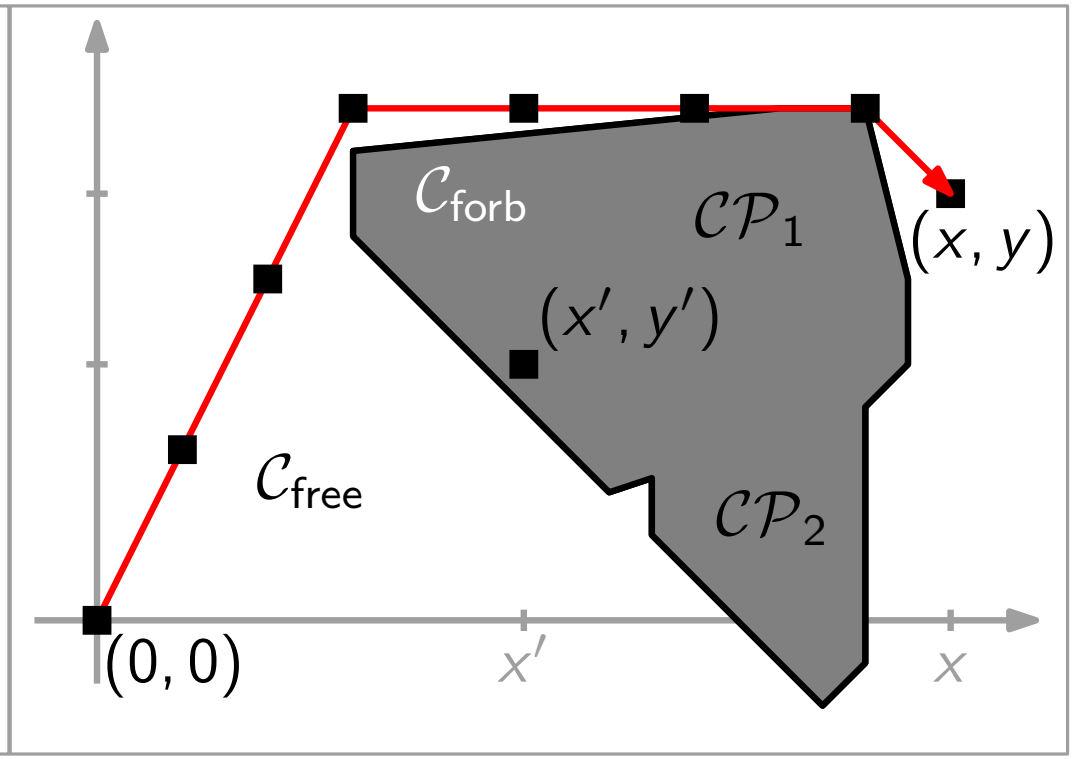

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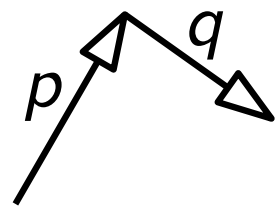
Some Linear Algebra

Vector sums



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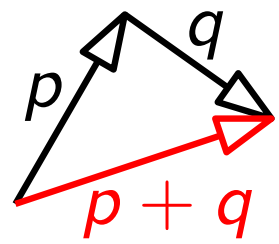

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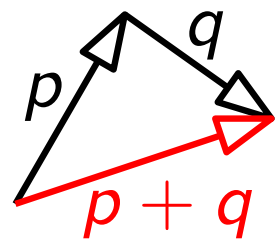
Vector sums

Algebra: $(p_x, p_y) + (q_x, q_y) = (p_x + q_x, p_y + q_y)$



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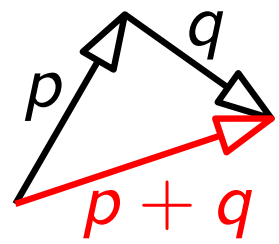

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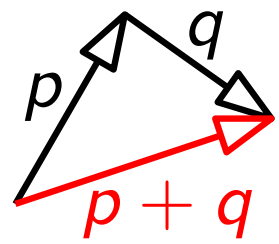
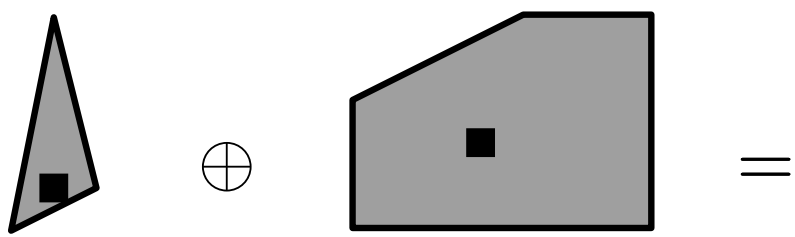


Minkowski sums



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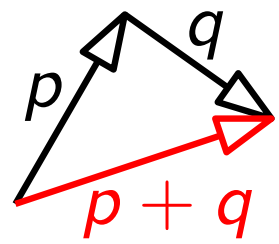

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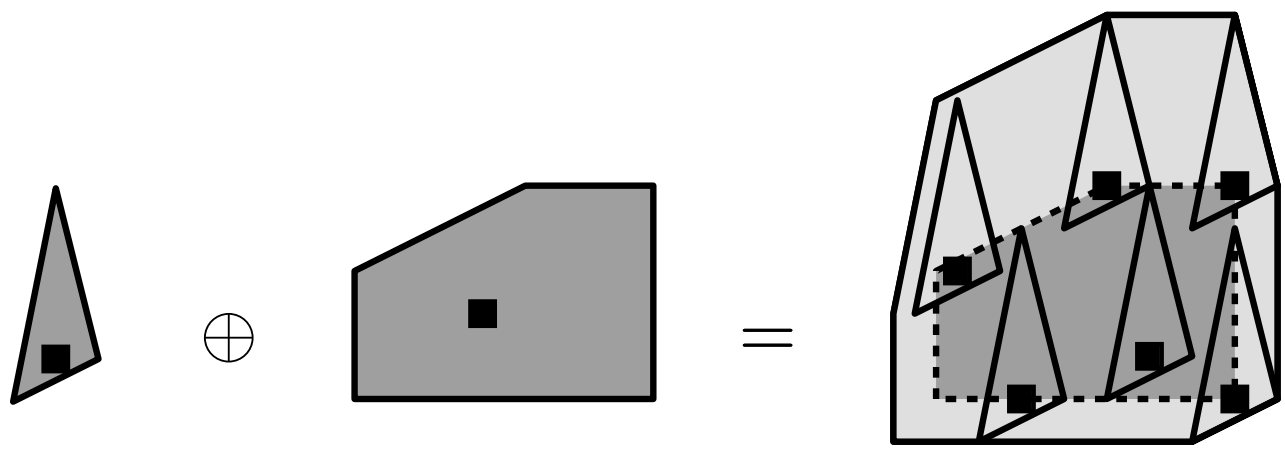

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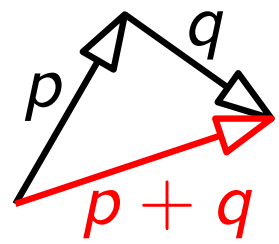

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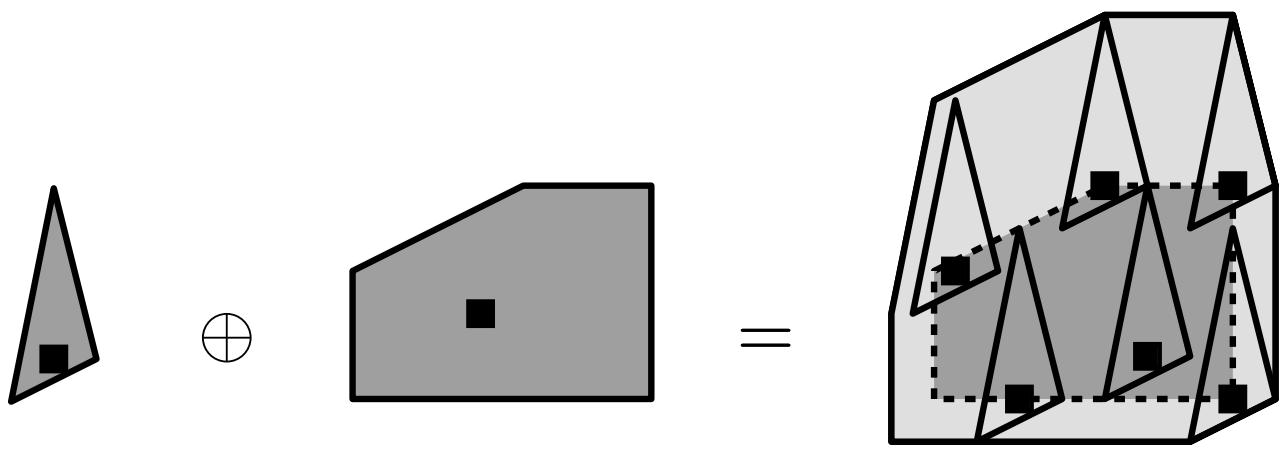

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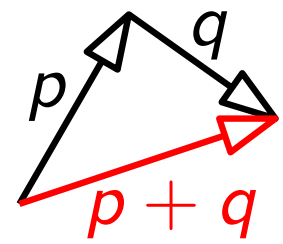

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Geometry: place copy of one shape
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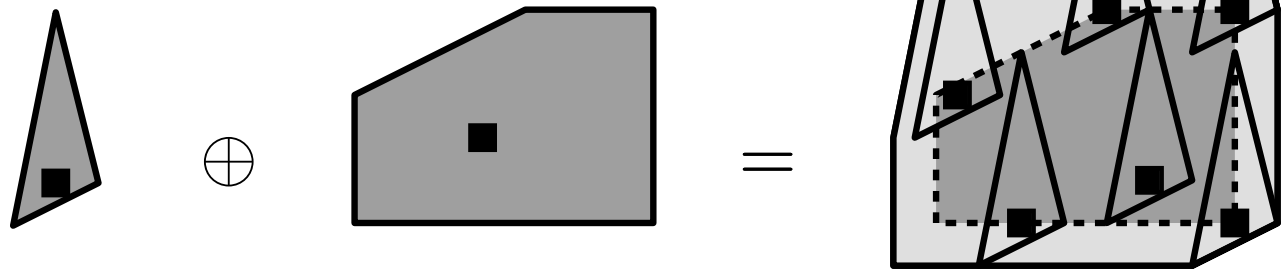

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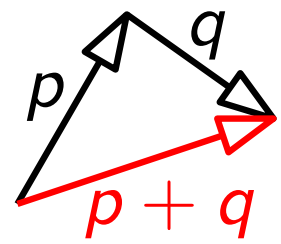

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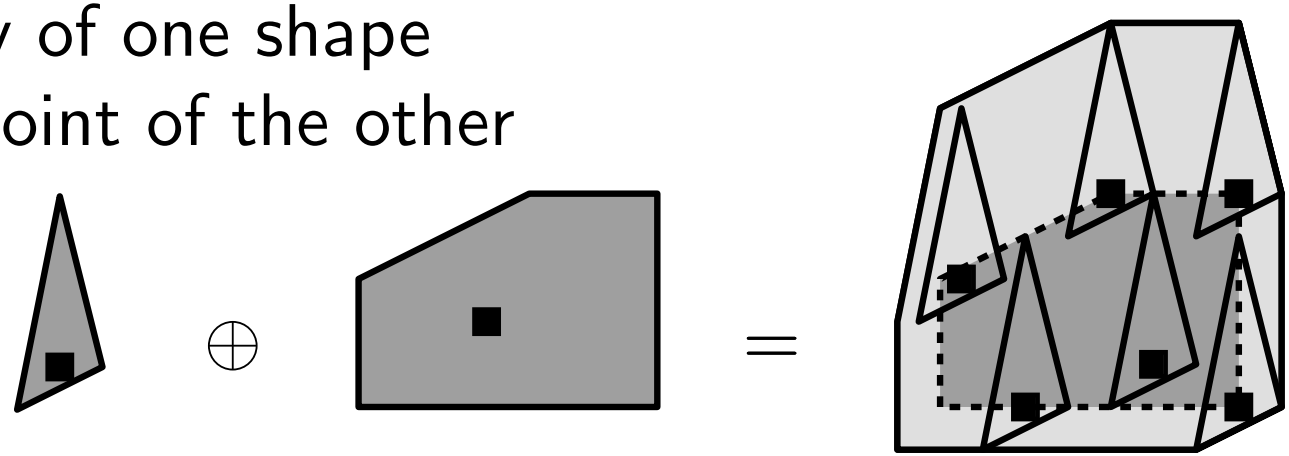
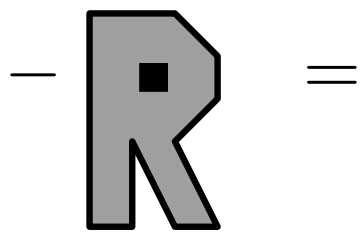


Inversion



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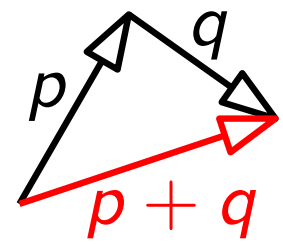

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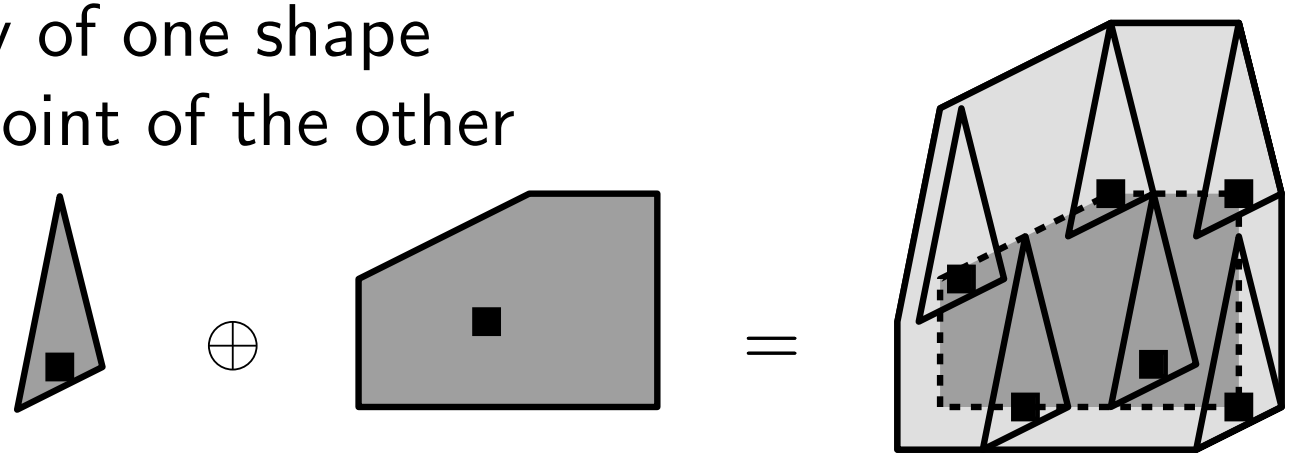

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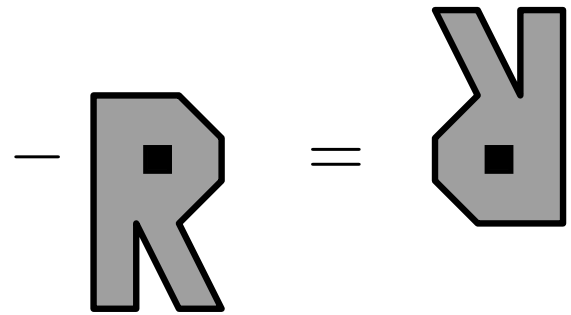

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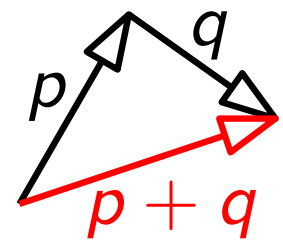

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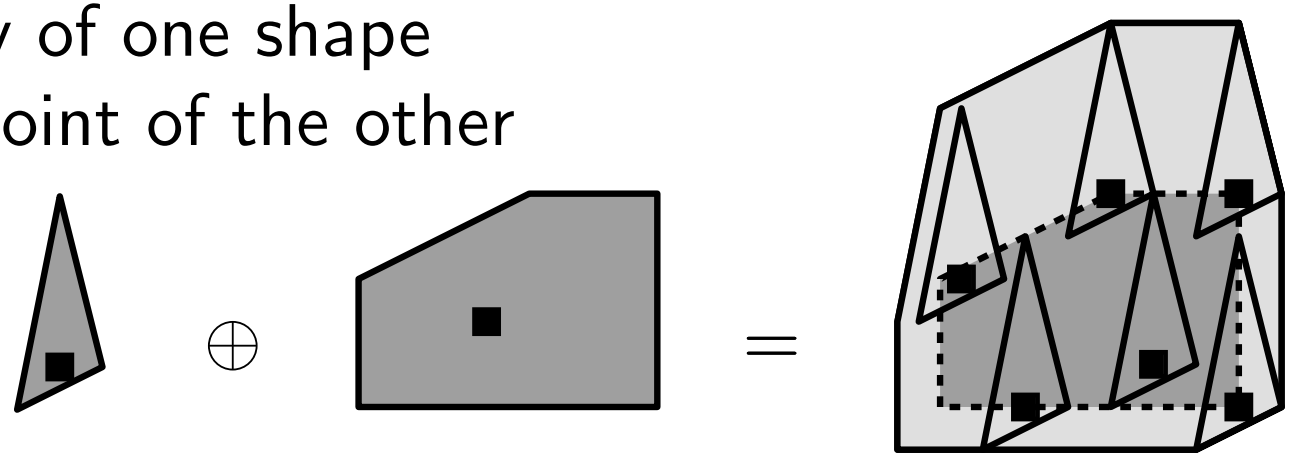
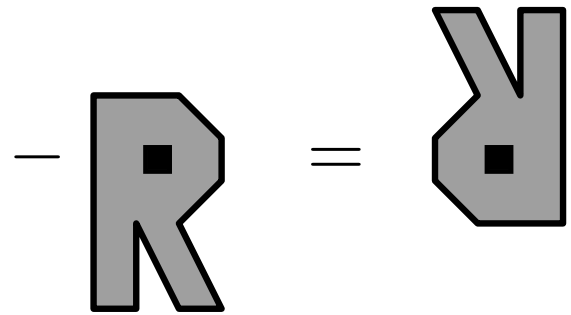


Inversion

Algebra: $-S = \{-p \mid p \in S\}$



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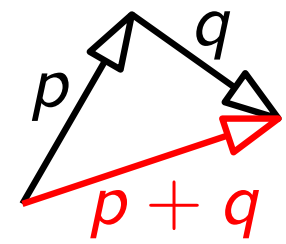

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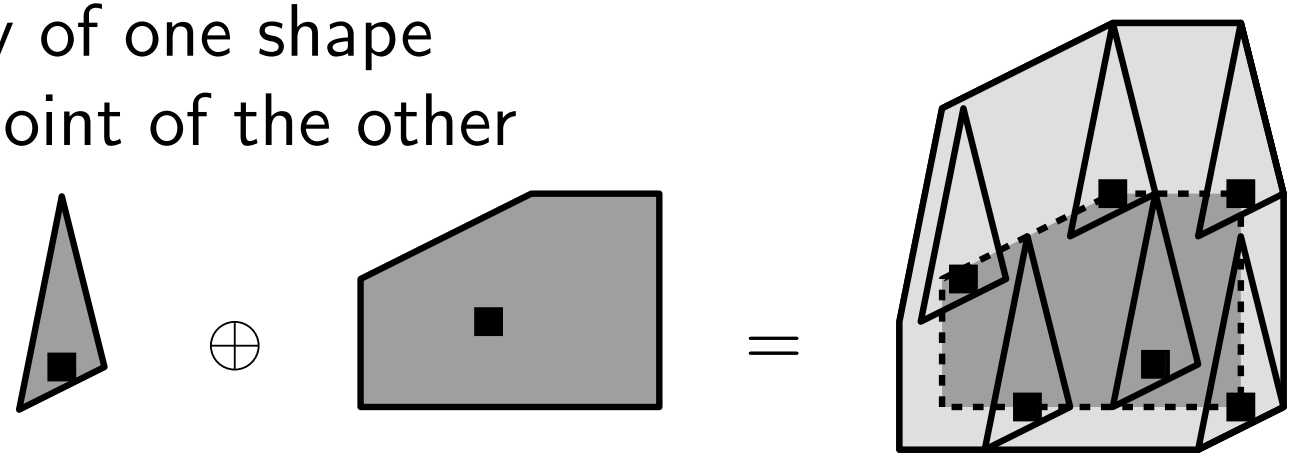
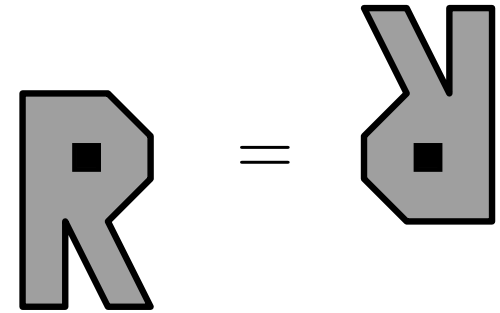


Inversion

Algebra: $-S = \{-p \mid p \in S\}$

Geometry: rotate 180° (point-mirror)
around reference point



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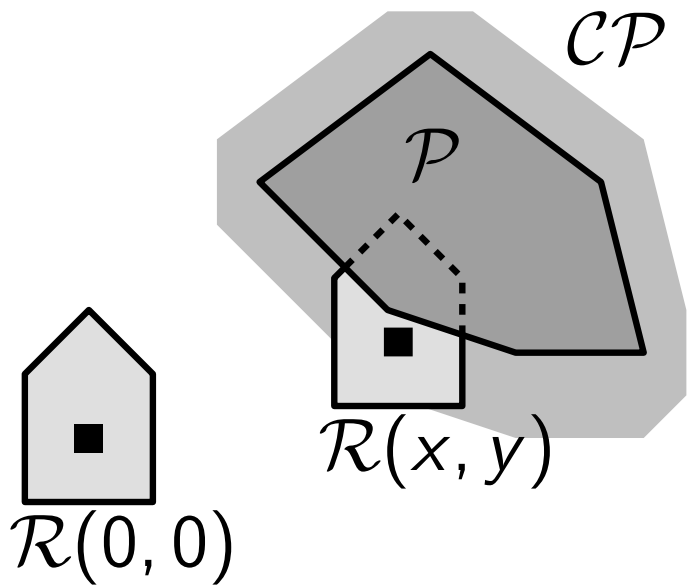

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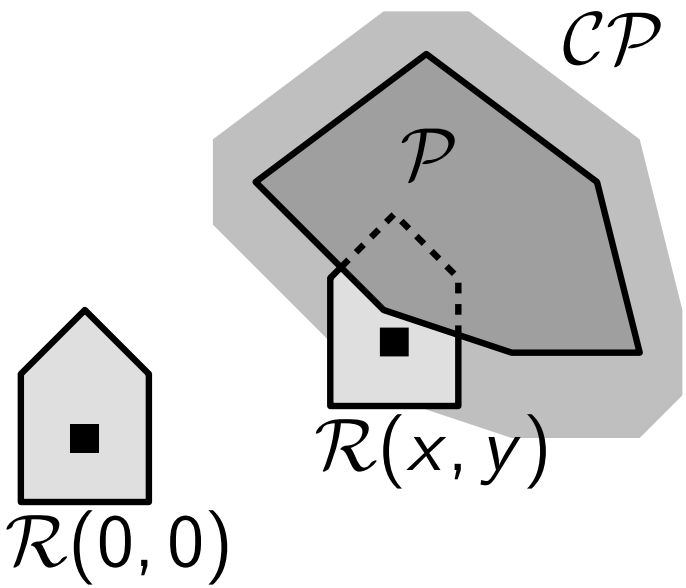

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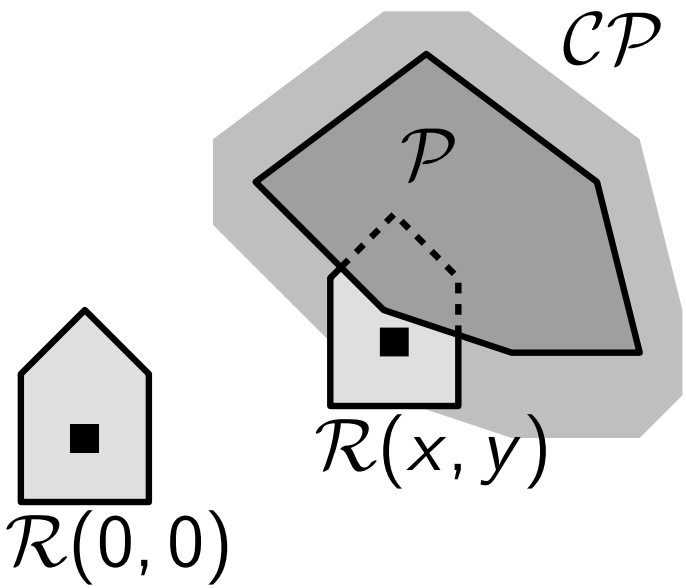

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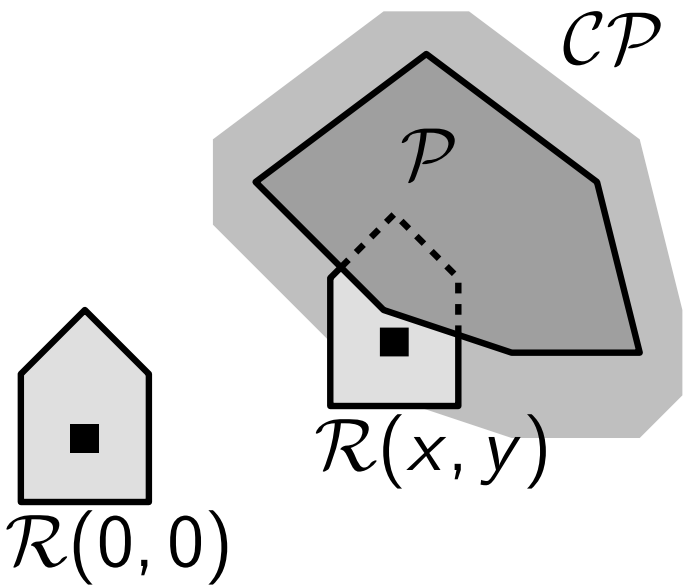

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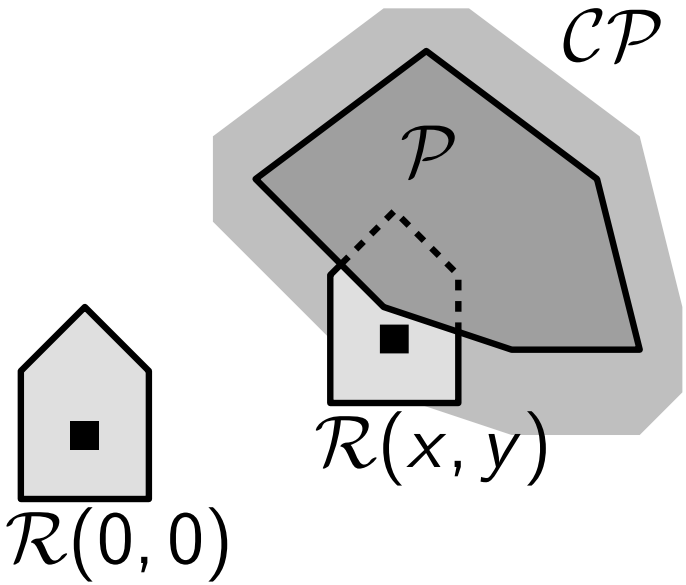

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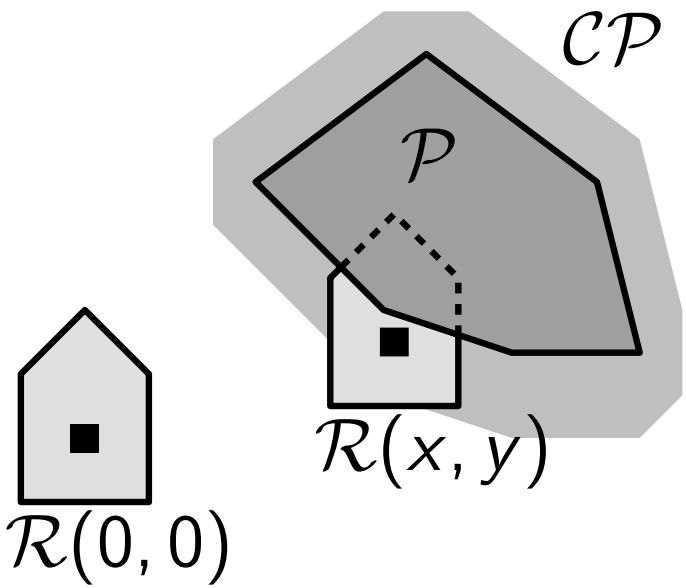
"$\Leftarrow$"



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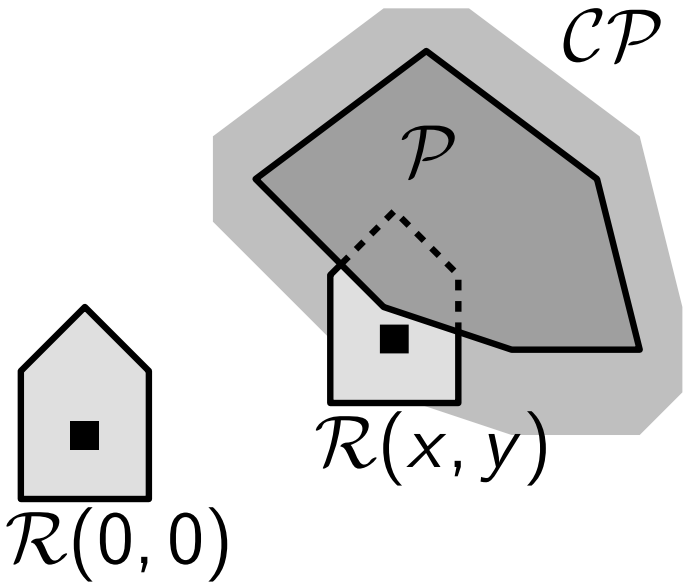

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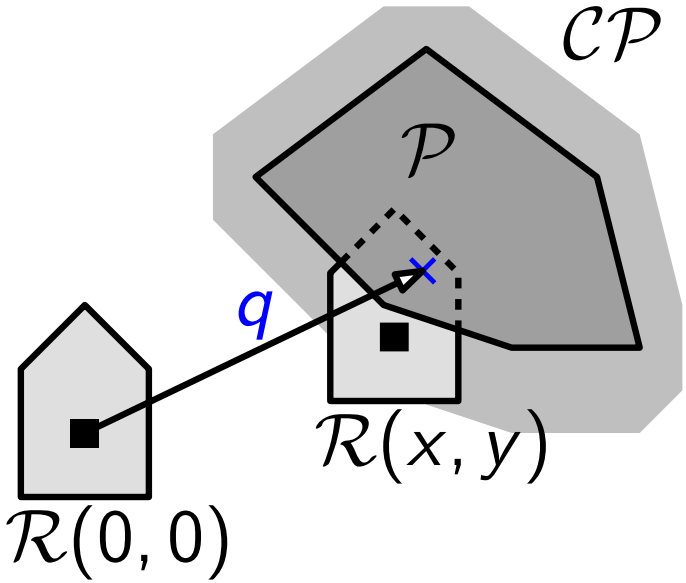

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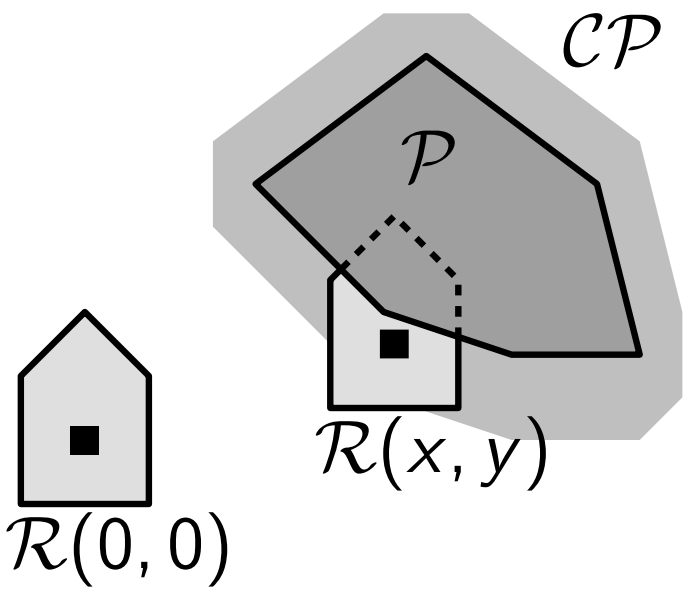

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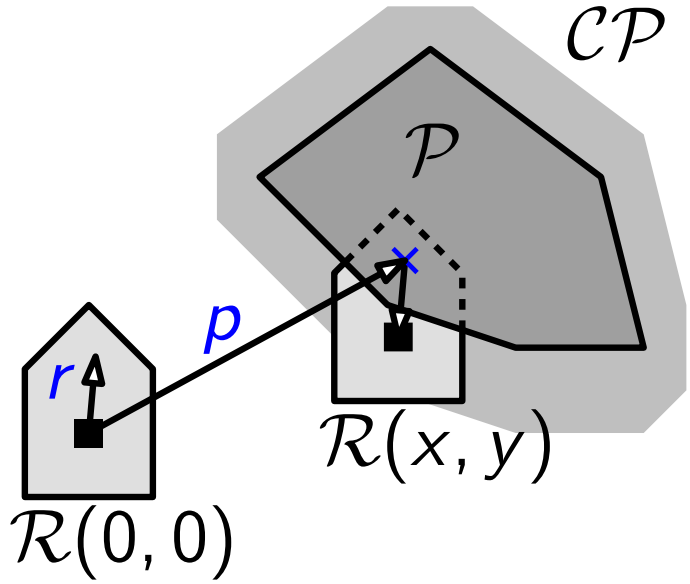
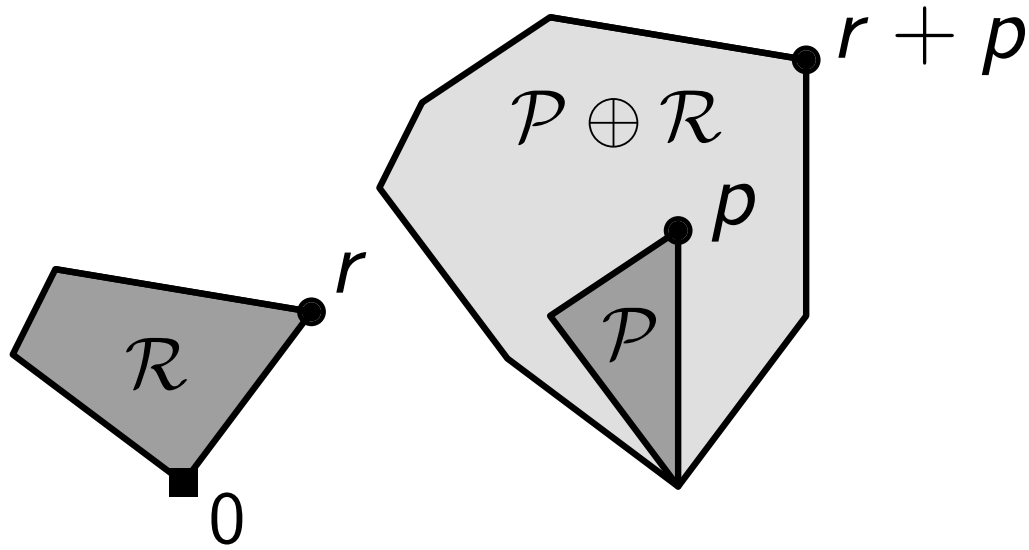
Then there are points
$r \in \mathcal{R}(0, 0)$ and $p \in \mathcal{P}$
such that ...



Minkowski Sums: Complexity

Theorem: If $\mathcal{P}$ and $\mathcal{R}$ are convex polygons with n and m edges, respectively, then $\mathcal{P} \oplus \mathcal{R}$ is a convex polygon with at most $n + m$ edges.



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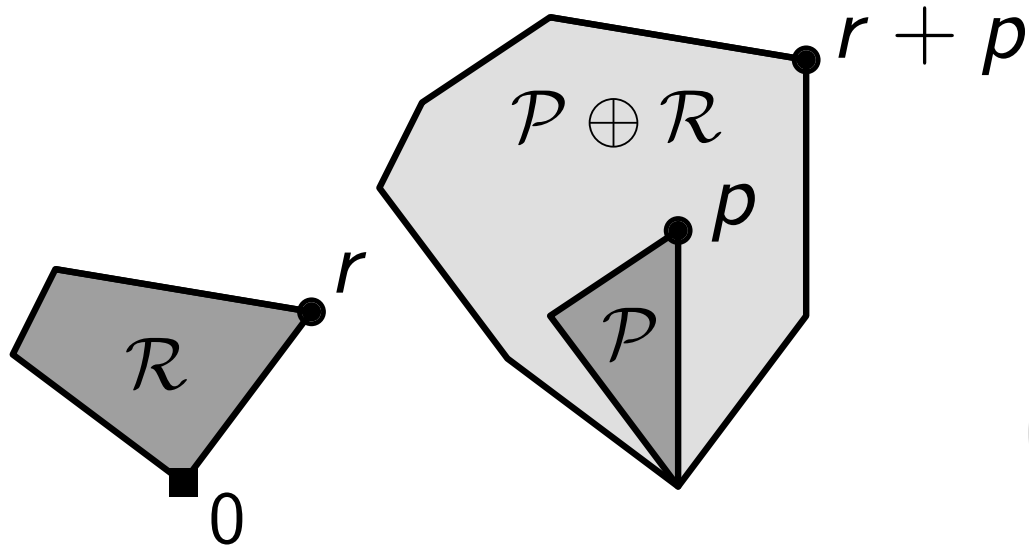

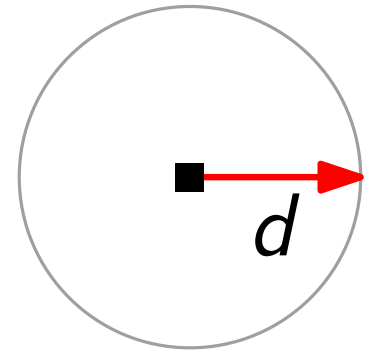

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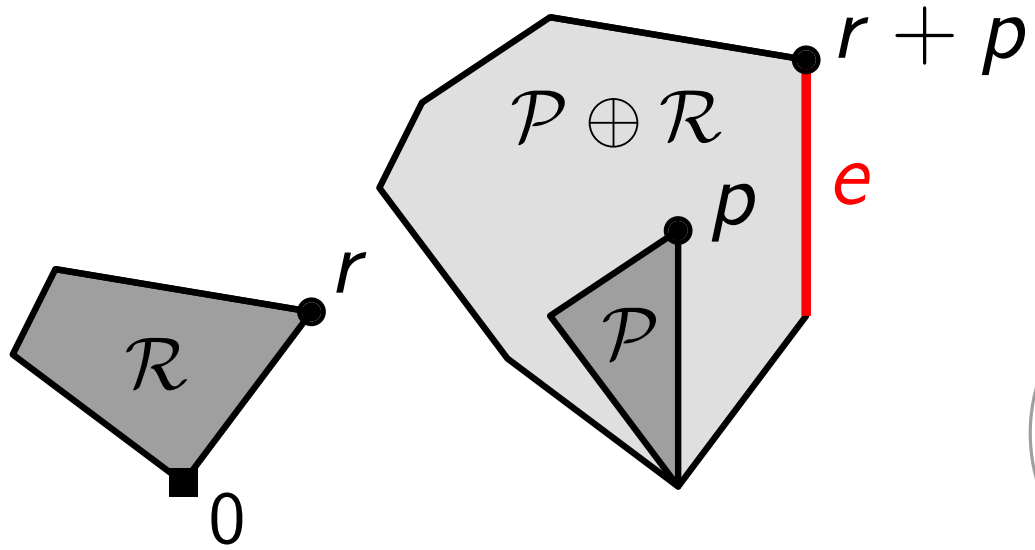

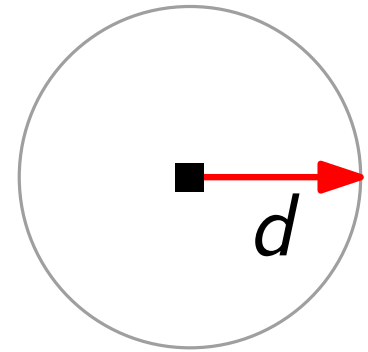

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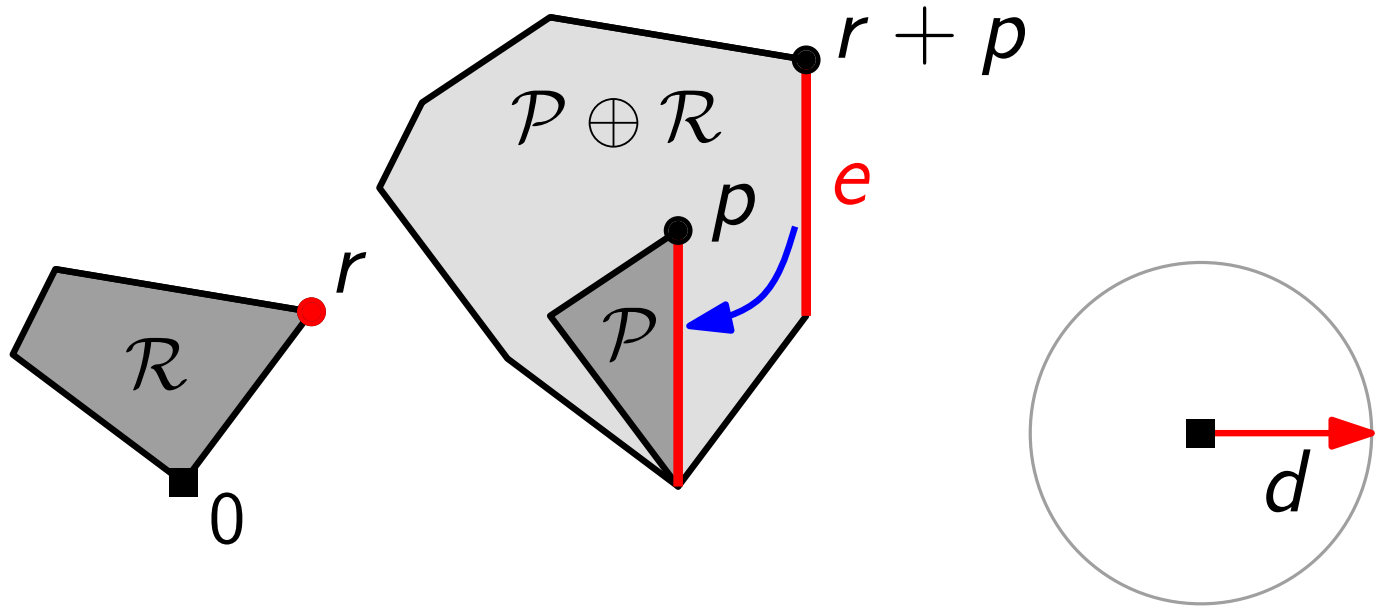

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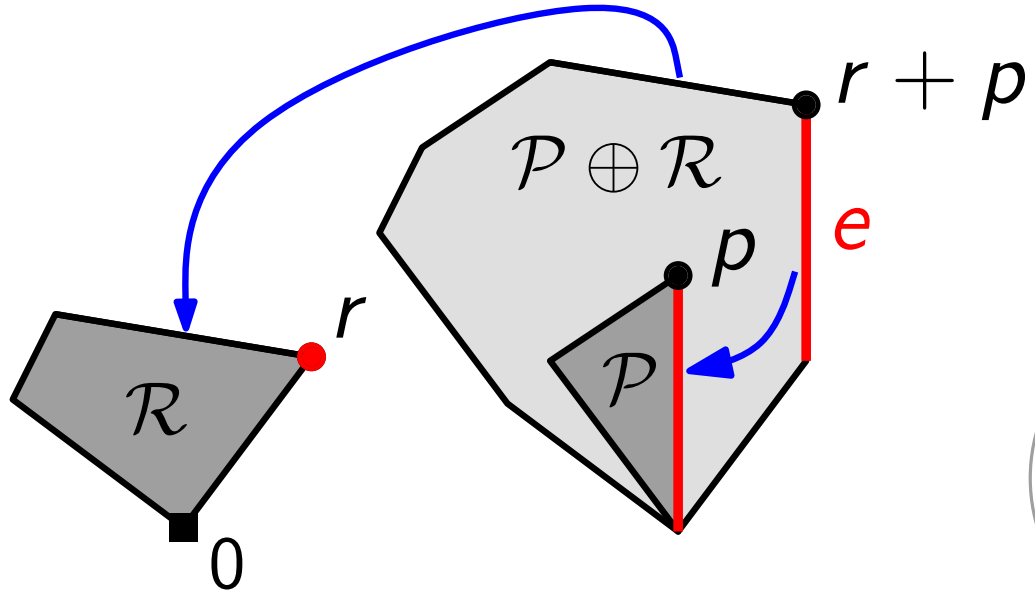

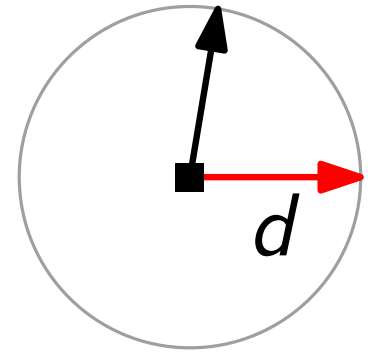

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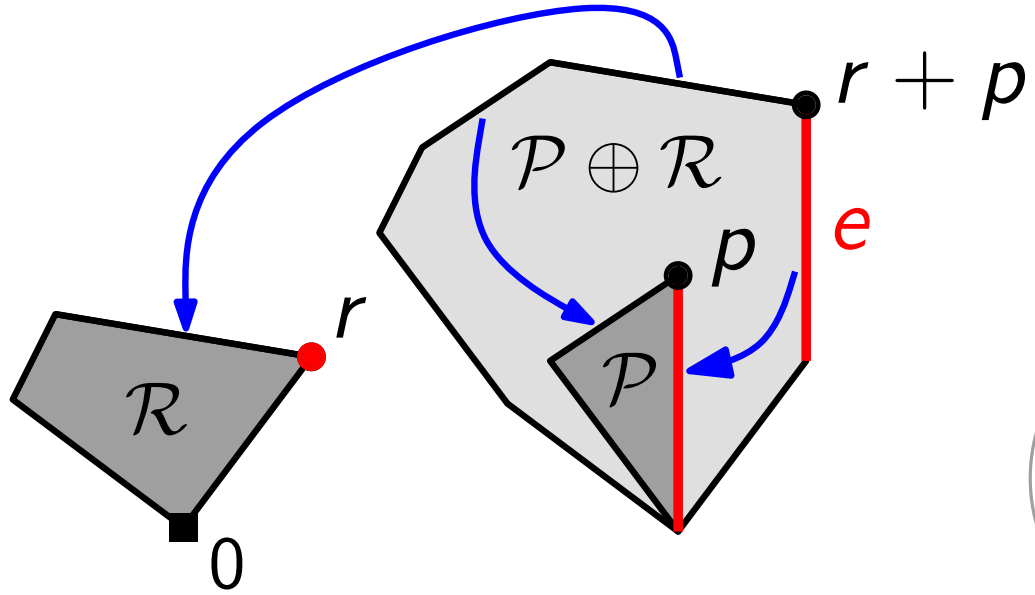

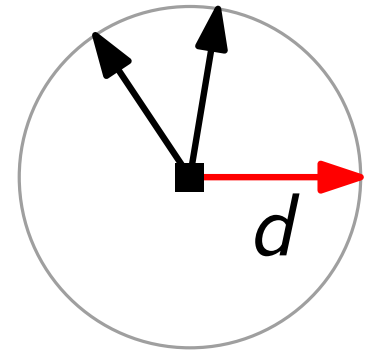

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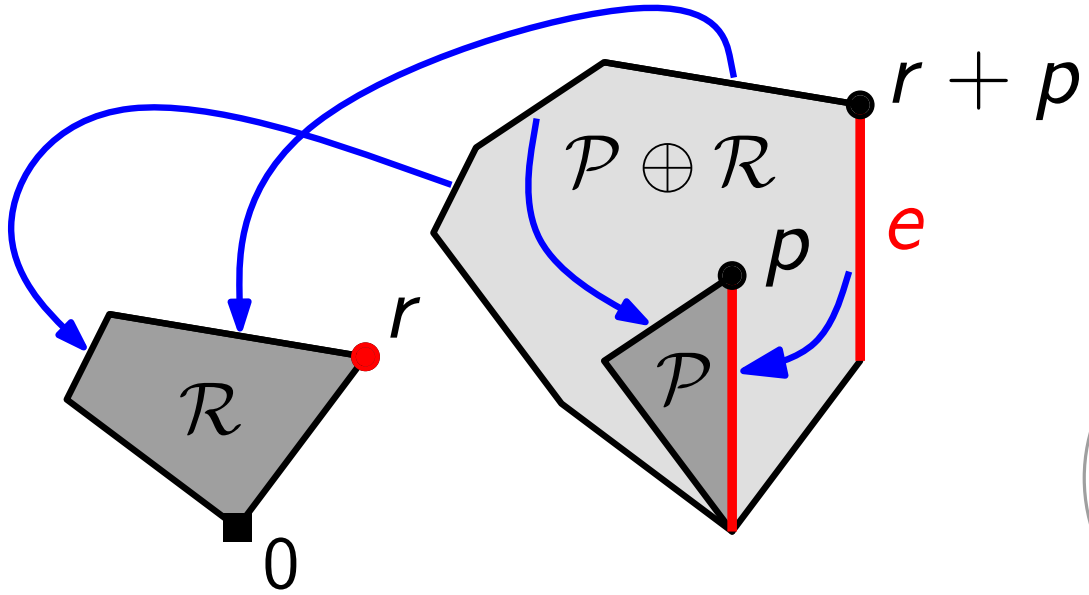

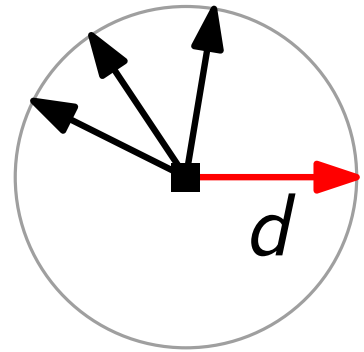

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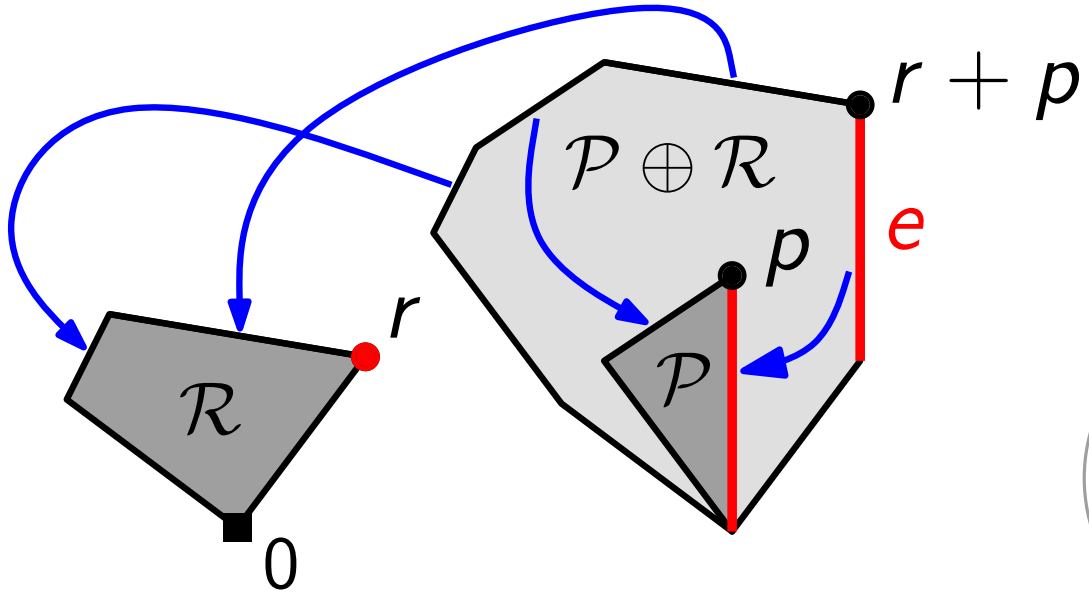
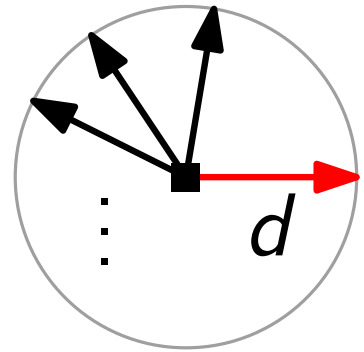
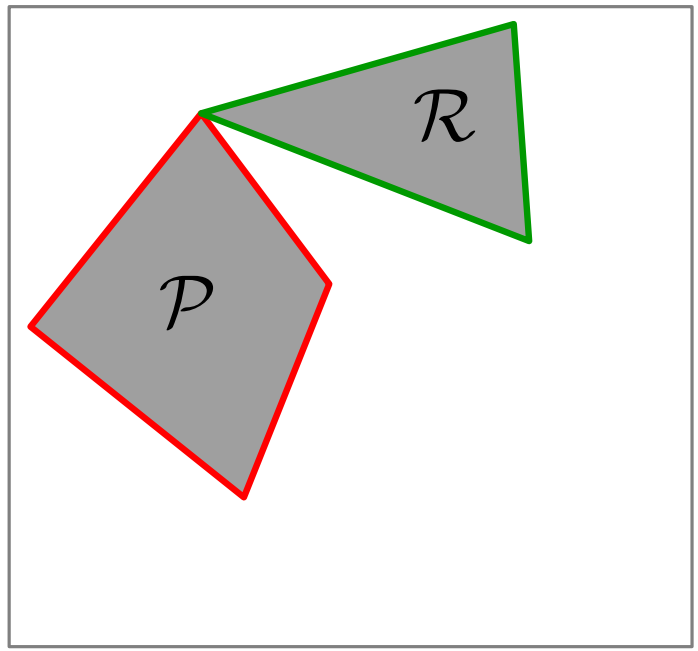
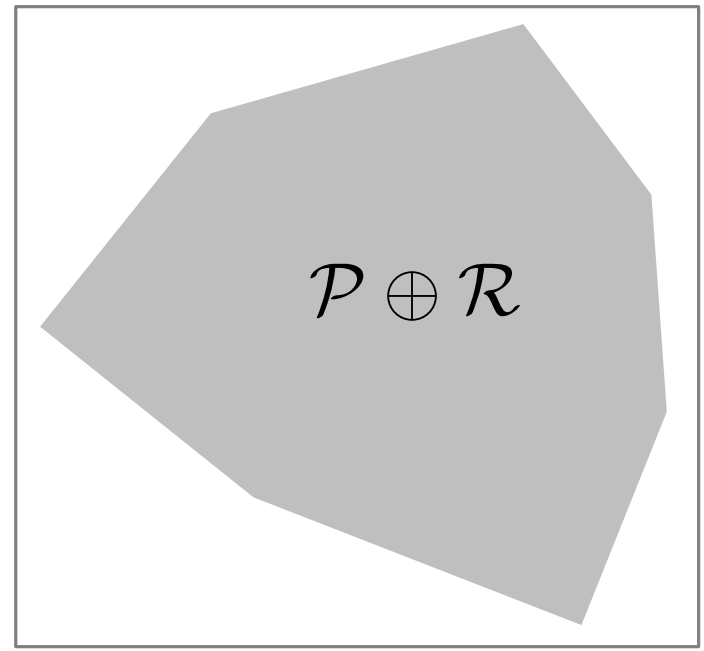




Minkowski Sums: Computation





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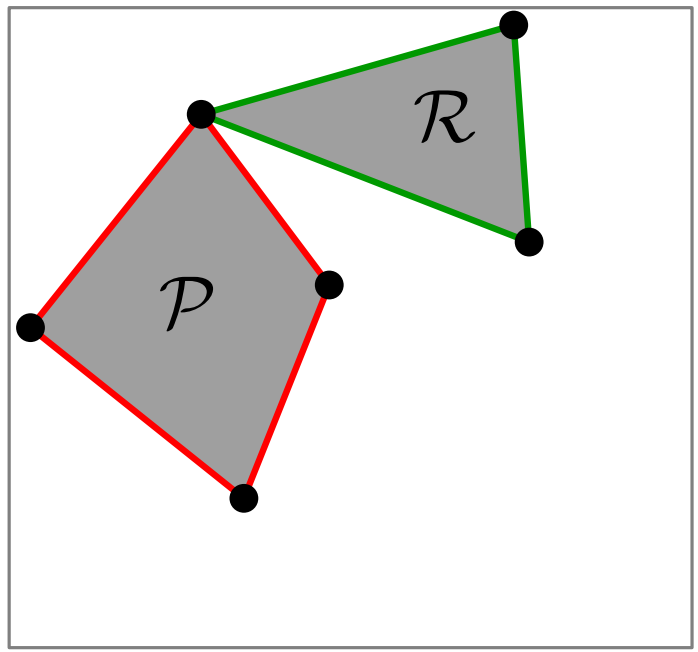
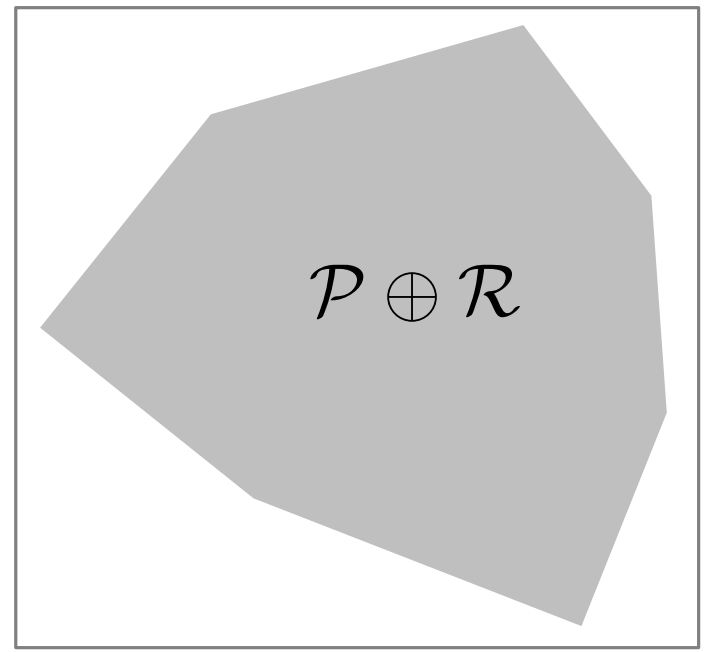
Task: How would you compute $\mathcal{P} \oplus \mathcal{R}$ given $\mathcal{P}$ and $\mathcal{R}$?





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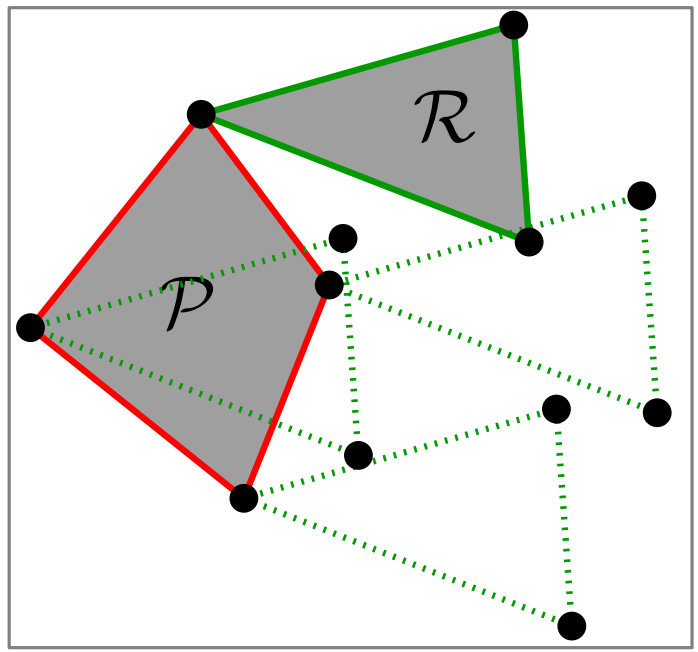

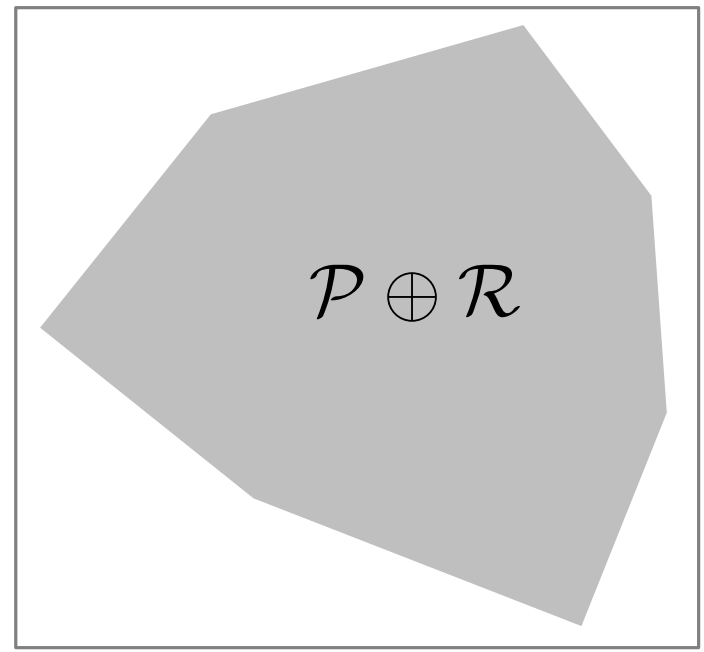

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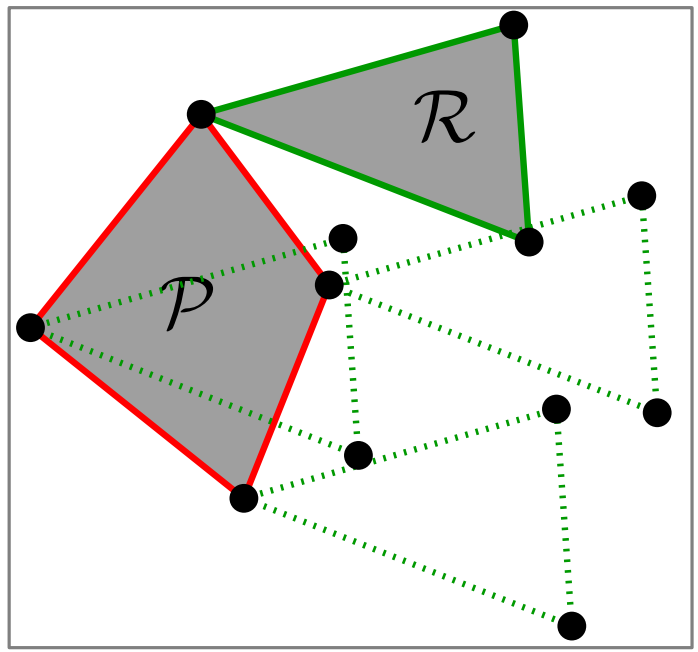
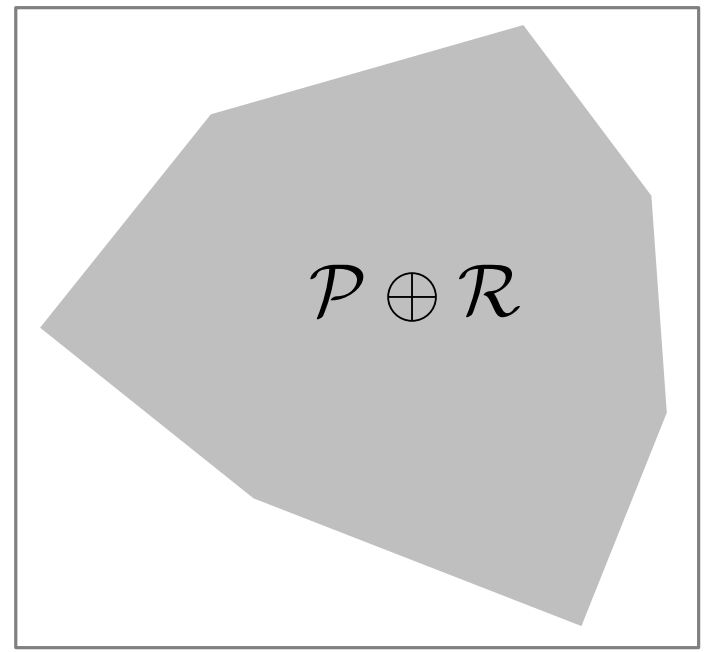
Idea: $\mathcal{P} \oplus \mathcal{R} = \mathsf{CH}\big(\{p + r \mid p \in \mathcal{P}, r \in \mathcal{R}\}\big)$ *(Proof?)*





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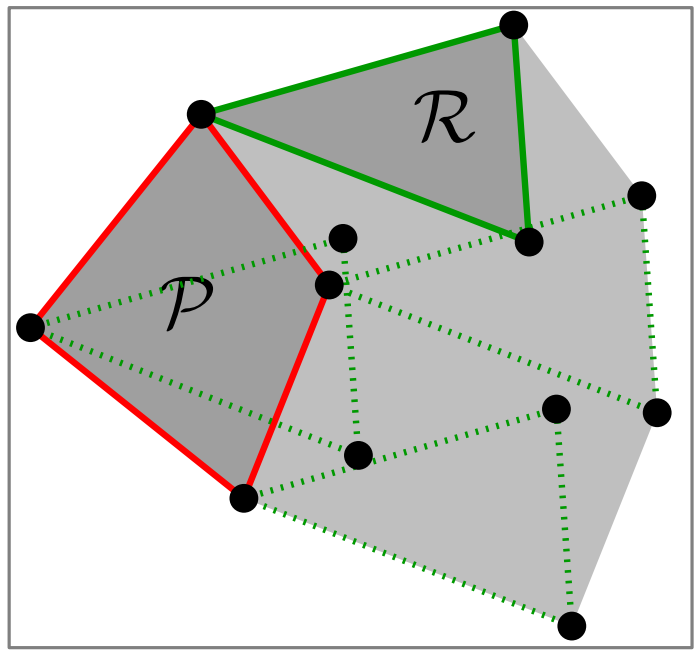

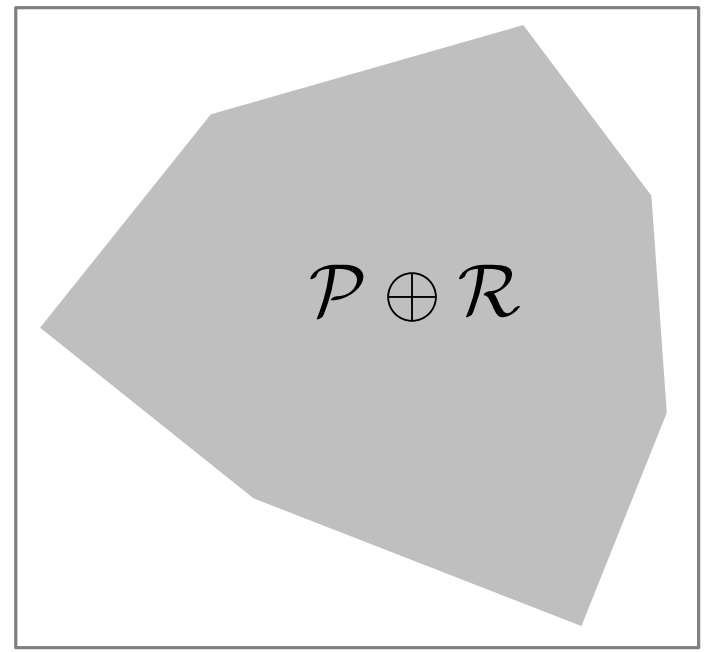

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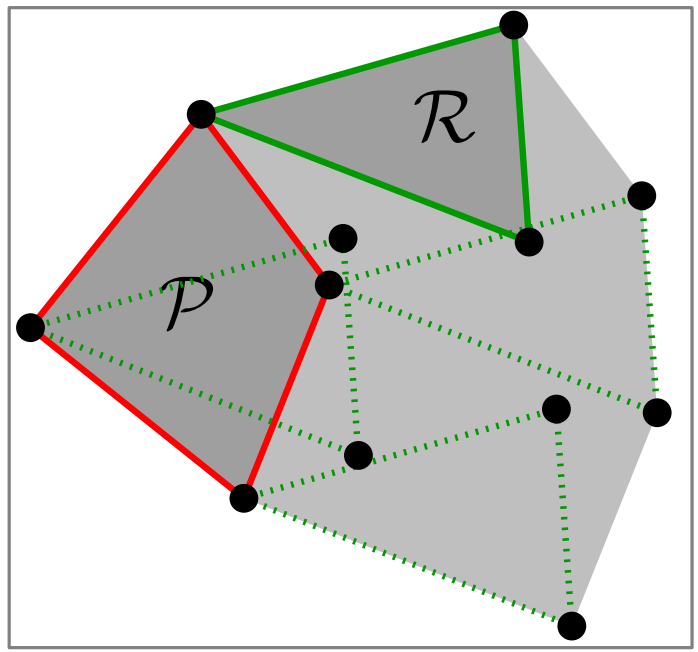
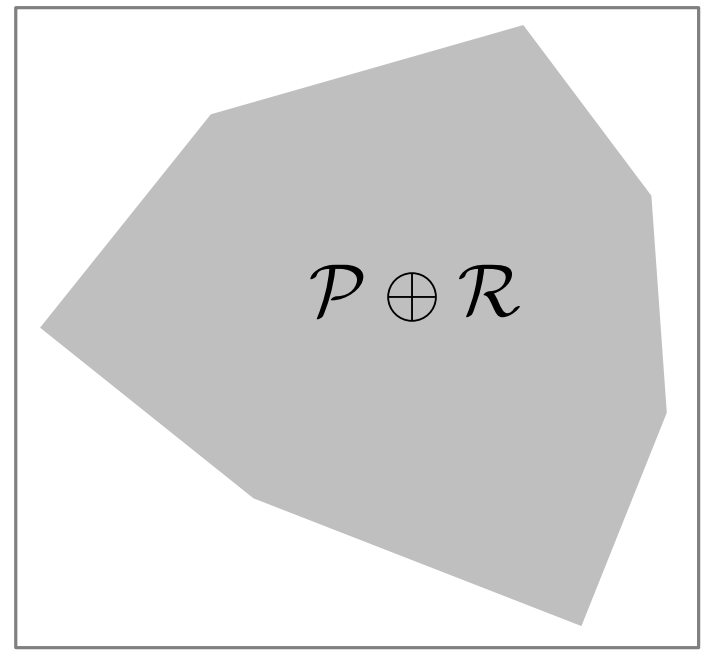
Problem:





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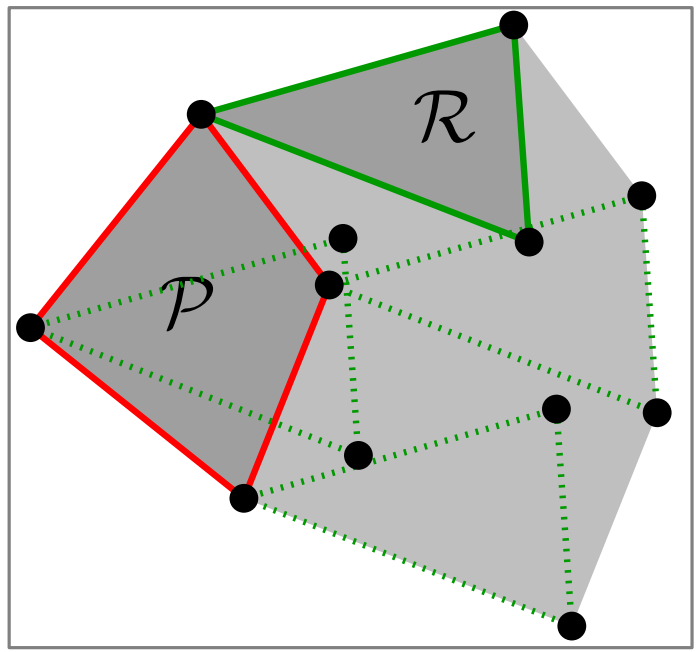

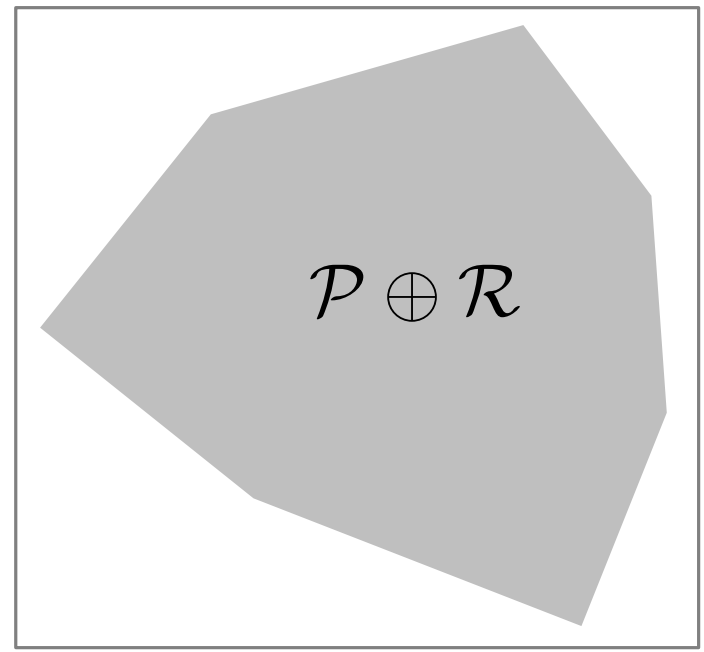

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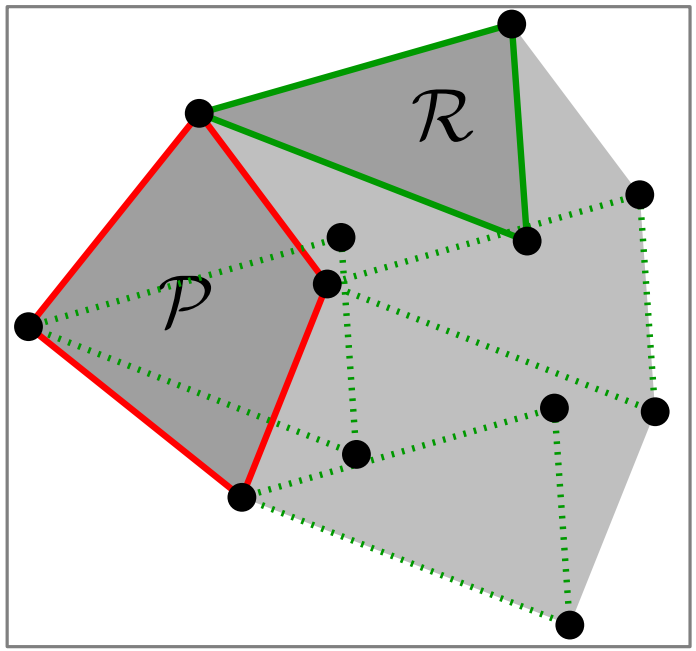
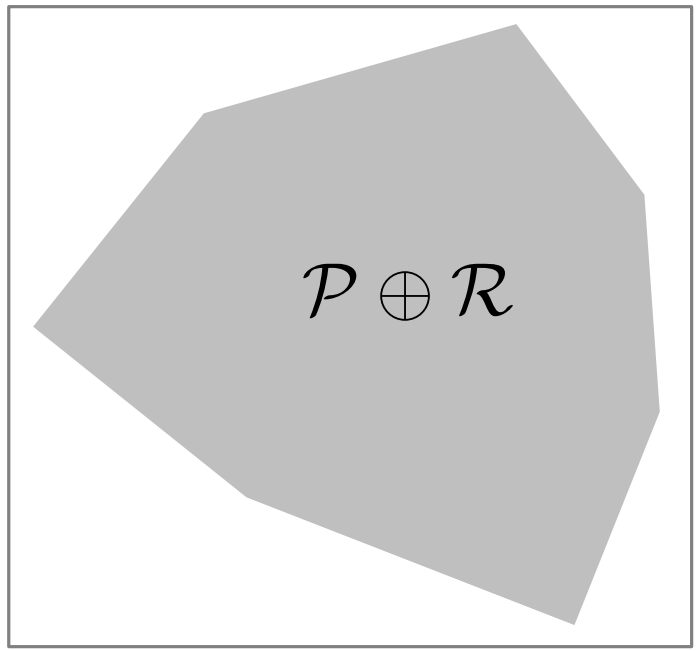
Problem: complexity $\in \Theta(|\mathcal{P}| \cdot |\mathcal{R}|)$ **:-(**





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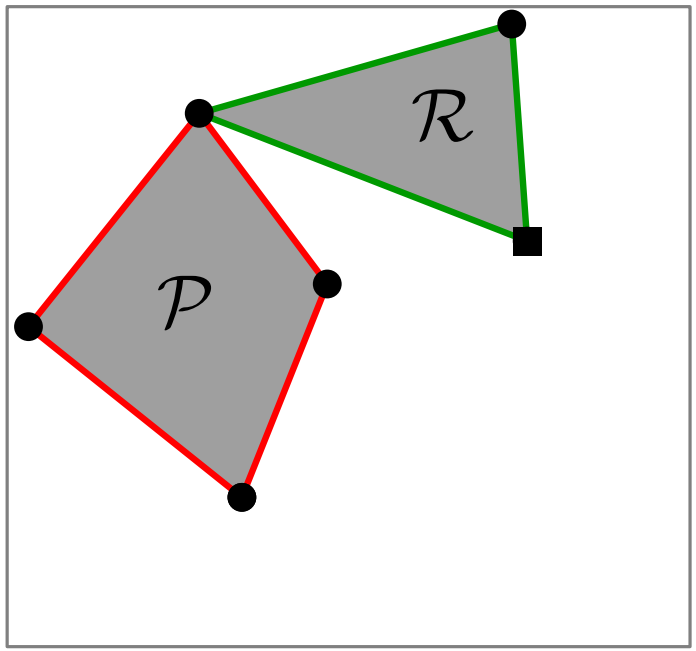

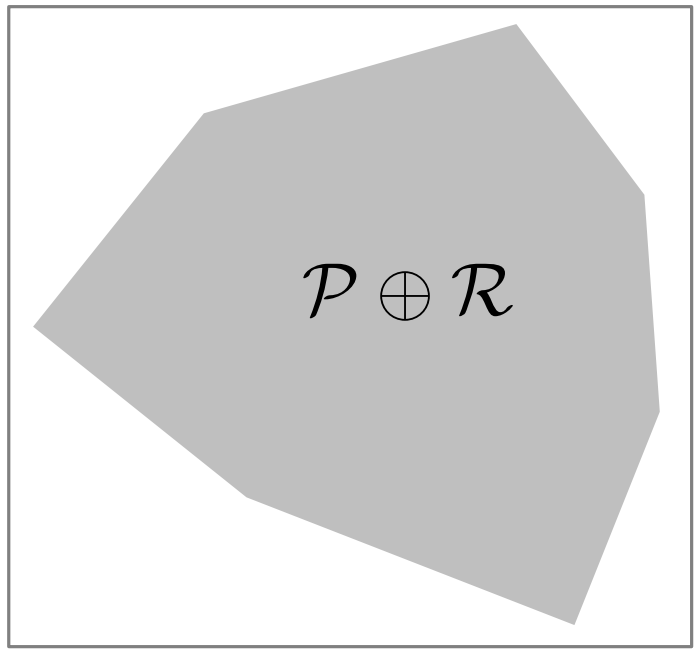

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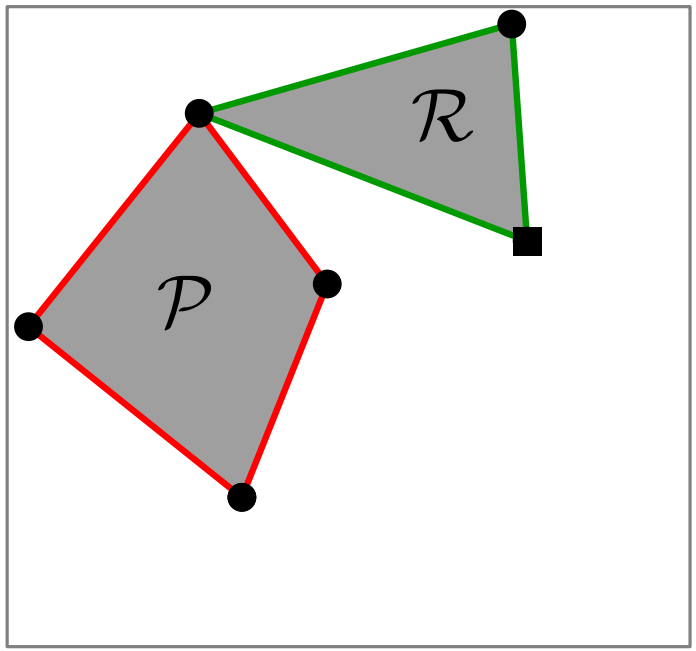

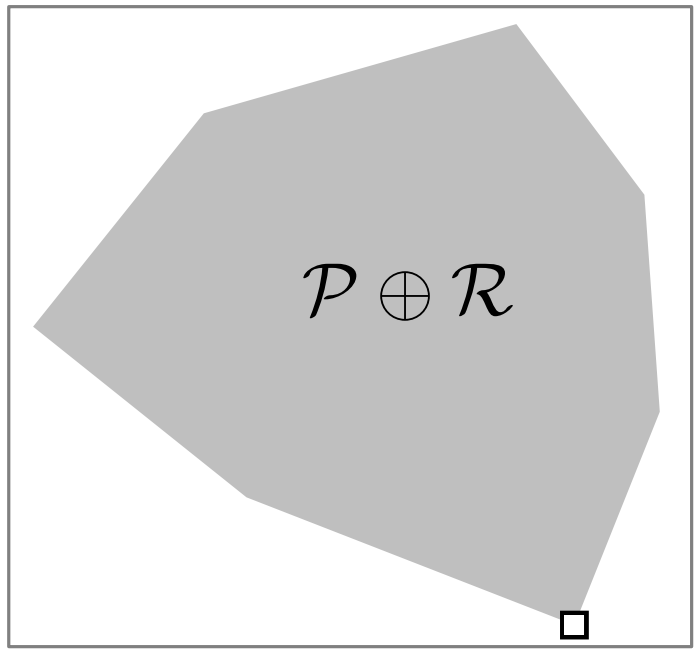

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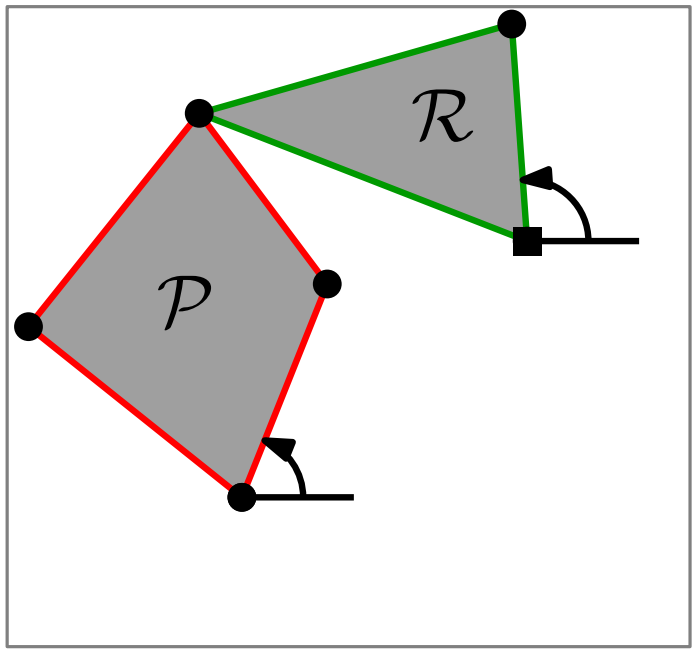

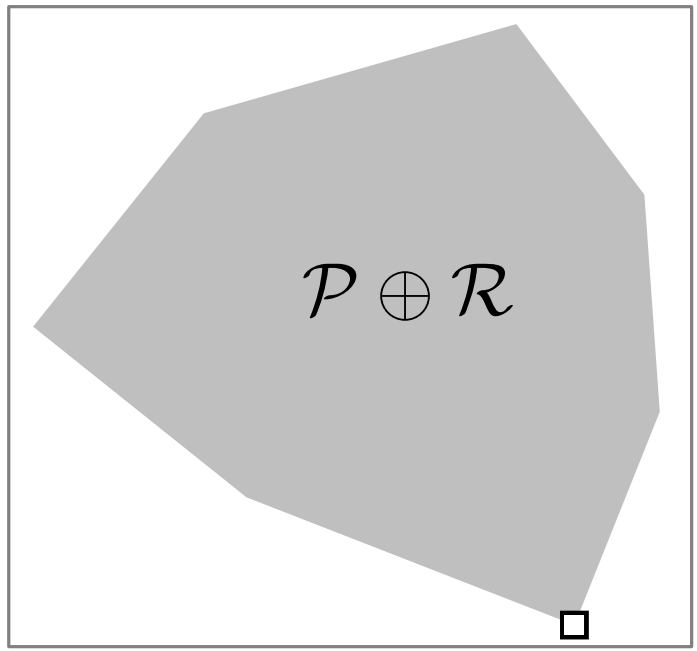

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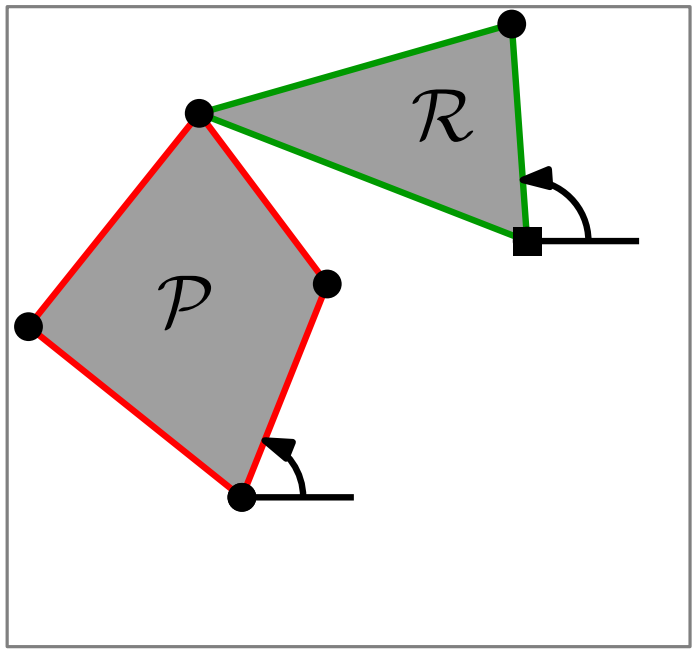

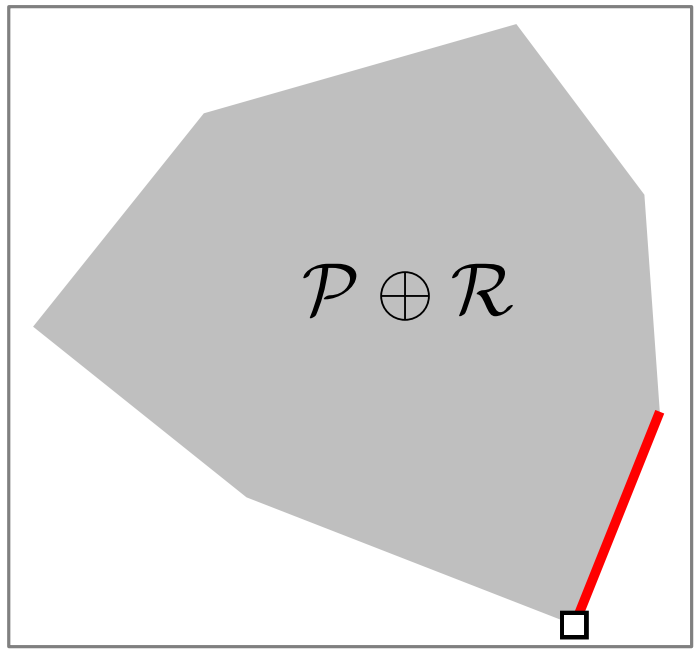

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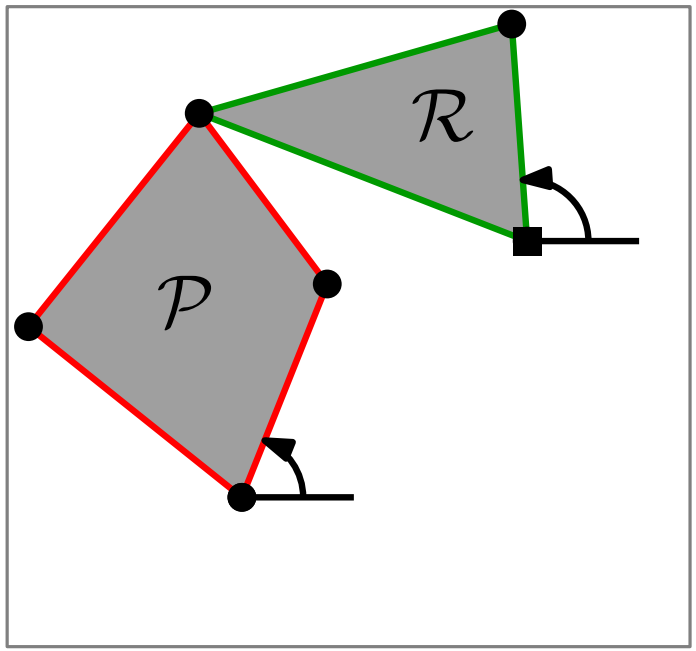

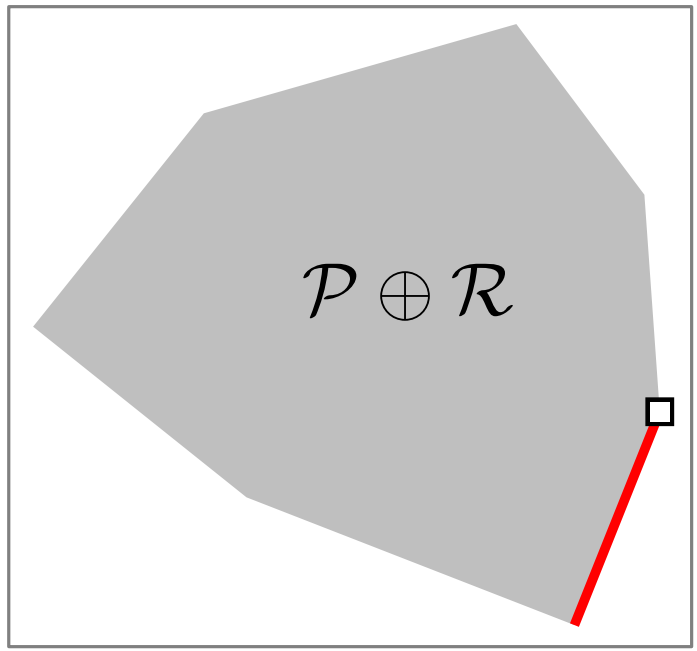

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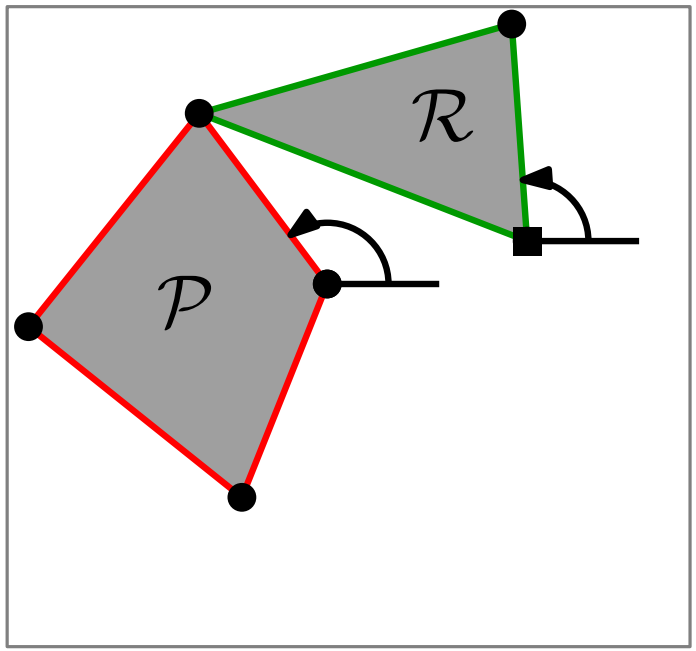

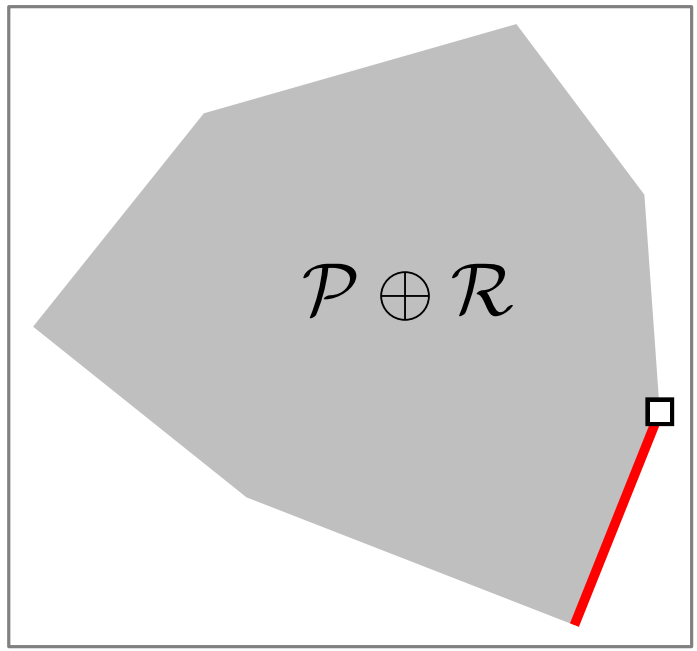

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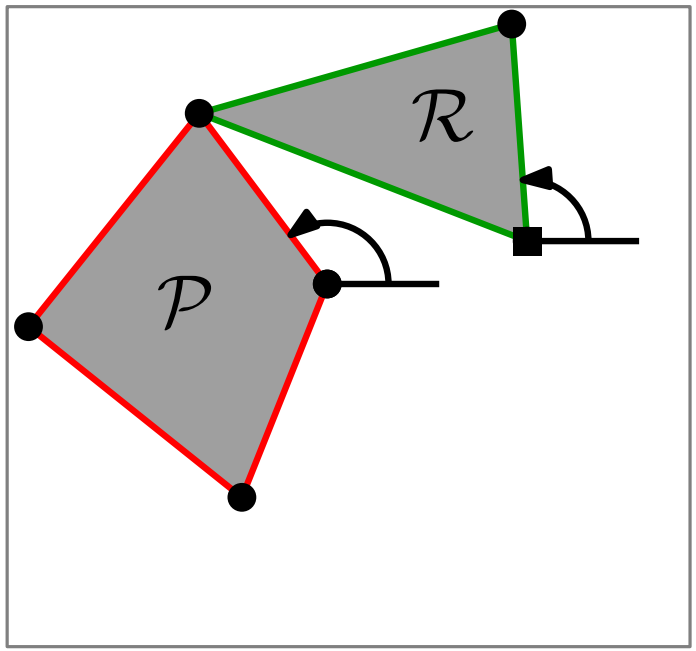

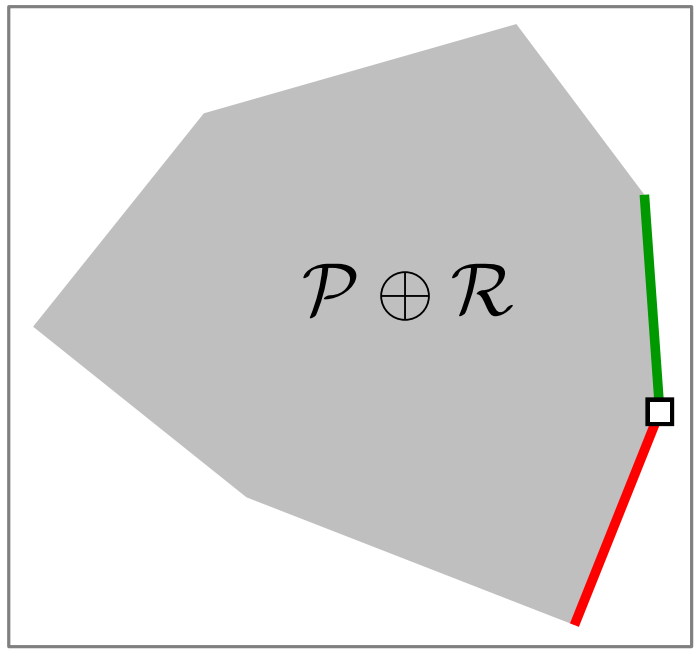

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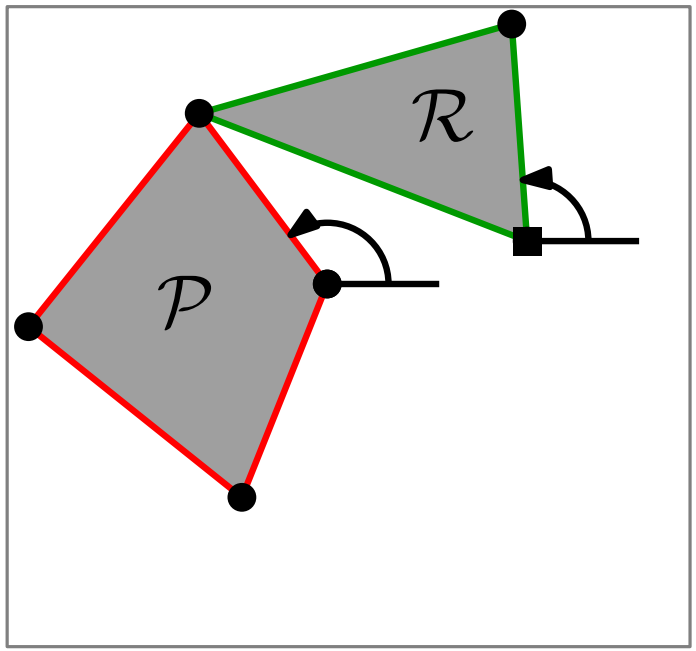

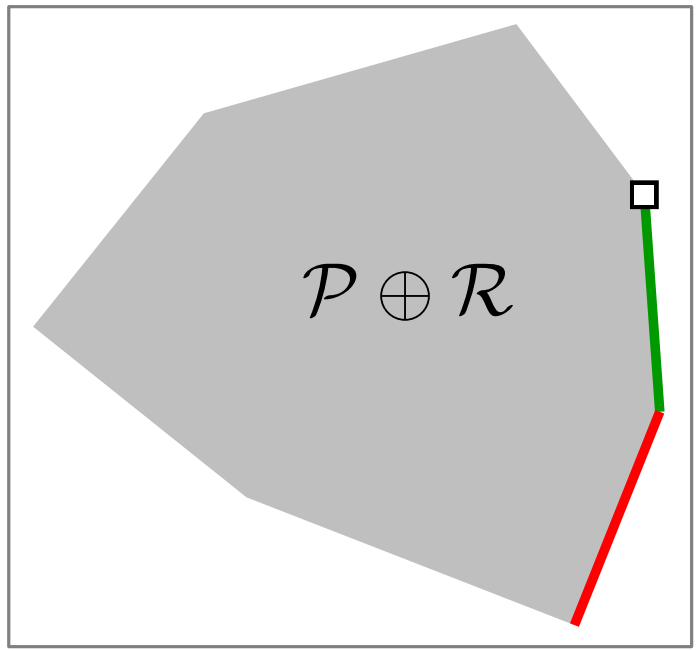

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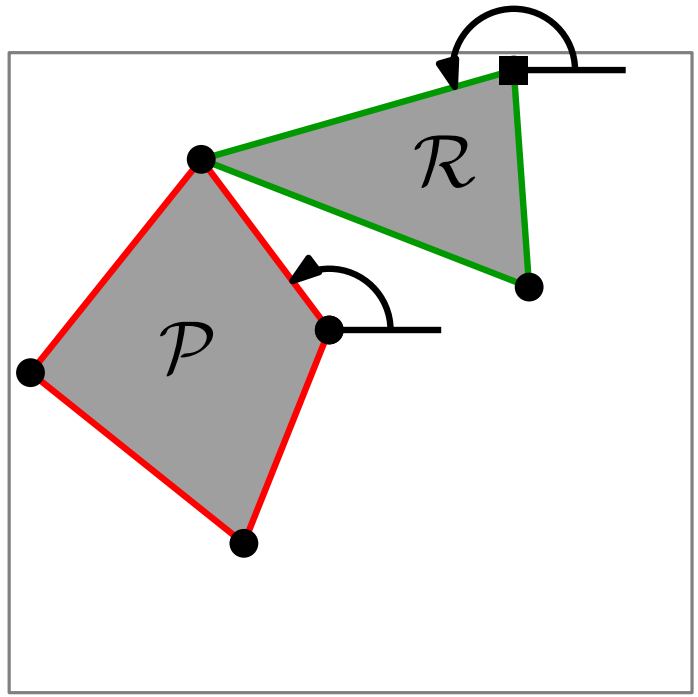

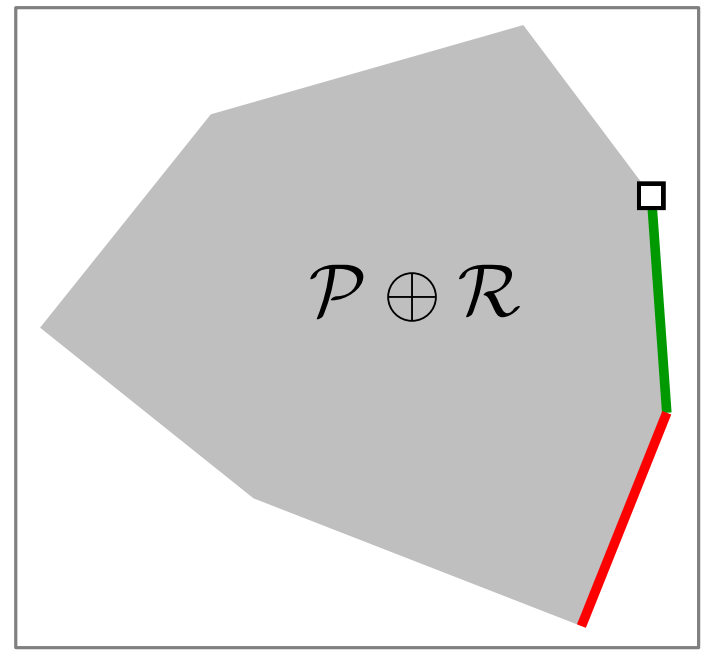

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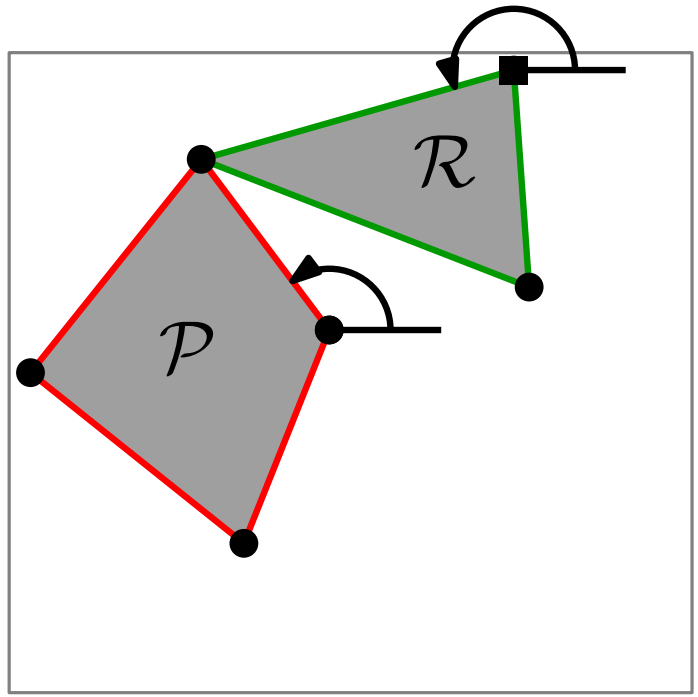

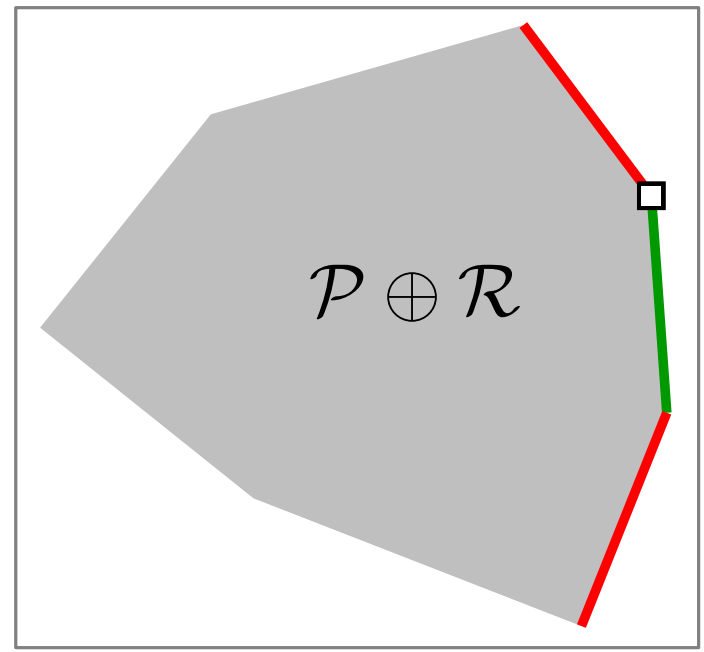

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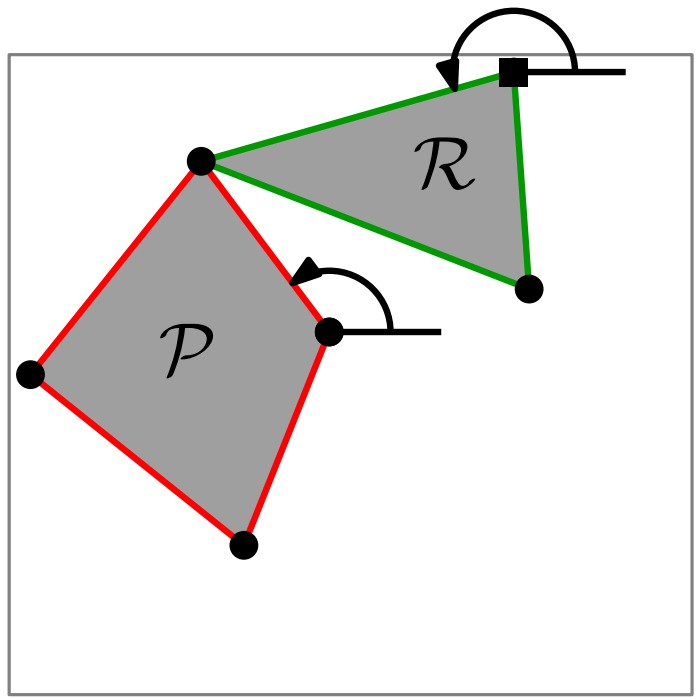

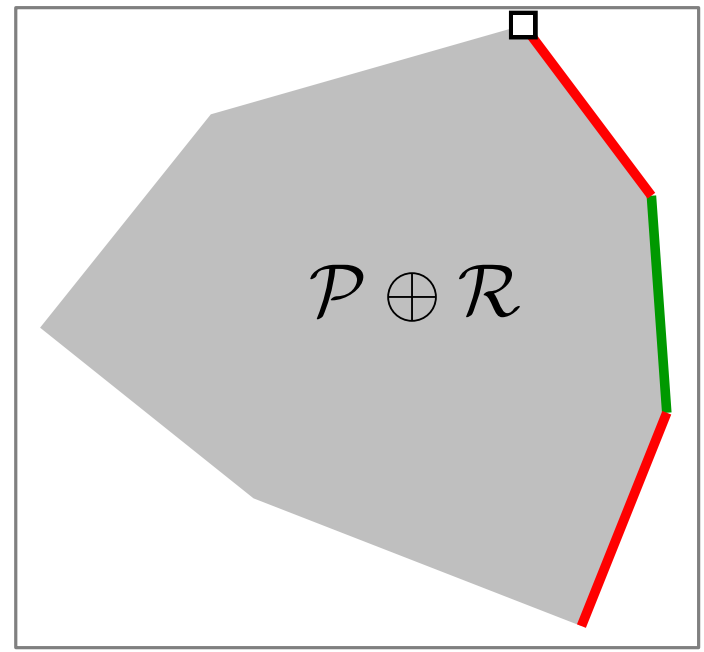

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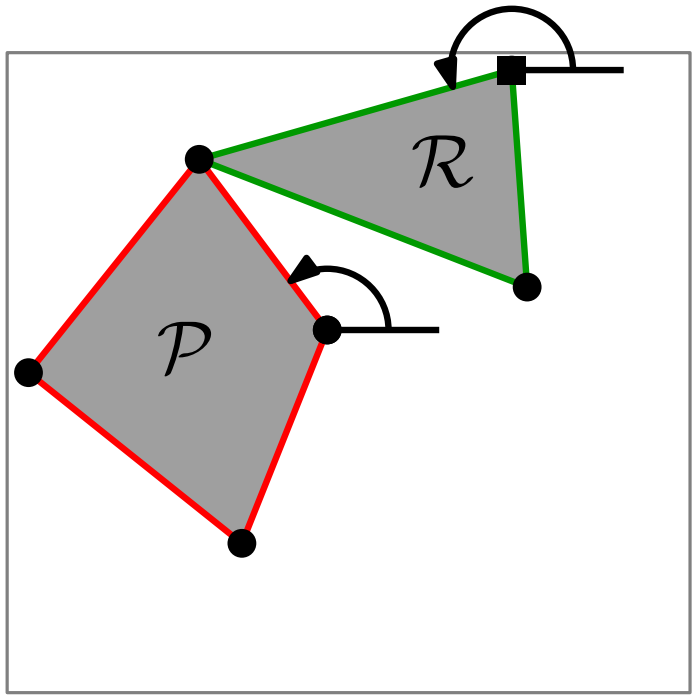

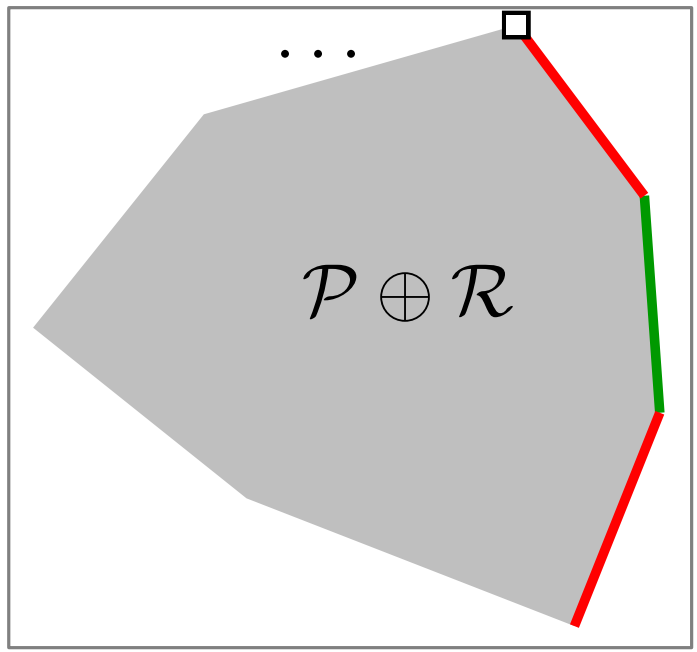

Einschub: Vorlesungsumfrage

Nutzen Sie die Vorlesungsbefragung – zu *konstruktiver* Kritik!

Von besseren Vorlesungen profitieren

- Sie (wenn Sie die nächste Veranstaltung bei mir hören)
- Ihre Nachfolger (im nächsten Wintersemester)
- ich (Gute Lehre macht Spaß! Gute Klausurergebnisse auch!)

Vielen Dank!

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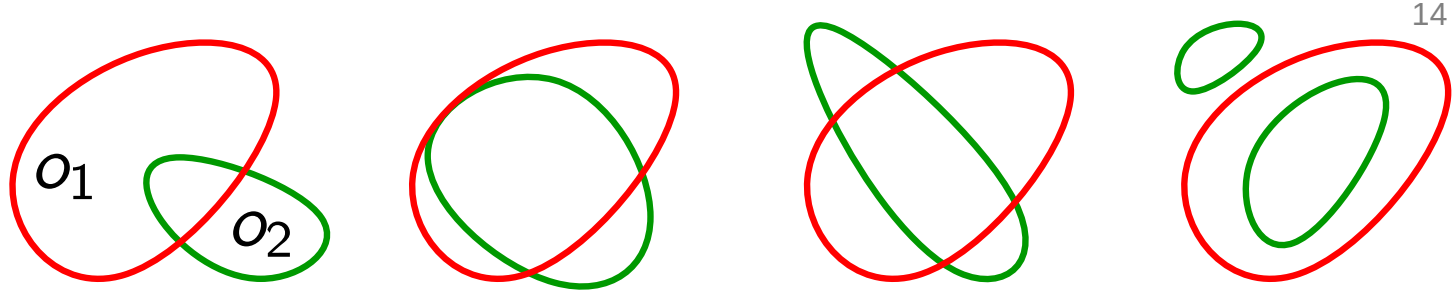
Suchen Sie in Ihrem stud-mail-Konto nach einer Email von EvaSys! Folgen Sie dem Link und geben Sie die TAN ein.

Vielen Dank!

Pseudodisks



Definition: A pair of planar objects o_1 and o_2 is a pair of pseudodisks if:

- $\partial o_1 \cap \text{int}(o_2)$ is connected, and
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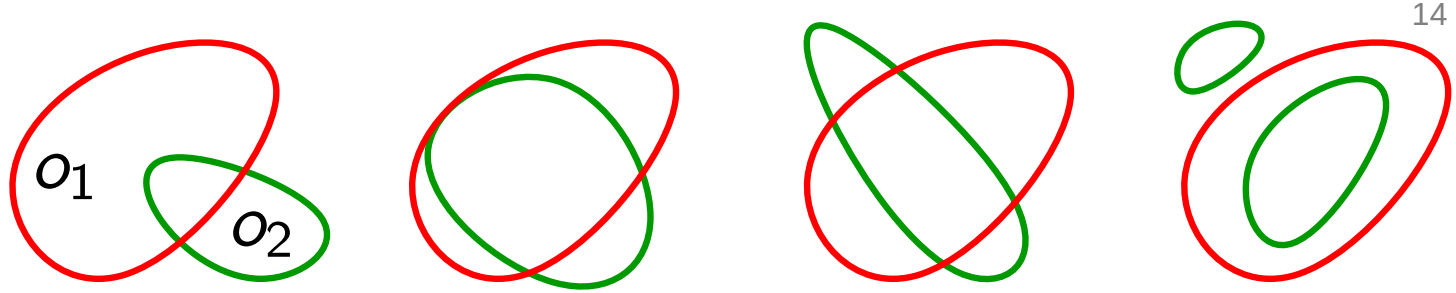

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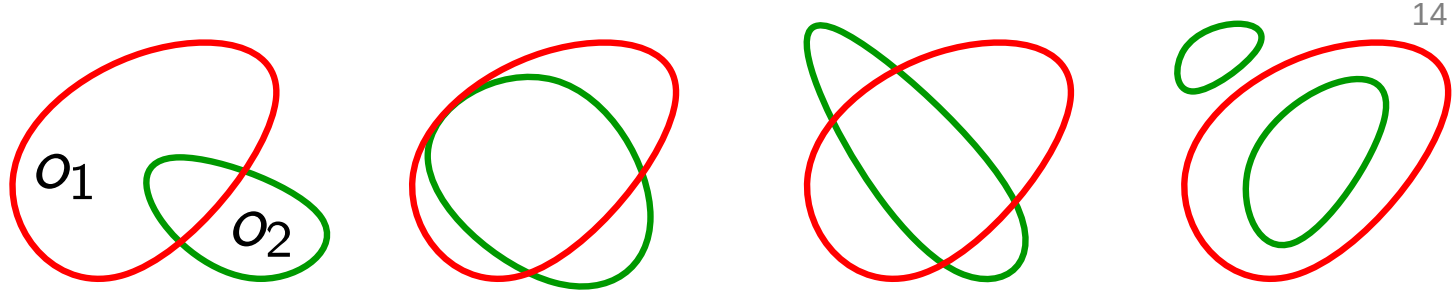

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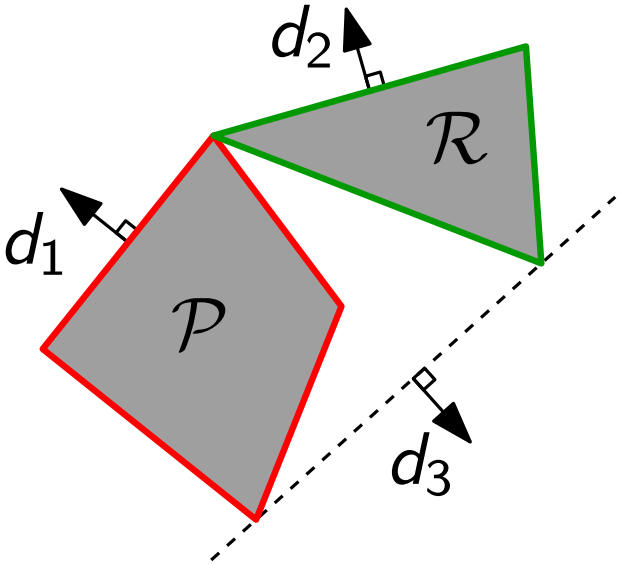
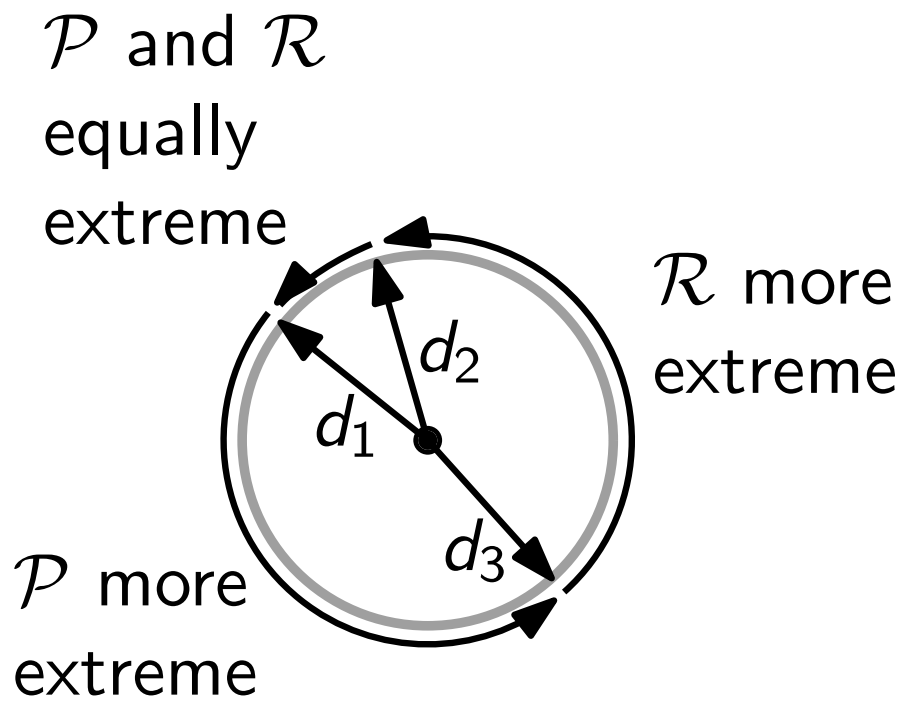
Observation: A pair of polygonal pseudodisks defines at most two boundary crossings.

Extreme directions

Observation: Let $\mathcal{P}$, $\mathcal{R}$ be interior-disjoint convex polygons. Let d_1 and d_2 be directions in which $\mathcal{P}$ is more extreme than $\mathcal{R}$.
Then $\mathcal{P}$ is more extreme than $\mathcal{R}$ either in $[d_1, d_2]$ or in $[d_2, d_1]$.





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Theorem: If $\mathcal{P}_1$ and $\mathcal{P}_2$ are convex polygons with disjoint interiors, and $\mathcal{R}$ is another convex polygon, then $\mathcal{P}_1 \oplus \mathcal{R}$ and $\mathcal{P}_2 \oplus \mathcal{R}$ is a pair of pseudodisks.

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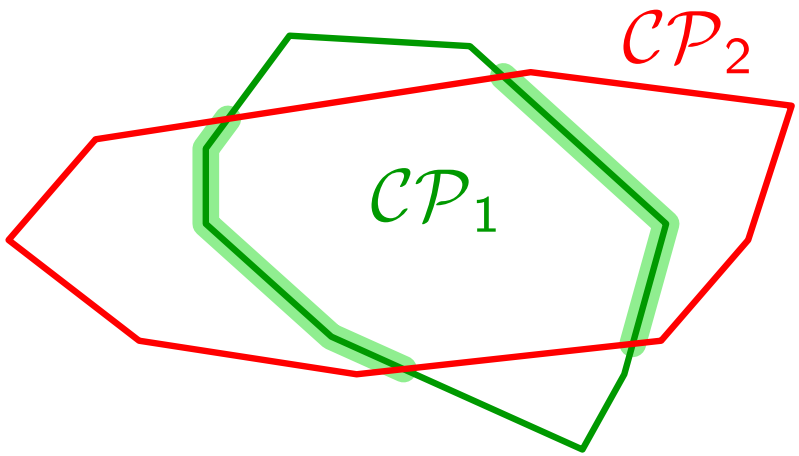

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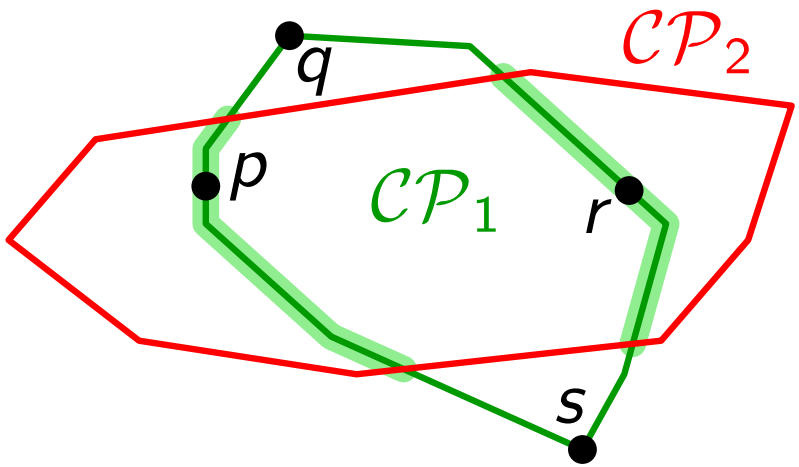

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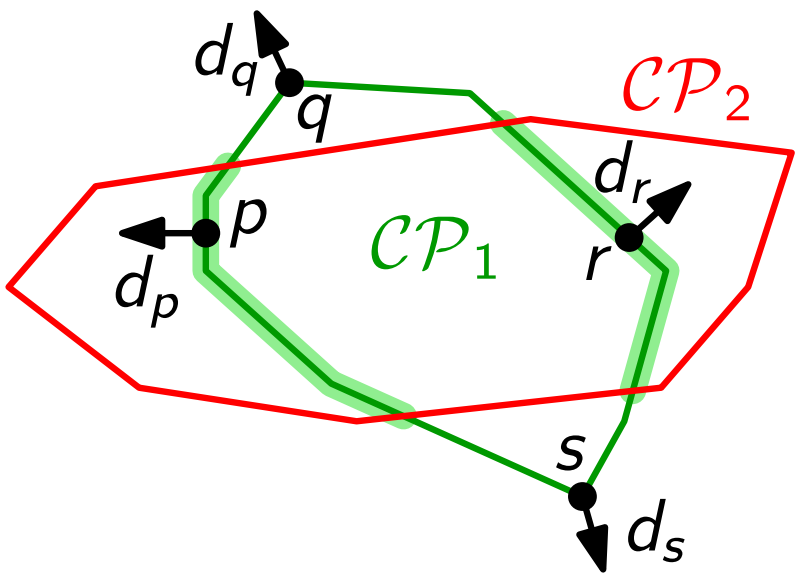

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Theorem: If $\mathcal{P}_1$ and $\mathcal{P}_2$ are convex polygons with disjoint interiors, and $\mathcal{R}$ is another convex polygon, then $\underbrace{\mathcal{P}_1 \oplus \mathcal{R}}_{\mathcal{CP}_1}$ and $\underbrace{\mathcal{P}_2 \oplus \mathcal{R}}_{\mathcal{CP}_2}$ is a pair of pseudodisks.

Proof. It suffices to show: $\partial\mathcal{CP}_1 \cap \text{int } \mathcal{CP}_2$ is connected.

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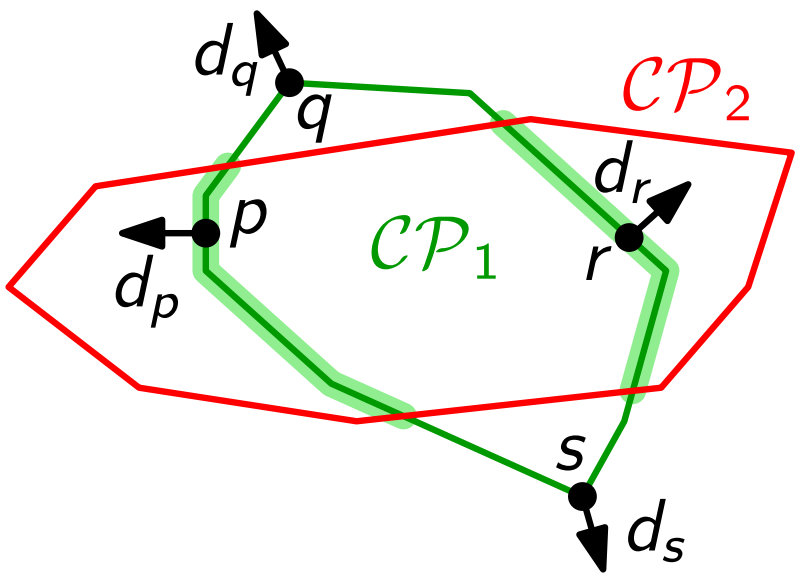

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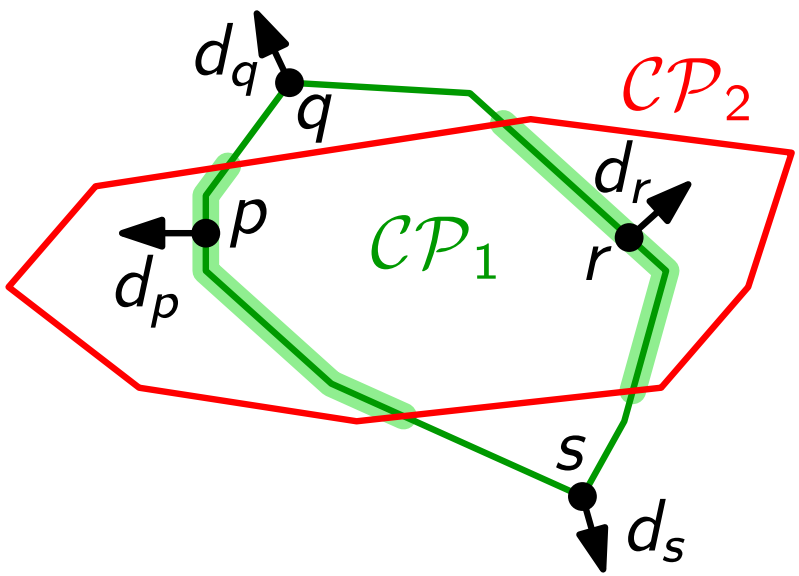

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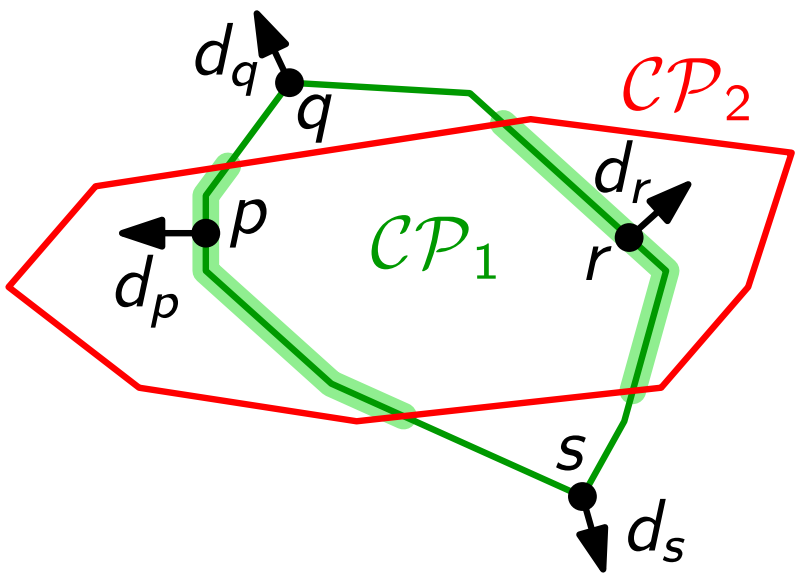

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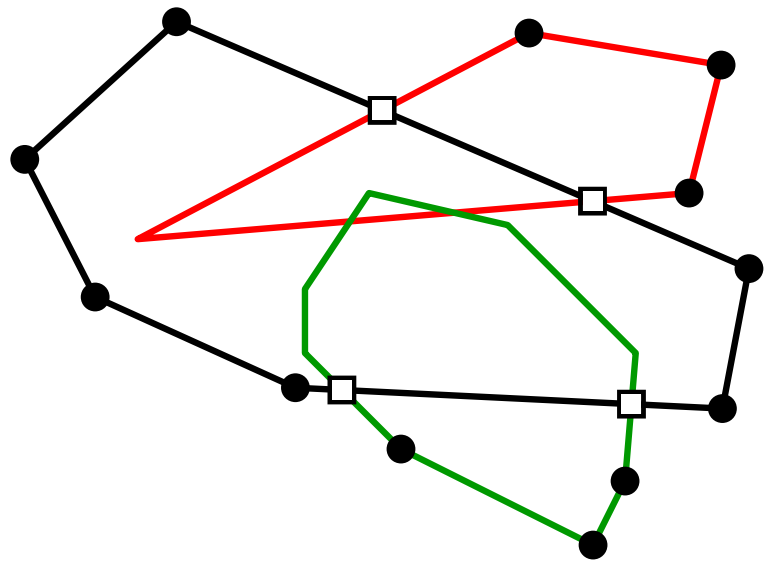
Theorem: A collection S of convex polygonal pseudodiscs with n vtc in total has a union with $\leq 2n$ vtc.



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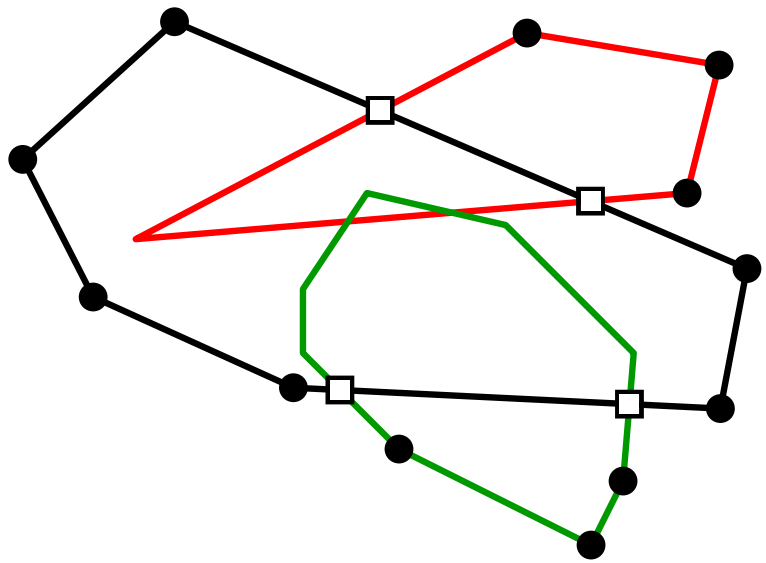

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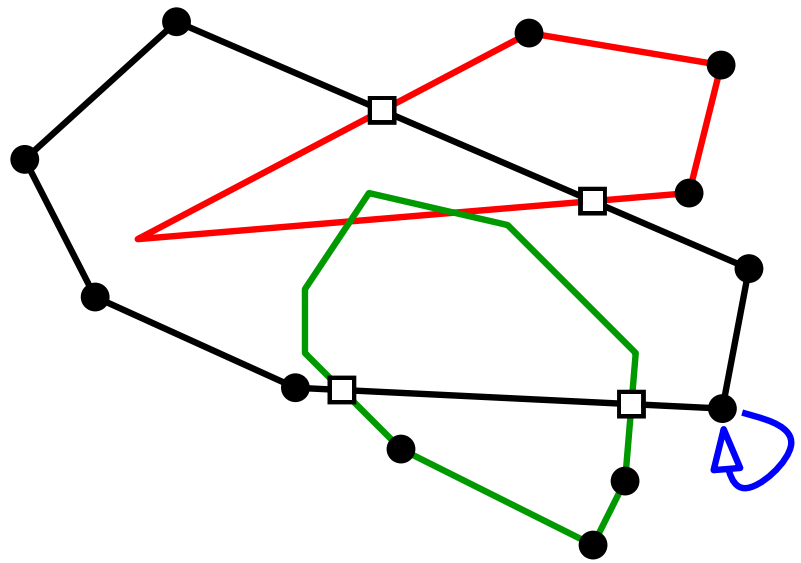

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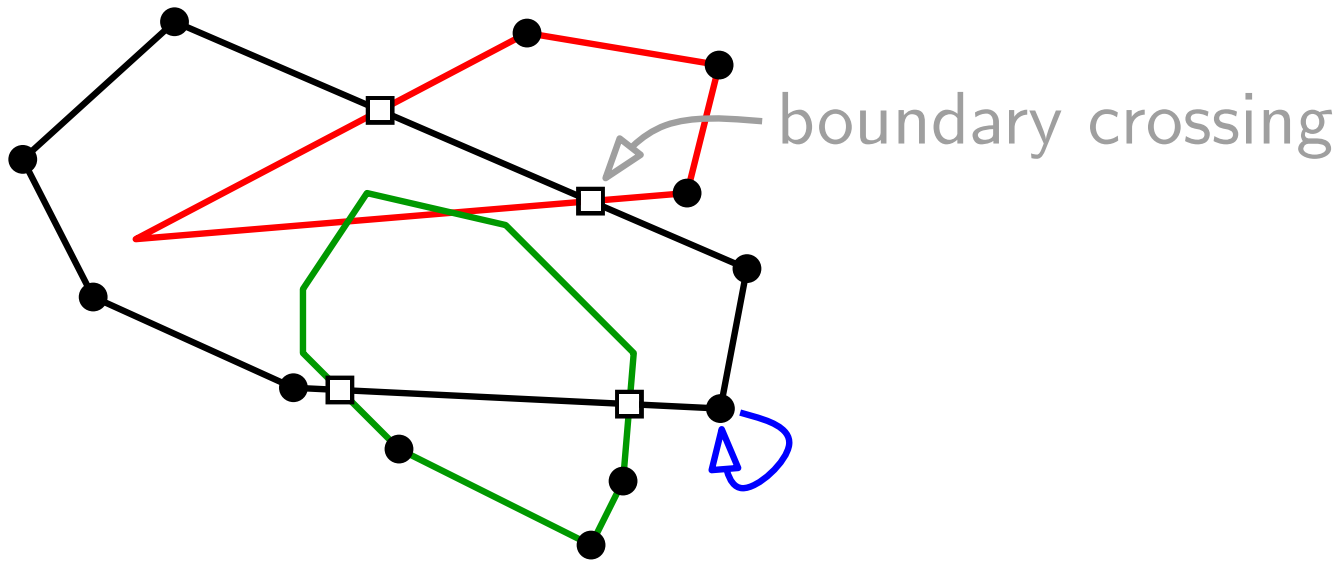

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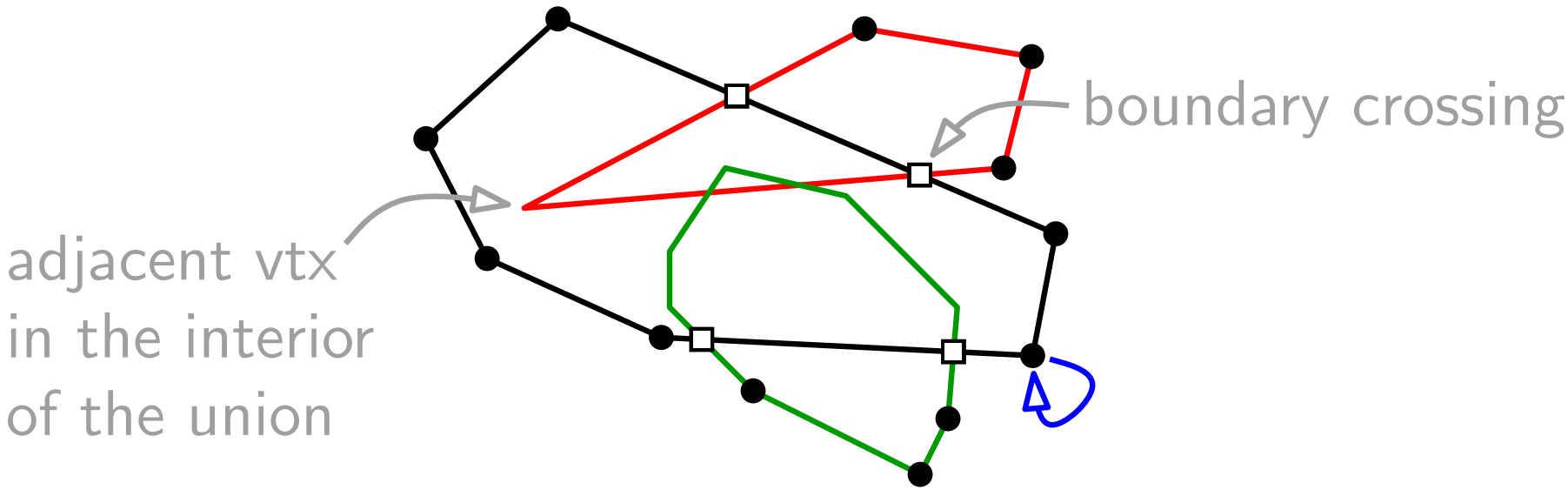

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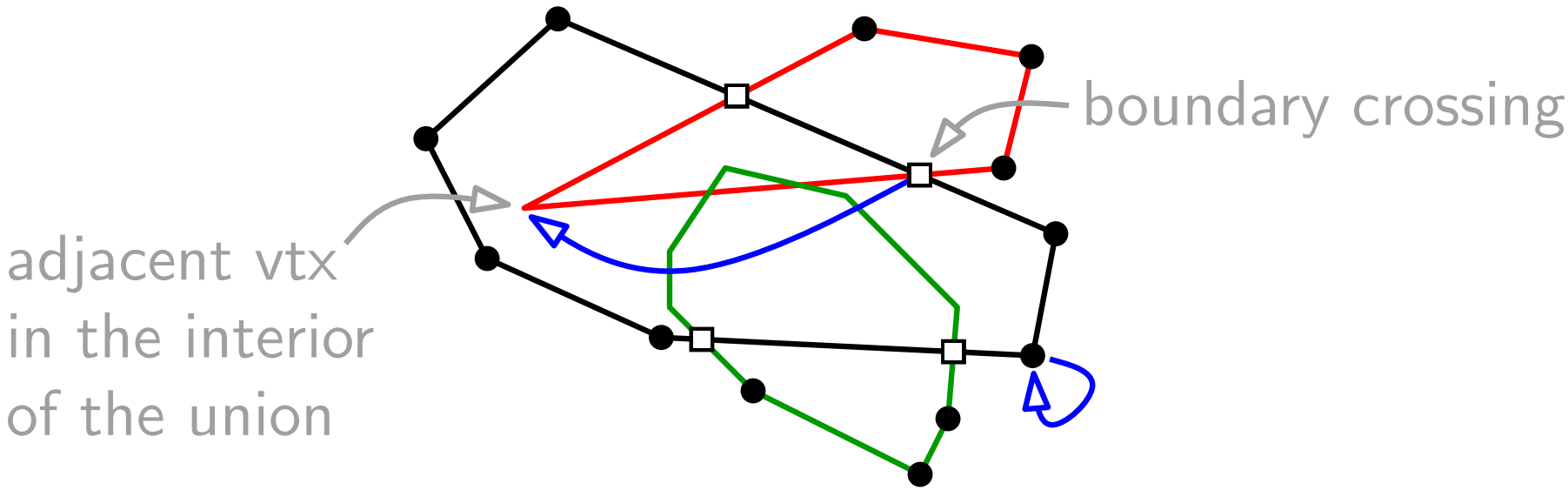

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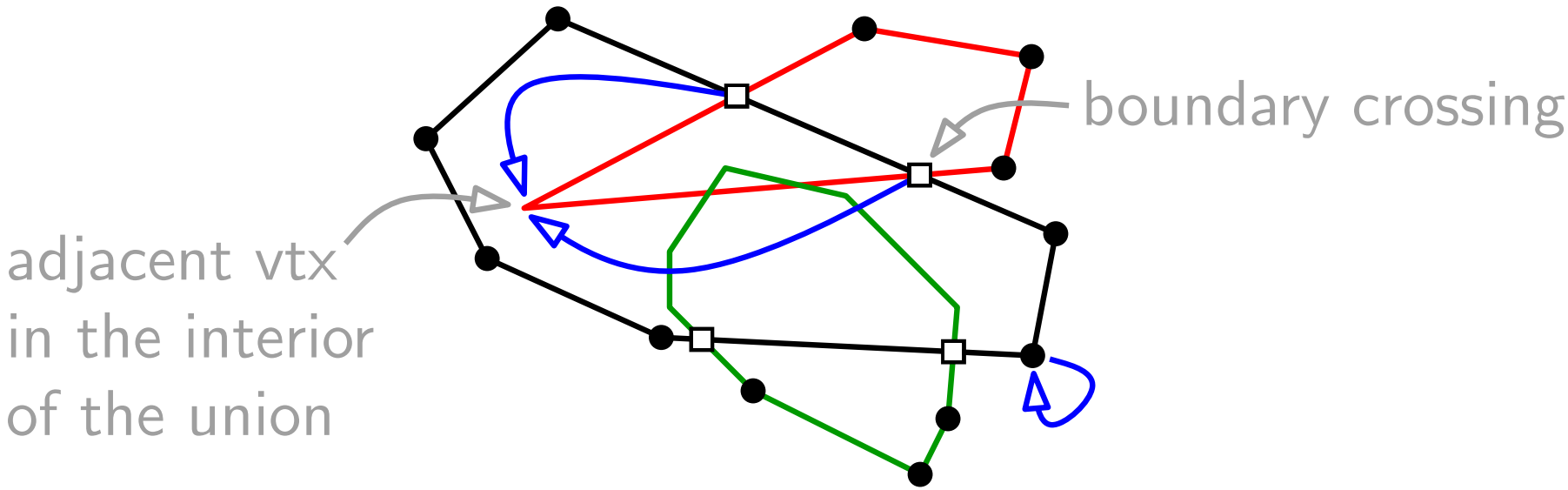

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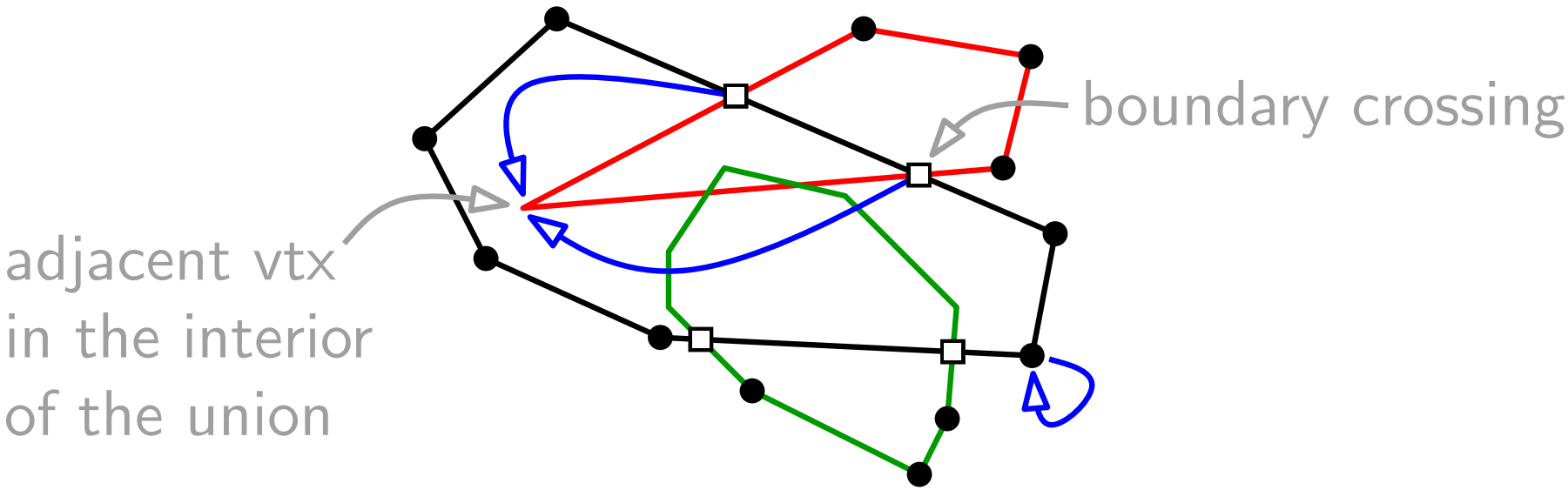

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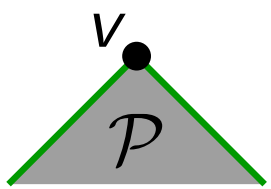

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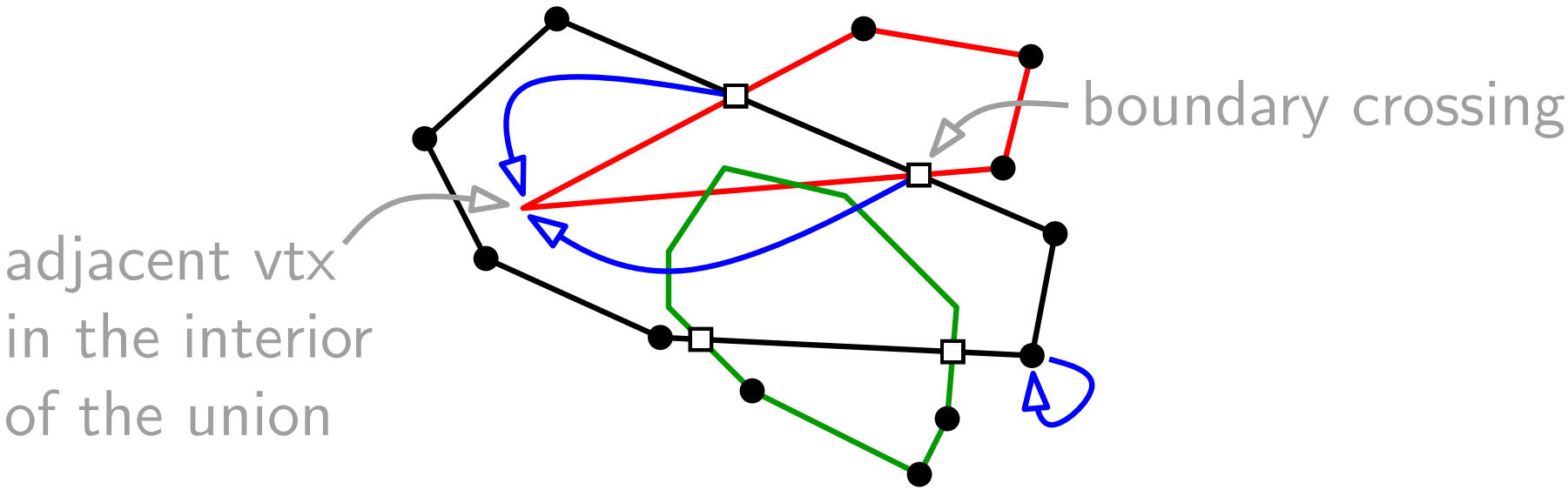

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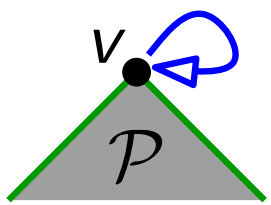

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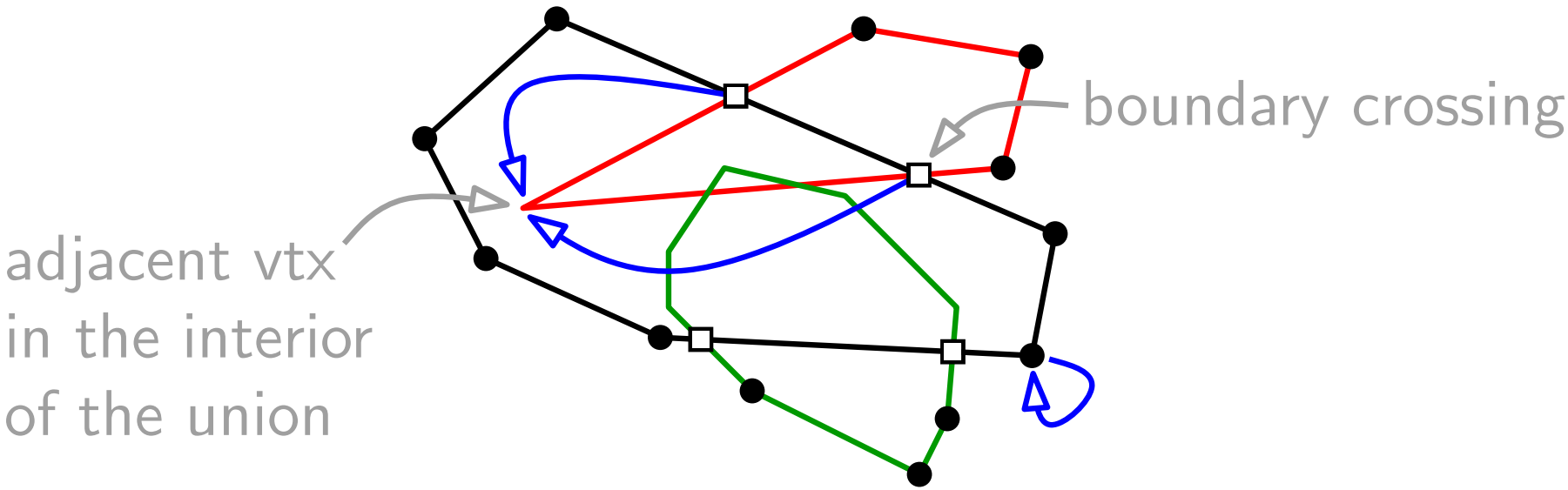

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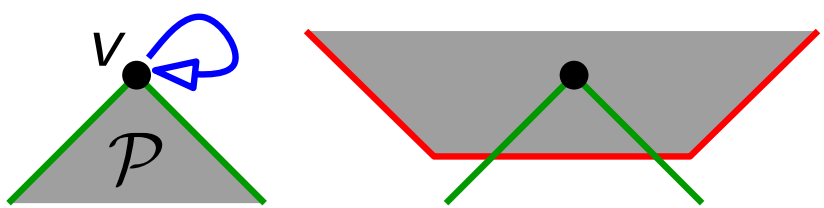

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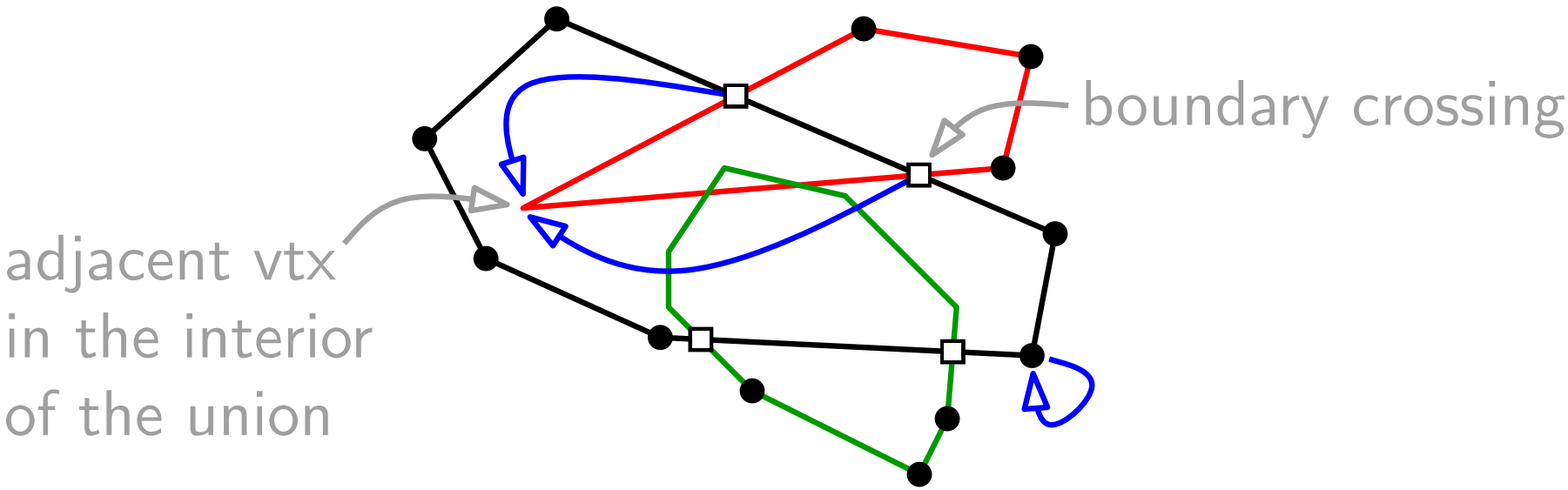

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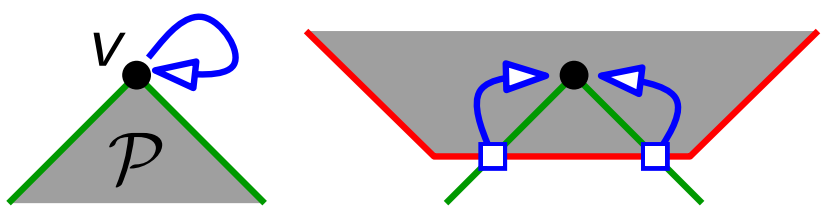

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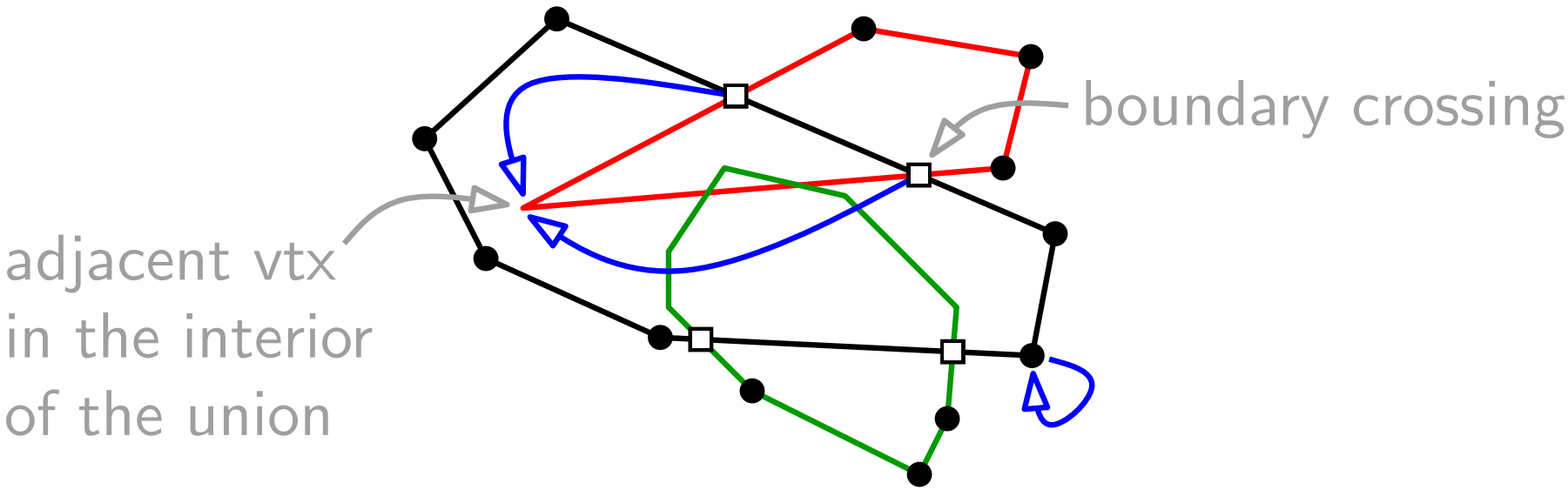

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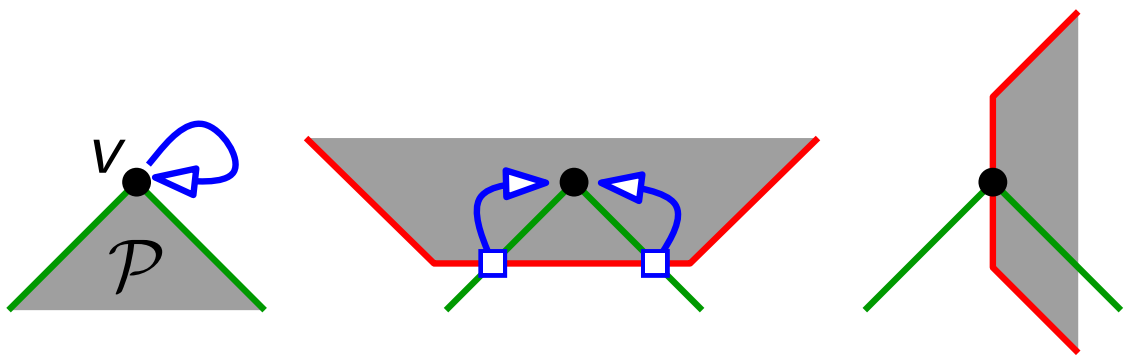

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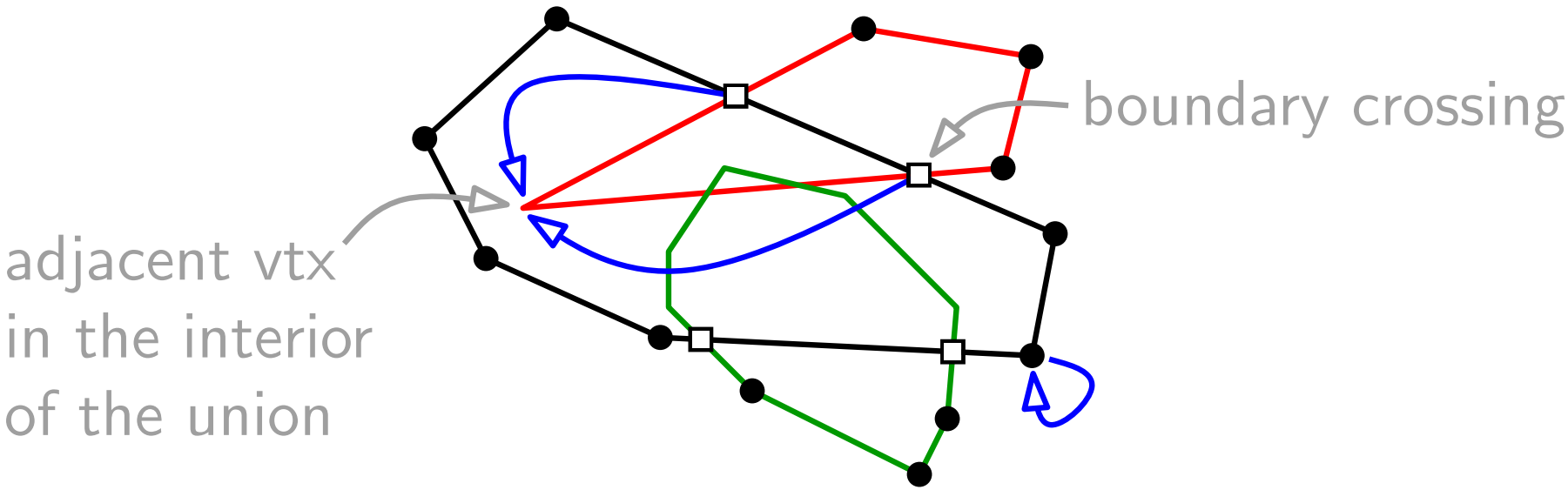

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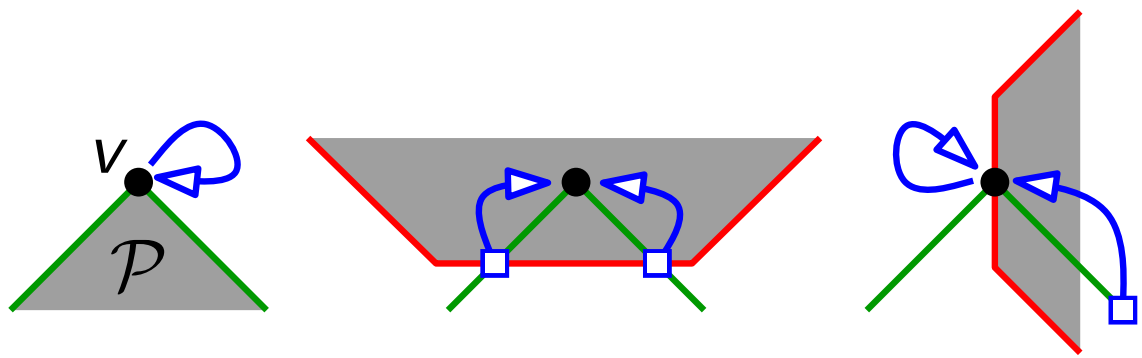

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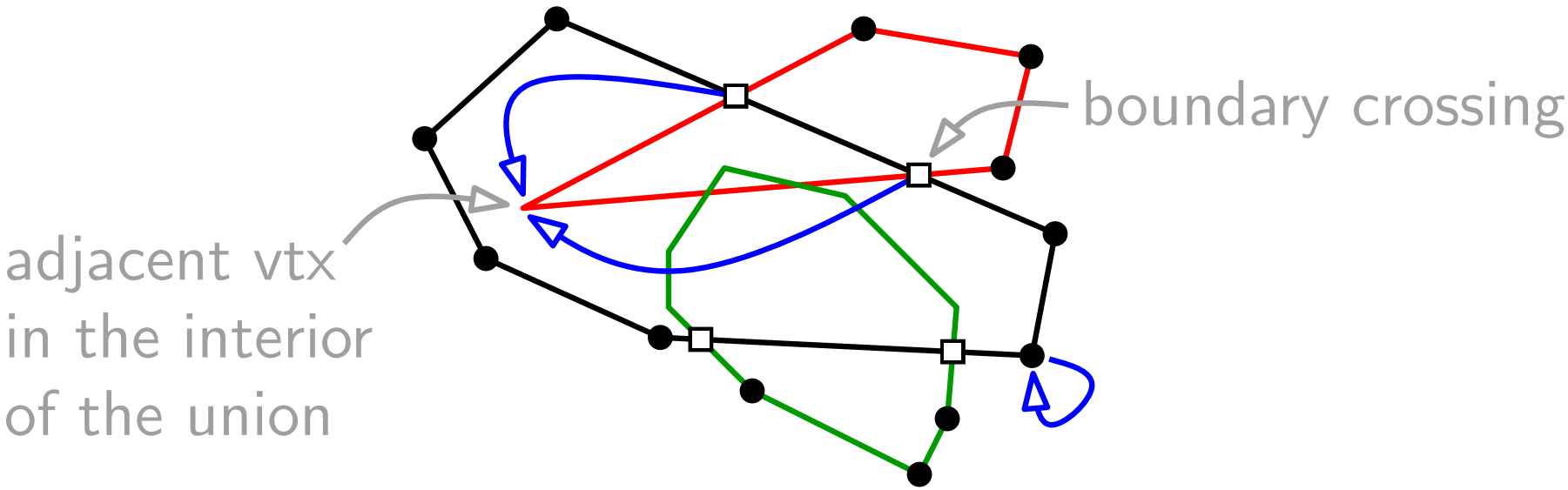

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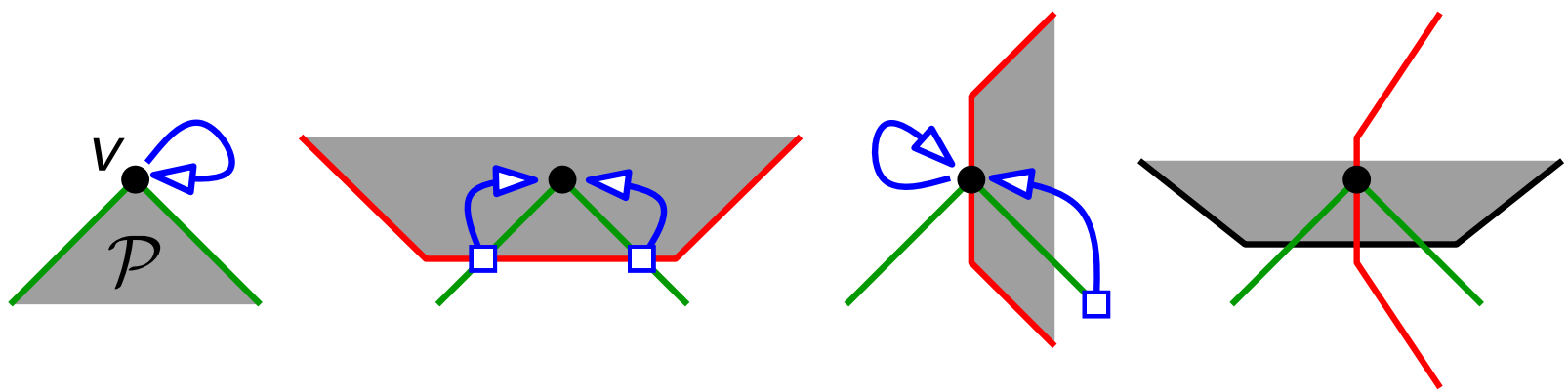

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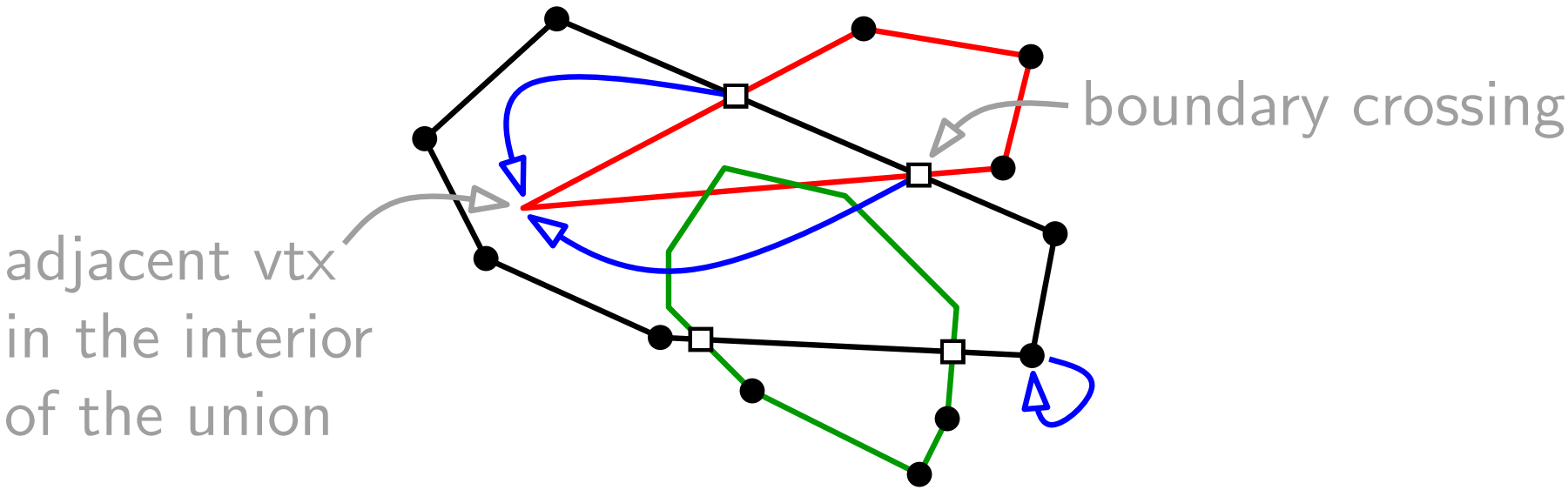

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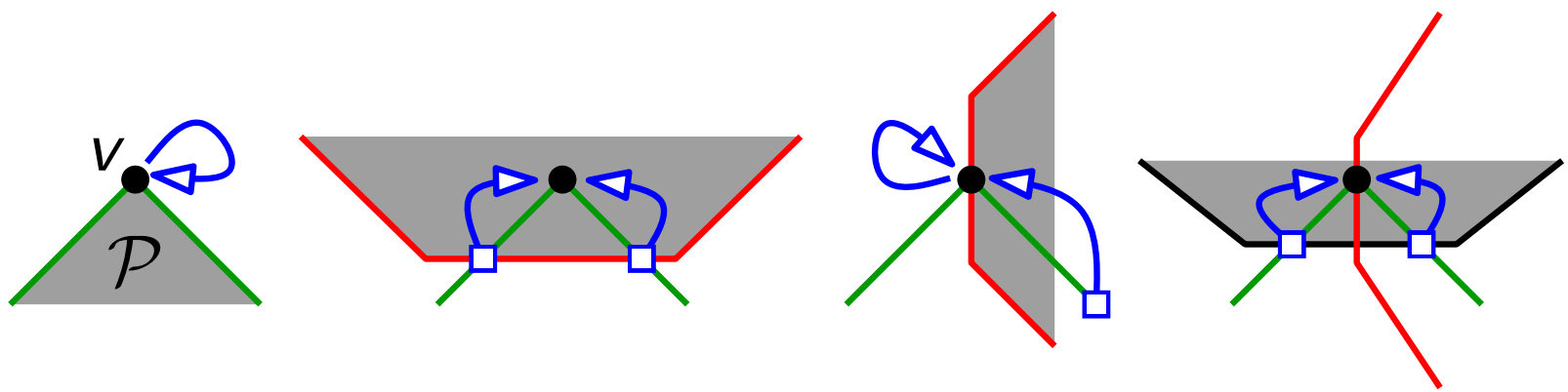

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Theorem: Let $\mathcal{R}$ be a constant-complexity convex robot, translating among a set S of disjoint polygonal obstacles with n edges in total.

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