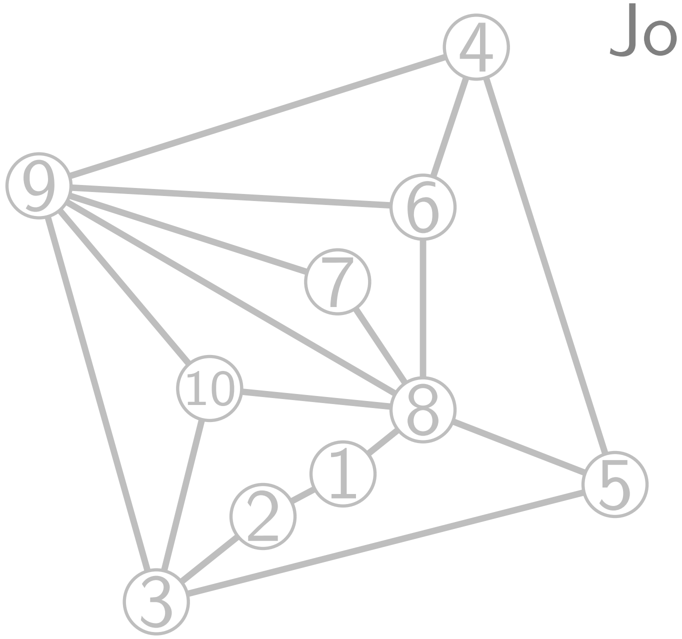
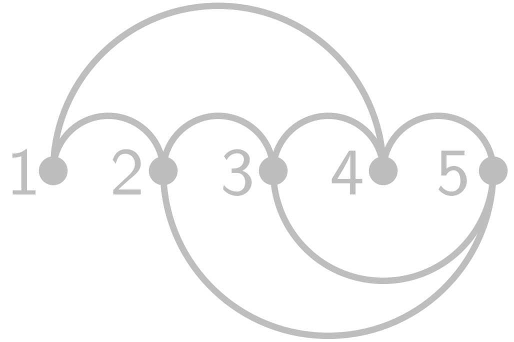
Visualisation of graphs

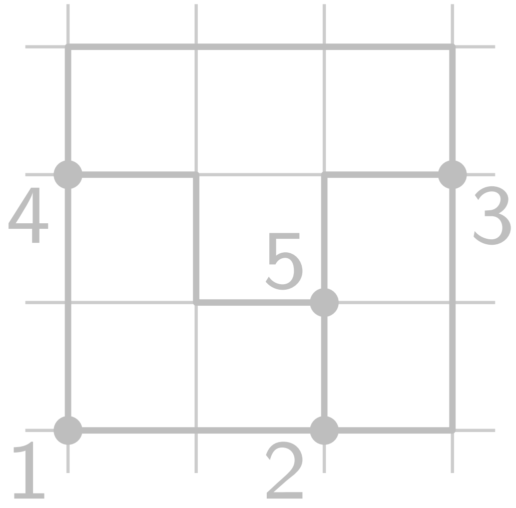
Introduction

The graph visualisation problem

Jonathan Klawitter · Summer semester 2020







Graphs and their representations

What is a graph?

- graph $G = (V, E)$
- vertices $V = \{v_1, v_2, \dots, v_n\}$
- edge $E = \{e_1, e_2, \dots, e_m\}$

Representation?

- Set notation

$$
\begin{aligned}
V &= \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\} \\
E &= \{\{v_1, v_2\}, \{v_1, v_8\}, \{v_2, v_3\}, \{v_3, v_5\}, \{v_3, v_9\}, \\
&\quad \{v_3, v_{10}\}, \{v_4, v_5\}, \{v_4, v_6\}, \{v_4, v_9\}, \{v_5, v_8\}, \\
&\quad \{v_6, v_8\}, \{v_6, v_9\}, \{v_7, v_8\}, \{v_7, v_9\}, \{v_8, v_{10}\}, \\
&\quad \{v_9, v_{10}\}\}
\end{aligned}
$$

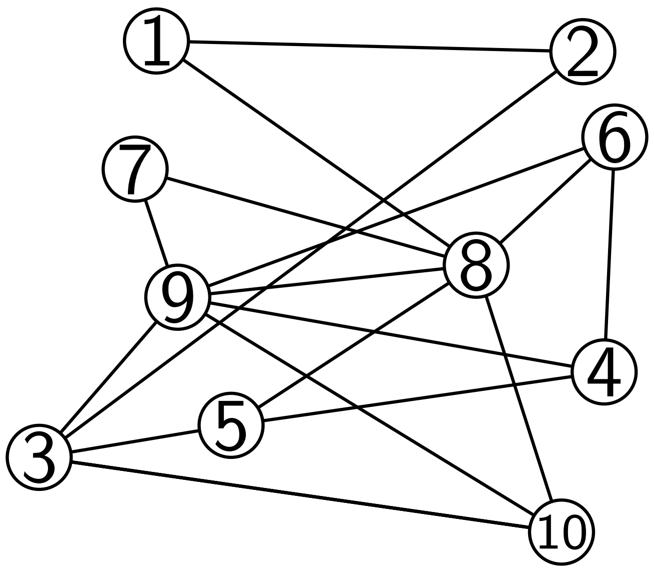
- Adjacency list

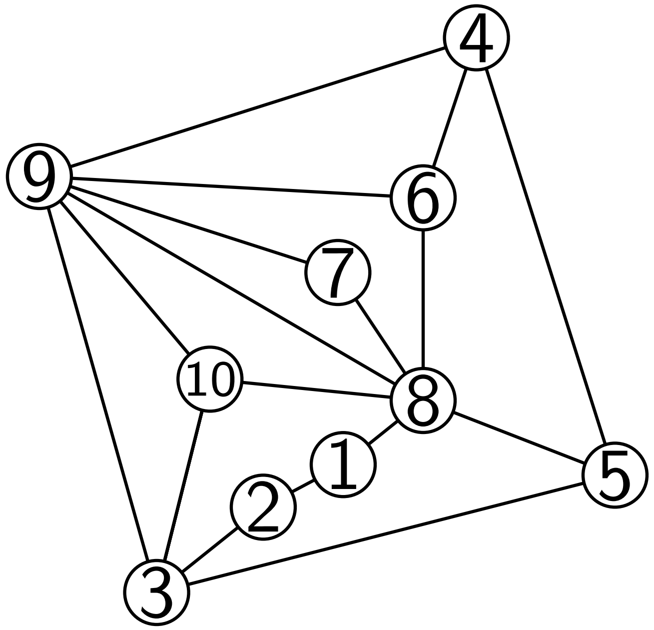
v_1:	v_2, v_8	v_6:	v_4, v_8, v_9
v_2:	v_1, v_3	v_7:	v_8, v_9
v_3:	v_2, v_5, v_9, v_{10}	v_8:	$v_1, v_5, v_6, v_7, v_9, v_{10}$
v_4:	v_5, v_6, v_9	v_9:	$v_3, v_4, v_6, v_7, v_8, v_{10}$
v_5:	v_3, v_4, v_8	v_{10}:	v_3, v_8, v_9

- Adjacency matrix

$$
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0
\end{pmatrix}
$$

- Drawing





Why draw graphs?

Graphs are a mathematical representation of real physical and abstract networks.

Abstract networks

- Social networks
- Communication networks
- Phylogenetic networks
- Metabolic networks
- Class/Object Relation Digraphs (UML)
- ...

Physical networks

- Metro systems
- Road networks
- Power grids
- Telecommunication networks
- Integrated circuits
- ...

Why draw graphs?

Graphs are a mathematical representation of
real physical and abstract networks.

- **People think visually** – complex graphs are hard to grasp without good visualisations!

Why draw graphs?

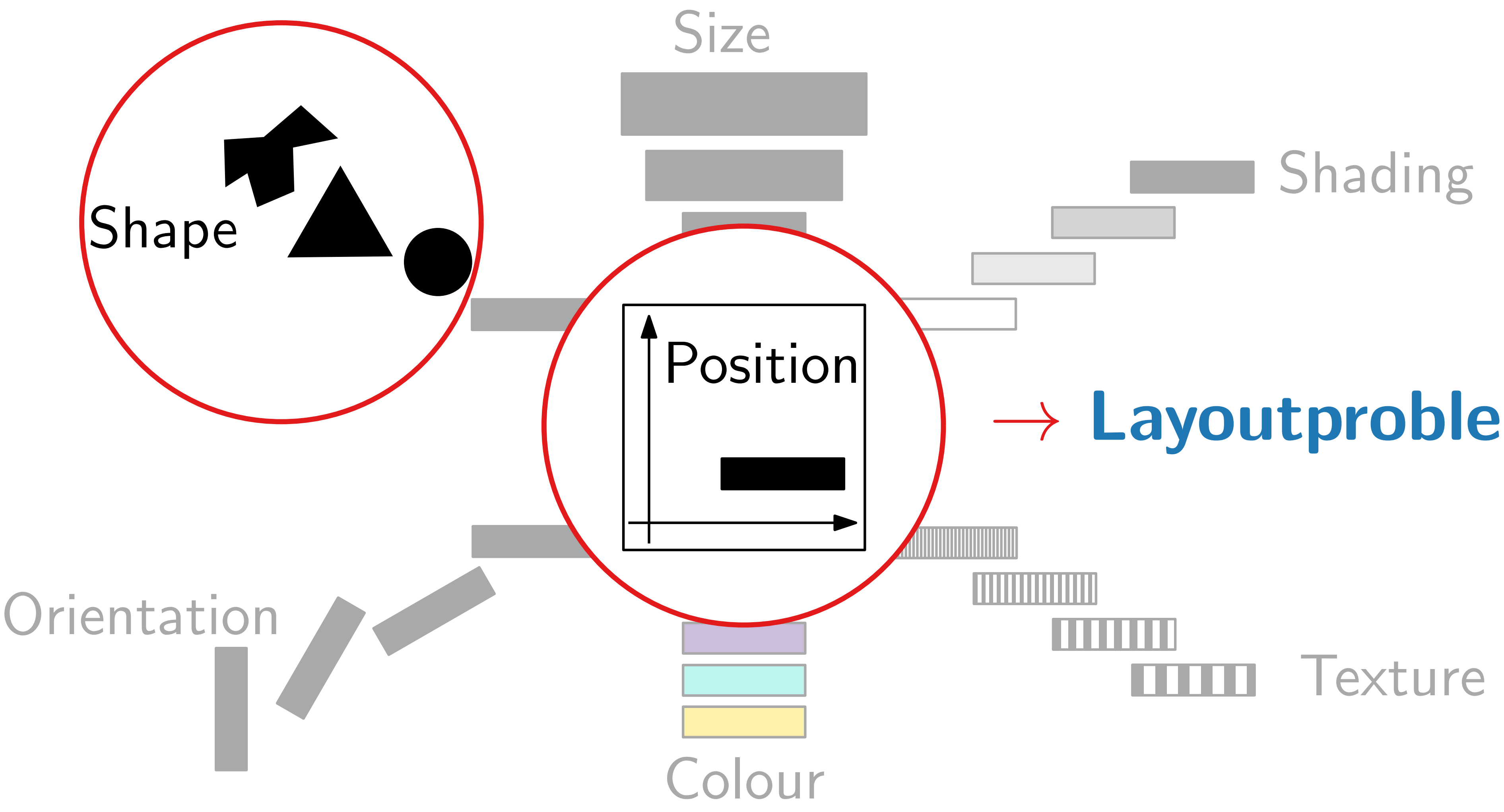
Graphs are a mathematical representation of real physical and abstract networks.

- **People think visually** – complex graphs are hard to grasp without good visualisations!
- Visualisations help with the **communication** and **exploration** of networks.
- Some graphs are too big to draw them by hand.

We need algorithms that draw graphs automatically to make networks more accessible to humans.

What are we interested in?

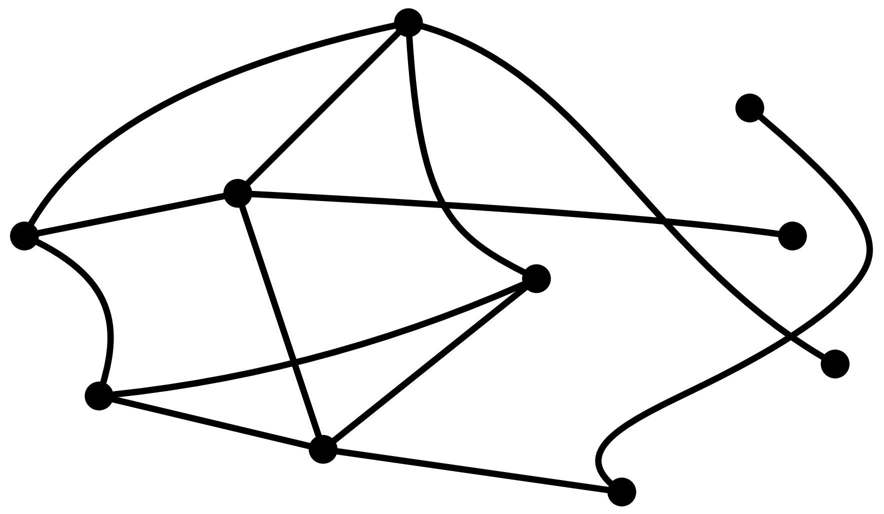
- Jacques Bertin defined visualising variables (1967)



→ **Layoutproblem**

The layout problem?

- *Here* restricted to the **standard representation**, so-called node-link diagrams.



Graph visualisation problem

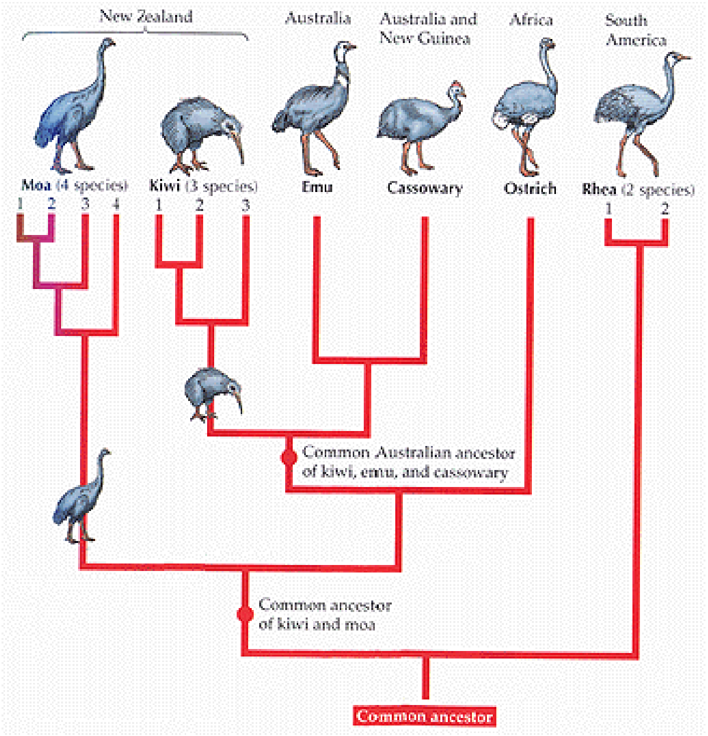
in: Graph $G = (V, E)$

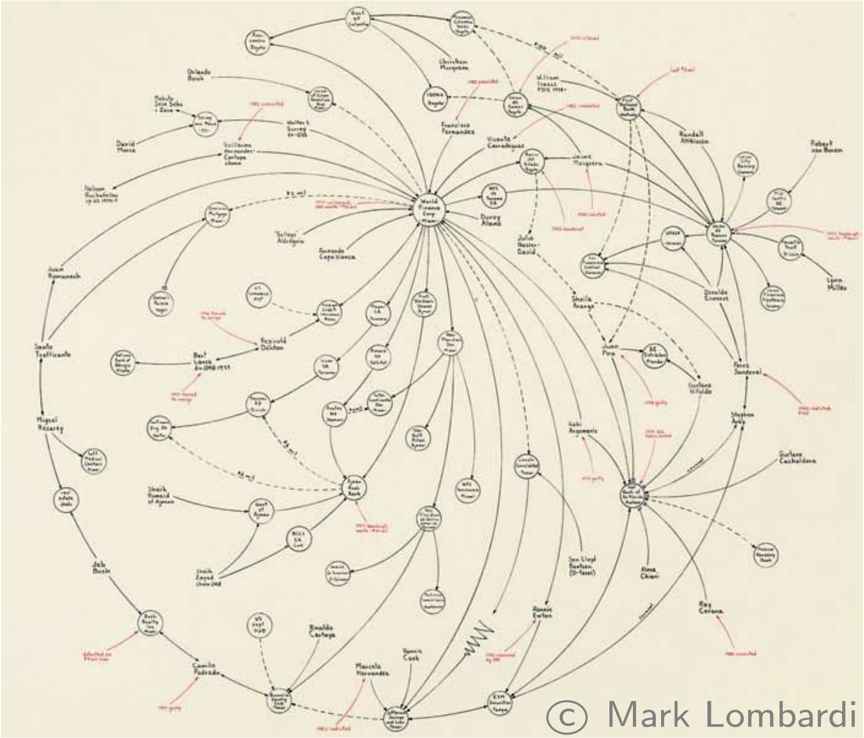
out: **nice** drawing Γ of G

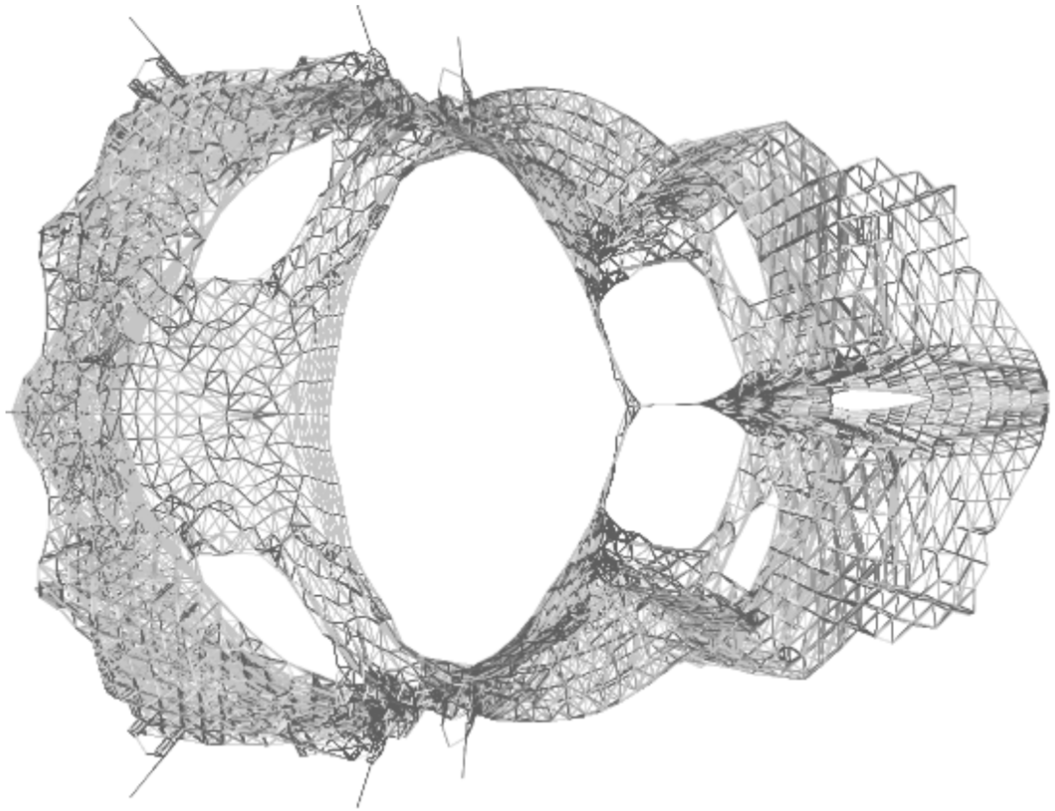
- $\Gamma \colon V \to \mathbb{R}^2$, vertex $v \mapsto$ point $\Gamma(v)$
- $\Gamma \colon E \to$ curves in $\mathbb{R}^2$, edge $\{u, v\} \mapsto$ simple, open curve $\Gamma(\{u, v\})$ with endpoints $\Gamma(u)$ und $\Gamma(v)$

But what is a **nice** drawing?

Examples







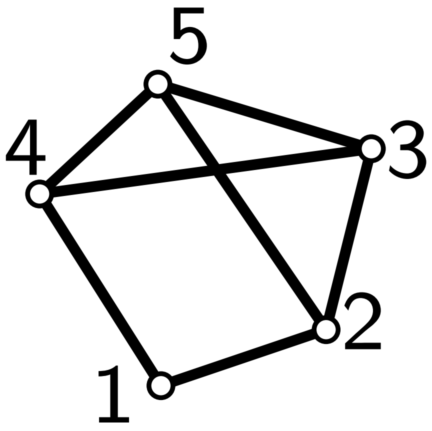
- See slides (and video) with more examples.

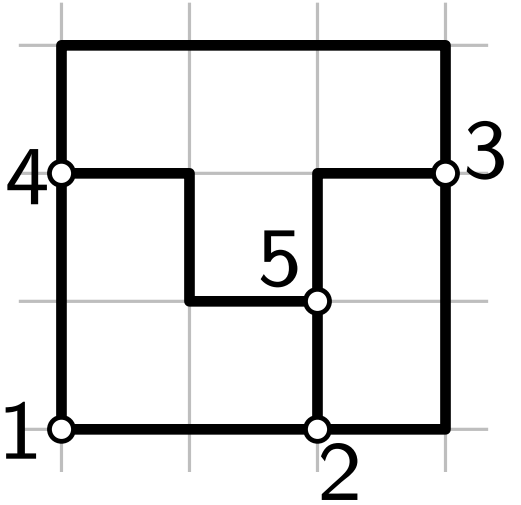
Requirements of a graph layout

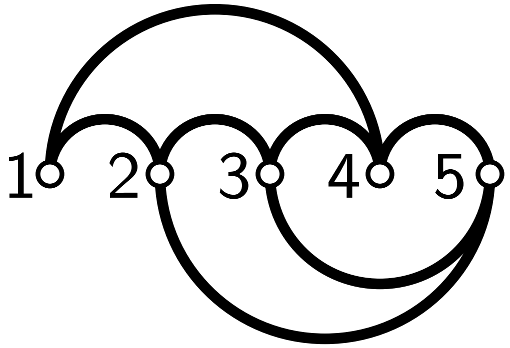
1. Drawing conventions and requirements, e.g.,

Requirements of a graph layout

1. Drawing conventions and requirements, e.g.,
   - straight edges with $\Gamma(uv) = \overline{\Gamma(u)\Gamma(v)}$
   - orthogonal edges (i.e. with bends)
   - grid drawings
   - without crossing
2. Aesthetics to be optimised, e.g.

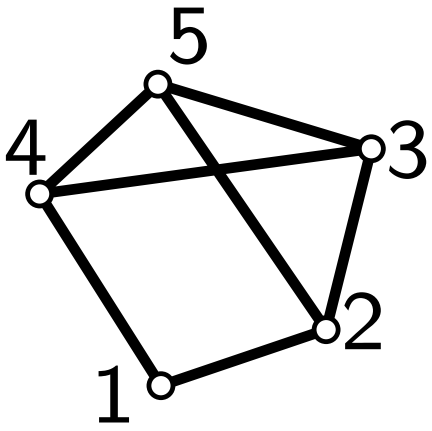


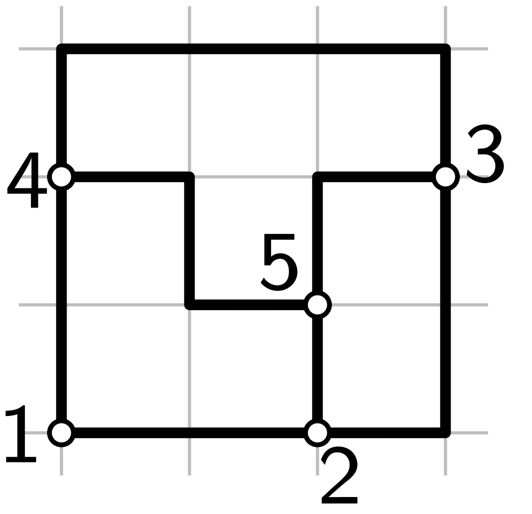


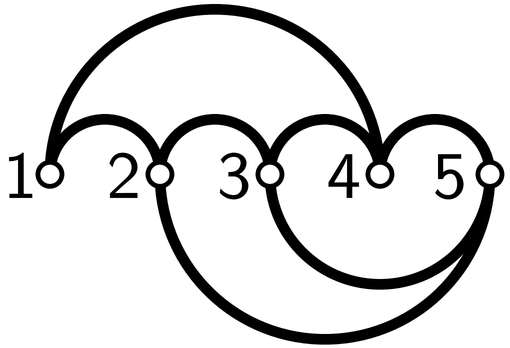


Requirements of a graph layout

1. Drawing conventions and requirements, e.g.,
   - straight edges with $\Gamma(uv) = \overline{\Gamma(u)\Gamma(v)}$
   - orthogonal edges (i.e. with bends)
   - grid drawings
   - without crossing
2. Aesthetics to be optimised, e.g.
   - crossing/bend minimisation
   - edge length uniformity
   - minimising total edge length/drawing area
   - angular resolution
   - symmetry/structure
3. Local Constraints, e.g.

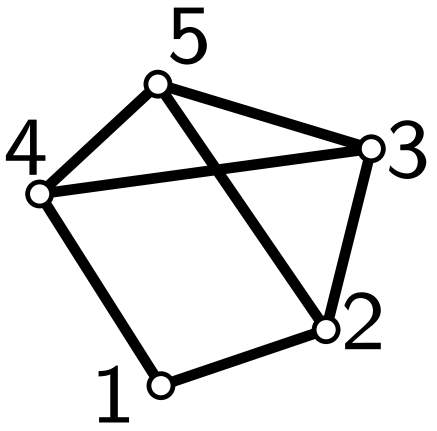


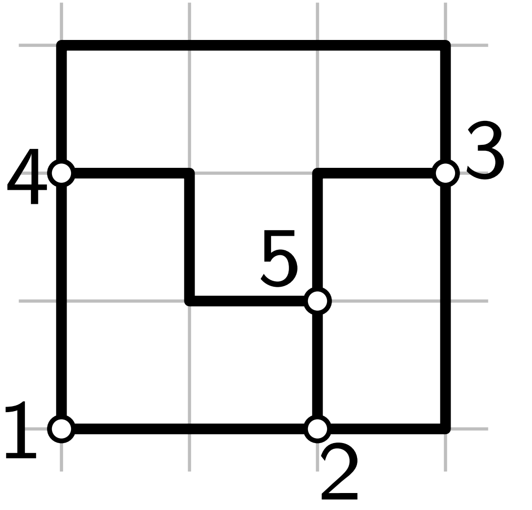




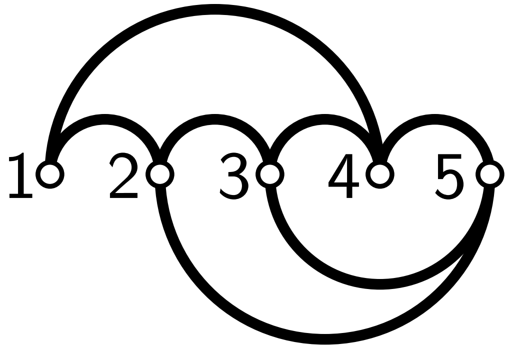
$\rightarrow$ lead to NP-hard optimization problems
$\rightarrow$ such criteria are often inversely related

Requirements of a graph layout





1. Drawing conventions and requirements, e.g.,
   - straight edges with $\Gamma(uv) = \overline{\Gamma(u)\Gamma(v)}$
   - orthogonal edges (i.e. with bends)
   - grid drawings
   - without crossing



2. Aesthetics to be optimised, e.g.
   - crossing/bend minimisation
   - edge length uniformity
   - minimising total edge length/drawing area
   - angular resolution
   - symmetry/structure

$\rightarrow$ lead to NP-hard optimization problems

$\rightarrow$ such criteria are often inversely related

3. Local Constraints, e.g.
   - restrictions on neighbouring vertices (e.g., “upward”).
   - restrictions on groups of vertices/edges (e.g., “clustered”).

The layout problem

Graph visualisation problem

in: Graph $G = (V, E)$

out: Drawing Γ of G such that

- **drawing conventions** are met,
- **aesthetic criteria** are optimised, and
- some **additional constraints** are satisfied.

- Many algorithmically interesting questions arise.
- Rendering problem downstream is ignored.