Lecture Outline - July 30, 2013

1.- Formal Definition of a Discrete Filter
   Linearity Property of Discrete Systems
   Shift-Invariance Property of Discrete Systems

2.- Object Domain Filtering
   Linear Convolution

3.- Spectral Domain Filtering
   Multiplication of Spectra

4.- Fast Spectral Domain Filtering
   Zero Padding Operation
   Cyclic Convolution
   Object Domain Fourier Convolution Theorem
LAB2E1: RC-filters and Ideal Filters

The impulse response of a RC first order filter, which is also a causal filter, is given by

\[ h(t) = T\{\delta(t)\} = h_e e^{-\alpha t} u(t); \quad \alpha = \frac{1}{RC} \]

The product \( RC \) is called the time-constant of the RC-filter since it can be shown that when \( t = t_r \approx 10RC \), the impulse response \( h(t) \approx 0 \) due to the fact that \( e^{-10} \approx 0 \). The value \( t = t_r \approx 10RC \) is called the time rise of the RC-filter since it is the elapsed time from the minimum value \( y(t) = 0 \) to the maximum value \( y(t) = 1 \) of the step response:

\[ y(t) = T\{u(t)\} = \frac{h_e}{\alpha} (1 - e^{-\alpha t}) u(t) \]

The normalized magnitude of the frequency response of the RC-filter is given by the plot below, and its frequency response is denoted by \( H(f) \) and is given by the following equation:

\[ H(f) = \frac{1}{1 + j2\pi f RC} = \frac{1}{1 + j(2\pi f / \alpha)} = \frac{1}{1 + j(f / B)}; \quad B \equiv \frac{\alpha}{2\pi} = \frac{1}{2\pi RC} \]

The parameter \( B \equiv \frac{\alpha}{2\pi} = \frac{1}{2\pi RC} \) is called the bandwidth or cut-off frequency of the filter and it is the value of frequency at which the magnitude or intensity of the frequency response has experienced a decibel drop of about 3 dB. For this example, the selected bandwidth is:

\[ B \equiv f_m = \frac{\alpha}{2\pi} = \frac{1}{2\pi RC} = 2000 \text{ Hz} \]

It is important to compare the graph of a function with the graph of another function with related attributes. Compare the blue graph of the RC-filter with the red graph of the ideal low-pass filter shown in figure below. Describe the roll-off of the blue graph.

The dynamics of a first order RC filter is described by a first order differential equation. Place a graph on a new page with a different cut-off frequency for the RC filter (changing the R and/or C parameters) and the ideal filter (changing the cut-off frequency). Explain your results.
% CAUSAL FIRST ORDER RC-FILTER
% Ideal Low-pass filter versus RC-filter
%
% PARAMETER SETTINGS
Fs=10000; % Sampling Frequency
Ts=1/Fs; % Sampling Period or Sampling Time
N=300; % Length of each discrete signal or vector
V=N*Ts; % Time duration (in seconds)
for each signal
fm=2000; % Cuttoff frequency
Wn=2*fm/Fs; % Normalized frequency
M=300; % Length of the impulse response signal
R=10000; % Value of Resistor
C=1/(2*pi*fm*R); % Value of Capacitor
a=1/(R*C); % Time Constant Parameter
h0=a; % Initial Condition Parameter
t=0:Ts:V-Ts; % General time axis
th=0:Ts:M*Ts-Ts; % Time axis for plotting
impulse response signal
f=-Fs/2:Fs/M:Fs/2-Fs/M; % Frequency axis
%
**
% RC-FILTER
hRC=h0*exp(-a*th); % RC-Filter impulse response function
hmax=max(hRC); % Maximum value of impulse response function
hRC=(hRC/hmax); % Normalized impulse response function
fhRC=fft(hRC); % Fourier Transform of impulse response function
sfhRC=fftshift(fhRC); % Frequency shift for two sided spectrum plot
asfhRC=abs(sfhRC); %Absolute value calculation
Hmax=max(asfhRC); %Maximum value of frequency response function
asfhRC=(asfhRC/Hmax); %Normalized frequency response function

%***********************************************

**
%MATLAB FIR1 FILTER
Wn=(2*fm)/Fs; %Normalized cut-off frequency for ideal filter
h=fir1(N-1,Wn); %Impulse response function of ideal filter
fh=fft(h); %Fourier transform of impulse response function
sfh=fftshift(fh); %Frequency shift for two sided spectrum plot
msfh=abs(sfh); %Absolute value calculation

%***********************************************

**
%PLOTS
%******************************************************

**
plot(f,asfhRC,f,msfh,'r') %Plots of RC and Ideal filters
grid
xlabel('Frequency in Hz.') %Horizontal axis
ylabel('Magnitude') %Vertical axis
title('HL(f) and HRC(f)') %Title of plot
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LAB2E2: First Order RC-filter: Step Response

\[ h(t) = T\{\delta(t)\} = h_0 e^{-\alpha t} u(t); \quad \alpha = \frac{1}{RC} \]

Below we present the step response for \( x(t) = 60u(t) \):

**Script m-file: rcstep.m**

```matlab
Fs=10000;
Ts=1/Fs;
N=500;
V=N*Ts;
R=100;
C=.0001;
a=1/(R*C);
h0=100;
x0=60;
t=0:Ts:V-Ts;
x=x0*((t+1)./(t+1));
h=h0*exp(-a*t);
y=x0*(h0/a)*(1-exp(-a*t));
plot(t,h,t,x,t,y)
grid
xlabel('Time in Sec.')
ylabel('Magnitude')
title('Impulse Resp. h(t), input x(t)=u(t), and output y(t) of RC-Filter')
```

**Impulse Resp. h(t), input x(t)=u(t), and output y(t) of RC-Filter**
MATLAB m-script file:unresp.m

N=100;
n=0:1:N-1;
a=4;
uN=ones([1,N]);
b=0.5;
hRC=a*(b.^n);
y=conv(uN,hRC);
m=0:1:2*N-2;
subplot(2,1,1)
stem(m,y)
xlabel('Normalized Time Index (Ts=1 Sec.)')
ylabel('Amplitude')
title('Finite Step Response: a=4, b=0.5')
grid

b=0.8;
hRC=a*(b.^n);
y=conv(uN,hRC);
m=0:1:2*N-2;
subplot(2,1,2)
stem(m,y)
xlabel('Normalized Time Index (Ts=1 Sec.)')
ylabel('Amplitude')
title('Finite Step Response: a=4, b=0.9')
grid
1.- Obtain the differential equation of the continuous-time first order RC filter shown in the diagram above and use the following derivative approximation formula

\[ \frac{d}{dt} y(t) \bigg|_{t=[(n-1)T_s]} \approx \frac{y[nT_s - (n-1)T_s]}{T_s} \]

in order to covert this differential equation into a difference equation representing a discrete-time filter.

2.- Obtain the Laplace transform of the differential equation which represents the transfer function of the continuous-time filter.

3.- Obtain the Z-transform of the difference equation which represents the transfer function of the discrete-time filter.

4.- Draw a block diagram for the discrete-time filter from its transfer function or from its difference equation.
Use the following five commands to study the “zeropadding” effect on the magnitude of the discrete Fourier transform (DFT) of discrete signals and discuss your results:

- ones
- zeros (use this function with a parameter value)
- fft
- abs
- stem

Starting with the discrete signal \( x = [1,1,1,1] \), create a new signal with length twice the previous signal. The length increase is obtained by appending zeros to the previous signal. Finish this process when the length of the new signal reaches 128. Proceed then to take the magnitude of the spectrum of each signal and plot the result.
Compute the discrete-time Fourier transform (DTFT) of the signal \( x = [1, 1, 1, 1] \) and provide an analytic (closed form) solution. Proceed to compute the DTFT and to provide an analytic (closed form) solution for all the signals computed in the previous exercise.
Run the MATLAB scripts in the folder “drimaging” and discuss your results. Use a different *data image* from the ones provided and repeat the process.
Use the definition of the two-discrete convolution operation and the concept of approximating a tangent line by a secant line in order to mathematically describe the result of the filter use in the “Dx_Lena.m” script.