Domiyes

Algorithmen für Programmierwettbewerbe

Sommersemester 2021

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The Problem

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- Adjacent endpoints have the same number.

![Diagram of dominoes on a board]

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![Diagram of dominoes on a board with numbered endpoints]

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  - there is a node in $V$ for each domino endpoint.
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- In the *domino graph* $D = (V, E)$...
  - there is a node in $V$ for each domino endpoint.
  - $uv \in E$ iff $u$ is adjacent to $v$ and $uv$ is not on the domino
Modeling the Problem — II

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Maximum Matchings in Forests

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1. Find all leaves $L$.
2. For each edge $uv$ with $v \in L$...
   \[ M = M \cup \{uv\}. \]
   Delete $u$ and $v$. 

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Domino graphs can have cycles! $\Rightarrow$ They are not trees. $\Rightarrow$ Our $O(V)$ algorithm will not work here.
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**Question:** What is the maximum degree $\Delta$ in the domino graph?
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![Diagram of domino graphs]

1. Single domino
2. Connected dominoes
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Let us find out using an example...

$$\Rightarrow \Delta \leq 3$$
Max. Matchings in General Graphs

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  - Micali-Vazirani Algorithm – $\mathcal{O}(\sqrt{VE})$, way too complicated!
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- We know that in our domino graphs $\Delta \leq 3$. Can we specialise them further?
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- Hopefully, such a specialisation will give us faster and/or simpler algorithms!
Domino Graph is Bipartite

Theorem. Any domino graph $D = (V, E)$ is bipartite.
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**Proof.** Domino graphs are subgraphs of the *infinite grid graph*.

The infinite grid graph can be two-coloured. Thus, we can divide $V$ into two edge-disjoint sets $A$ and $B$. 
Berge’s Theorem on Maximum Matchings

Let $M$ be a (not necessarily maximal) matching...
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An *augmenting path* is an *alternating path* that starts and ends in an $M$-free node.
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Theorem. (Berge)

$M$ is maximum matching $\iff \#\text{Augmenting path}$
Matching Algos using Berge’s Theorem

- Berge’s theorem immediately gives us an outline for a general maximum matching algorithm:
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\text{MaxMatching}(G = (V, E))
\]

\[
\begin{align*}
M &= \emptyset \\
\text{while } & \exists \text{ Augmenting path } P \text{ in } G \text{ do} \\
& \quad \text{Augment } M \text{ along } P \\
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- Why can we not implement this algorithm “directly”?
- There are \textit{many} paths that could be augmenting!
- Solution: Specialise the algorithm for bipartite graphs.
Reduction to Maximum Flow

- Recap from *Algorithmic Graph Theory*: Let $G = (A \cup B, E)$ be a bipartite graph.
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![Diagram of a bipartite graph with labeled sets A and B.]
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\[ |M| = 3 \]
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- Idea: Find \textit{augmenting paths} from an M-free \(a \in A\) to an M-free \(b \in B\) until there are none left.
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![Graph showing augmenting paths between sets A and B]
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The Domiyes Algorithm

\texttt{DOMIYES(\textit{Domino}[] \: D)}
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\[
P = \{ p_1 \mid (p_1, p_2) \in D \} \cup \{ p_2 \mid (p_1, p_2) \in D \}
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Let \( f : P \rightarrow \mathbb{N} \) be a bijection
The Domiyes Algorithm

**DOMIYES(Domino[] D)**

\[ P = \{p_1 | (p_1, p_2) \in D\} \cup \{p_2 | (p_1, p_2) \in D\} \]

Let \( f : P \rightarrow \mathbb{N} \) be a bijection

\[ A = \{f(p) | p \in P \land p.x \equiv p.y \ (\text{mod} \ 2)\} \]

\[ B = \{f(p) | p \in P \land p.x \not\equiv p.y \ (\text{mod} \ 2)\} \]
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- $P = \{p_1 \mid (p_1, p_2) \in D\} \cup \{p_2 \mid (p_1, p_2) \in D\}$
- Let $f : P \to \mathbb{N}$ be a bijection
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- $E = \{\{u, v\} \in \binom{P}{2} \mid u \text{ adj. to } v \text{ of diff. domino}\}$
The Domiyes Algorithm

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M = \text{MaxBipartiteMatching}(A, B, E)
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\[
k = 0
\]

**foreach** \( \{ a, b \} \in M \) **do**

\[
f^{-1}(a).\text{number} = k; \quad f^{-1}(b).\text{number} = k
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\[
k = k + 1
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**The Domiyes Algorithm**

Define the set $P$ as:

$$P = \{p_1 \mid (p_1, p_2) \in D\} \cup \{p_2 \mid (p_1, p_2) \in D\}$$

Let $f : P \to \mathbb{N}$ be a bijection.

Define the sets $A$ and $B$ as:

$$A = \{f(p) \mid p \in P \land p.x \equiv p.y \pmod{2}\}$$

$$B = \{f(p) \mid p \in P \land p.x \not\equiv p.y \pmod{2}\}$$

Define the set $E$ as:

$$E = \{\{u, v\} \in \binom{P}{2} \mid u \text{ adj. to v of diff. domino}\}$$

Define $M$ as the maximum bipartite matching function applied to $A, B, E$.

Let $k = 0$.

**foreach** $\{a, b\} \in M$ **do**

$$f^{-1}(a).number = k; \quad f^{-1}(b).number = k$$

$$k = k + 1$$

$n := D$.length

$O(n)$

$O(n^2)$

$O(V E)$

$O(n)$
The Domiyes Algorithm

\textbf{DOMIYES}(Domino[] \textit{D})

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P &= \{p_1 \mid (p_1, p_2) \in \textit{D}\} \cup \{p_2 \mid (p_1, p_2) \in \textit{D}\} \\
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\( n := \textit{D}.\text{length} \)

\( O(n) \)

\( O(n) \)

\( O(n^2) \)

\( O(n) \)

\( O(V E) \)

\( O(n^2 + V E) \)
$\text{MaxBipartiteMatching}(A, B, E \subseteq (\frac{A}{2}) \cup (\frac{B}{2}))$
\textbf{MaxBipartiteMatching}

\texttt{MaxBipartiteMatching}(A, B, E \subseteq (A^2) \cup (B^2))

\begin{itemize}
  \item $M = \emptyset$
  \item \textbf{foreach} $M$-free $a \in A$ \textbf{do}
  \item \hspace{1cm} return $M$
\end{itemize}
MaxBipartiteMatching

MaxBipartiteMatching((A, B, E \subseteq (A^2) \cup (B^2)))

\[ M = \emptyset \]

\textbf{foreach} \ M\text{-free} \ a \in A \ \textbf{do}

\textbf{if} \ \exists \ \text{aug. path} \ P \ \text{from} \ a \ \text{to} \ M\text{-free} \ b \in B \ \textbf{then}

\textbf{return} \ M
MaxBipartiteMatching

MaxBipartiteMatching(A, B, E ⊆ (A^2) ∪ (B^2))

M = ∅

foreach M-free a ∈ A do
  if ∃ aug. path P from a to M-free b ∈ B then
    foreach uv ∈ P do
      if {u, v} ∈ M then
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      else
        M = M ∪ {{u, v}}
  return M
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return M

This still runs in O(VE) time.
However...
MaxBipartiteMatching

MaxBipartiteMatching(A, B, E ⊆ (A₂) ∪ (B₂))

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\textbf{foreach} \ M\text{-free} \ a \in A \ \textbf{do}

\textbf{if} \ \exists \ \text{aug. path} \ P \ \text{from} \ a \ \text{to} \ M\text{-free} \ b \in B \ \textbf{then}

\textbf{foreach} \ uv \in P \ \textbf{do}

\textbf{if} \ \{u, v\} \in M \ \textbf{then}

\[ M = M \setminus \{\{u, v\}\} \]

\textbf{else}

\[ M = M \cup \{\{u, v\}\} \]

\textbf{return} \ M

This still runs in \( O(VE) \) time.

However...

\[ \Delta \leq 3 \Rightarrow |E| \leq 3V \]
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However...

\[ \Delta \leq 3 \Rightarrow |E| \leq 3V \]

\[ O(VE) = O(V \cdot 3V) = O(V^2). \]