Lehrstuhl für Informatik I
Effiziente Algorithmen und wissensbasierte Systeme
Universität Würzburg

Würzburg, December 3, 2014

Prof. Dr. Alexander Wolff
Dipl.-Inform. Philipp Kindermann

Homework Assignment #9
Computational Geometry (Winter Term 2014/15)

Exercise 1

In this exercise, we look at the number of different triangulations that a set of n points in the plane admits.

a) Prove that no set of n points can be triangulated in more than $2^{\binom{n}{2}}$ ways. **[2 points]**

b) Can you give an (asymptotically) sharper upper bound on the number of possible triangulations? **[2 extrapoints]**

c) Prove that there are sets of n points that can be triangulated in at least $2^{n-2\sqrt{n}+1}$ different ways. **[4 points]**

d) Can you give an (asymptotically) sharper lower bound on the number of possible triangulations? **[2 extrapoints]**

Exercise 2

Prove that any two triangulations of a planar point set can be transformed into each other by edge flips.

Hint: Start by showing that any two triangulations of a convex polygon can be transformed into each other by edge flips. **[5 extrapoints]**

Exercise 3

Prove that the smallest angle of any triangulation of a convex polygon whose vertices lie on a circle is the same. This implies that *any* completion of the Delaunay *graph* of a set of points to the Delaunay *triangulation* maximizes the minimum angle.

Hint: You can use the result of Exercise 2. **[4 points]**

Please turn over.

Exercise 4

Let P be a set of points in the plane and let $G_P = (P, E_P)$ be the graph with $\{p, q\} \in E_P$ if and only if p and q are the only points in P that are contained in the disk D_{pq} with diameter $\overline{pq}$, i.e., $D_{pq} \cap P = \{p, q\}$.

a) Prove that the Delaunay graph of P contains the graph G_P. **[3 points]**

b) Prove that $\{p, q\}$ is an edge of G_P if and only if the Delaunay edge between p and q intersects its dual Voronoi edge. **[3 points]**

c) Show how to compute G_P for a set P of n points in $O(n \log n)$ time. **[4 points]**

This assignment is due at the beginning of the next lecture, that is, on December 10 at 10:15. Solutions will be discussed in the tutorial on Friday, December 12, 14:00–15:30 in room SE I.