Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems

Institute for Informatics

Computational Geometry

Line-Segment Intersection
or
Map Overlay

Lecture #2

Dr. Philipp Kindermann

Winter Semester 2018


Dawson
Whitehorse
Echo Bay
Ft. Simpson
Yellowknife
Uranium City
Prince Rupert
Dawson Creek
Lynn Lake
Edmonton
Vancouver
Victoria
Saskatoon
Calgary
Regina


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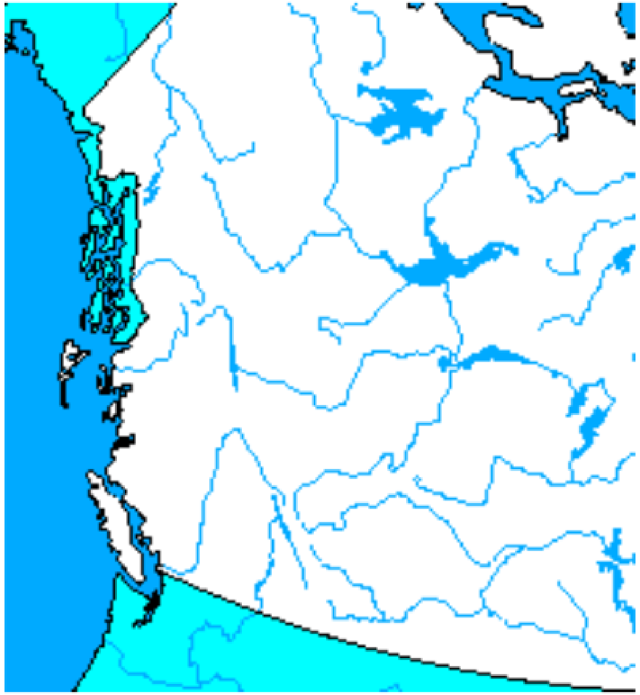

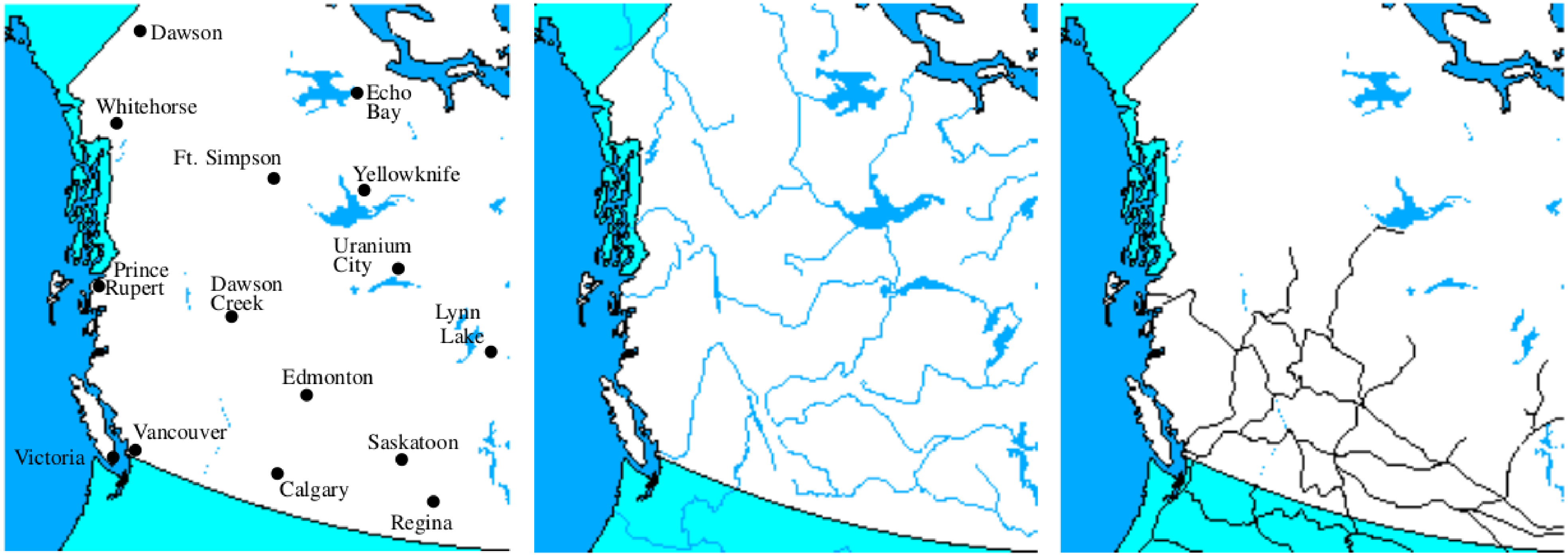
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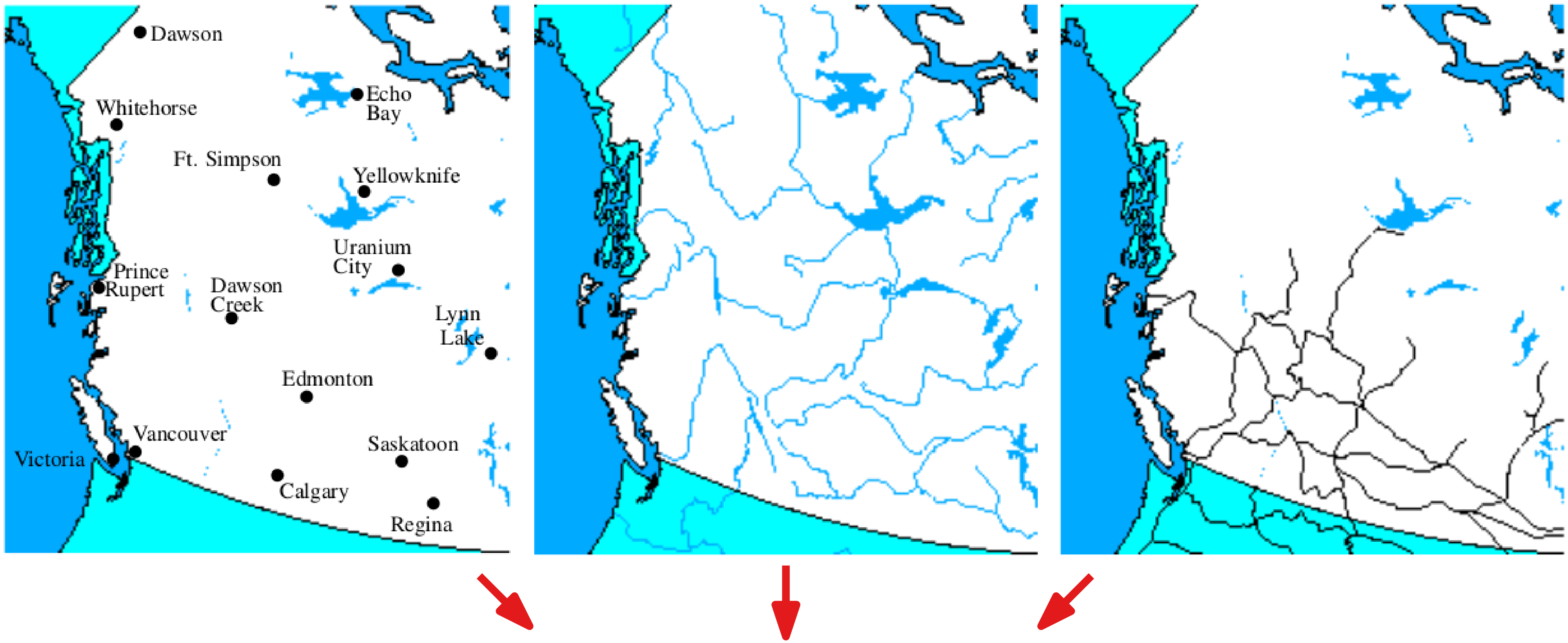
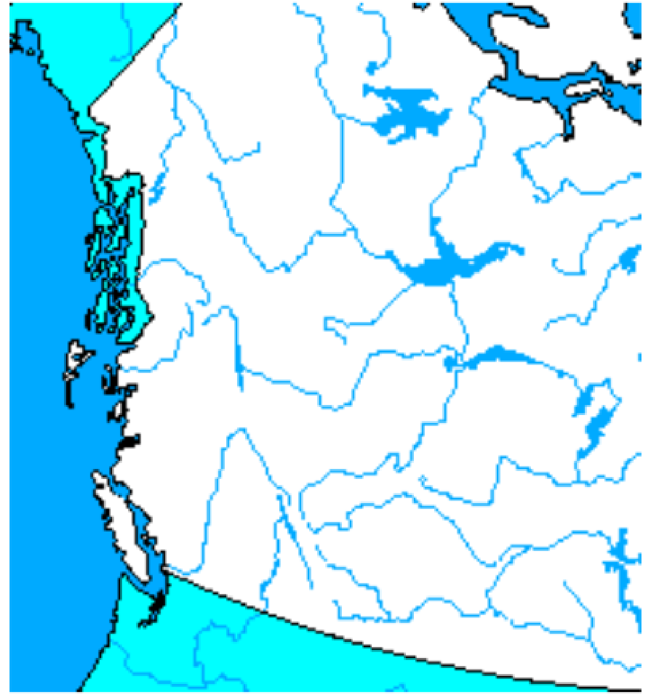


Map Overlay in Geographic Information Systems (GIS)



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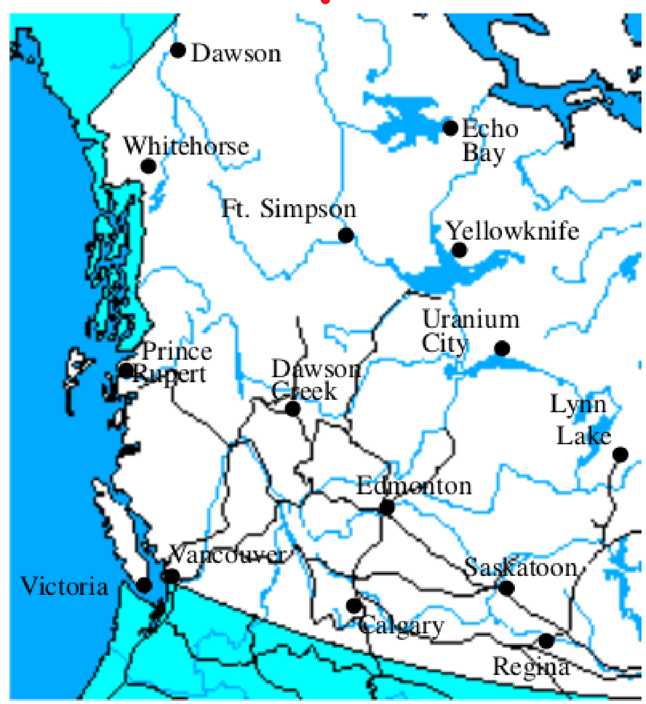

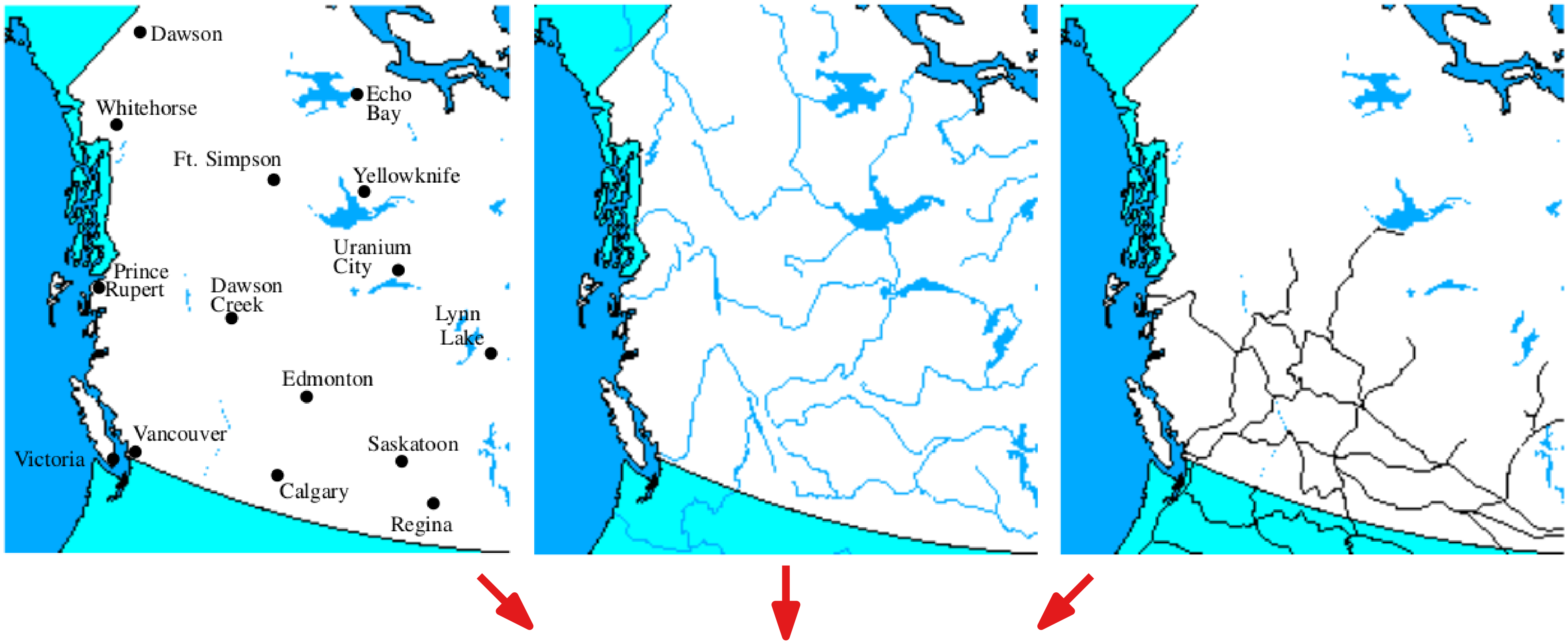

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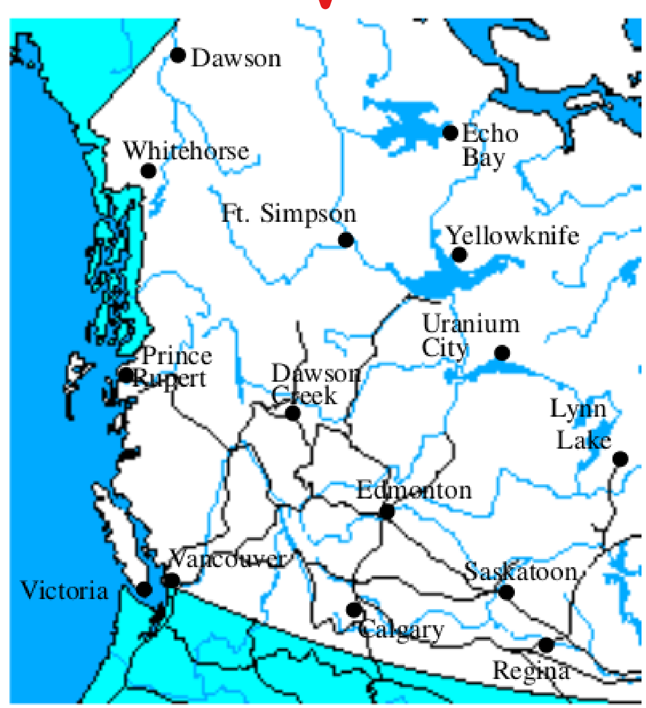

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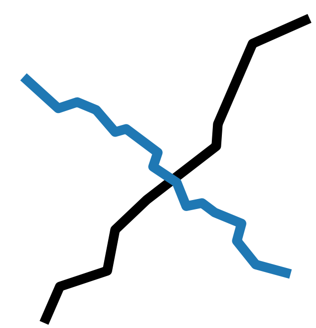

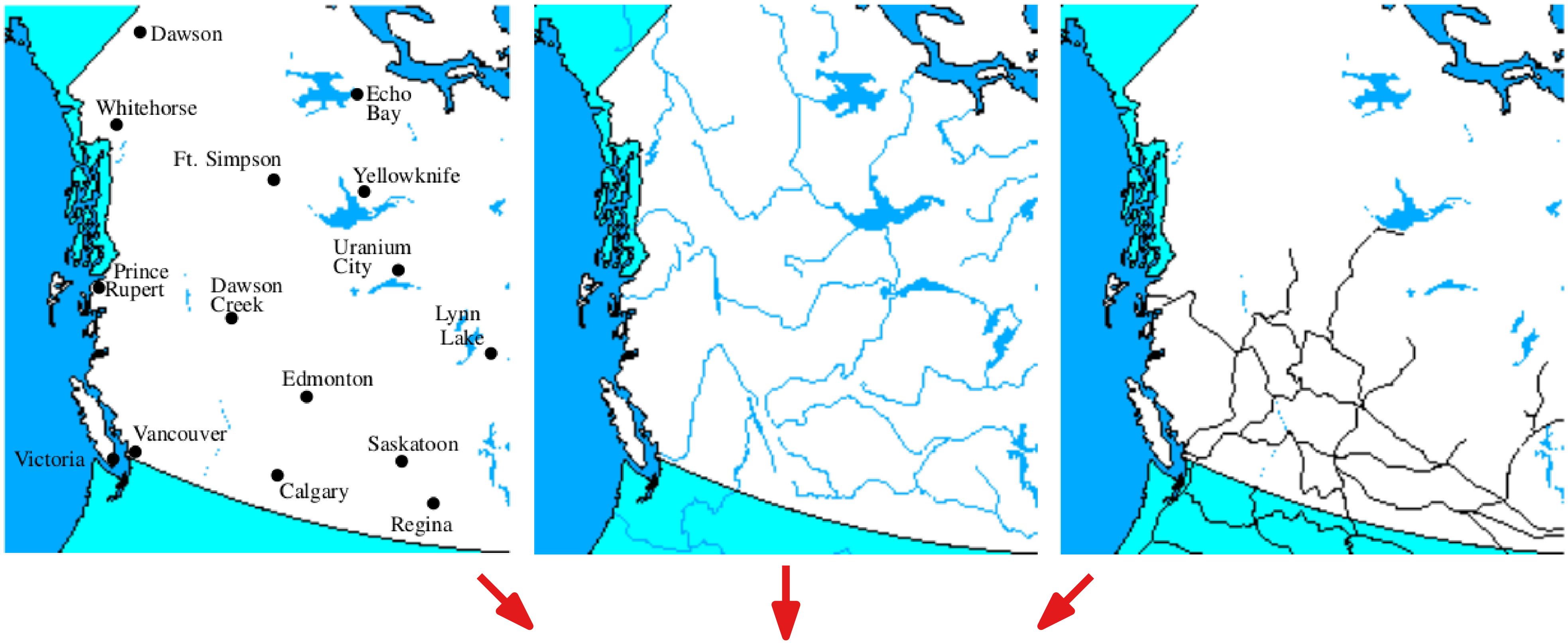

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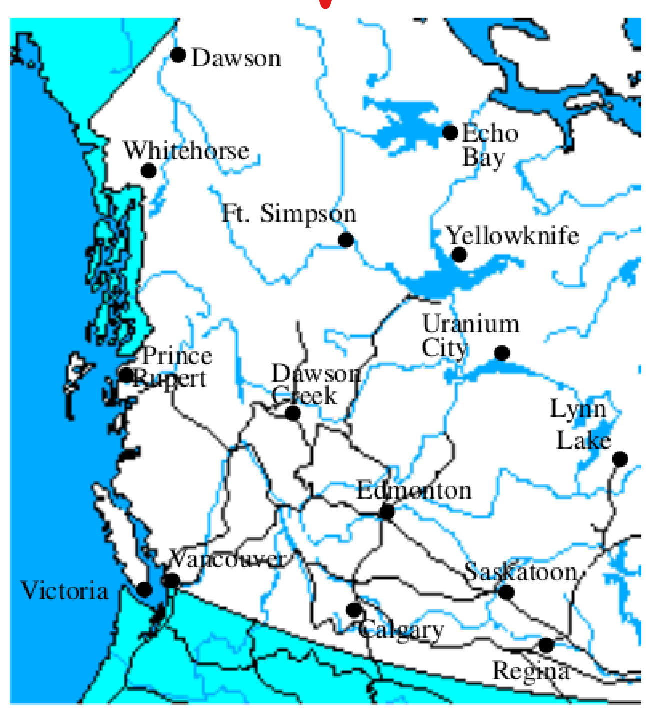

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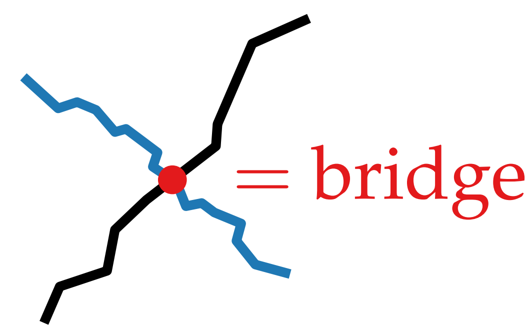
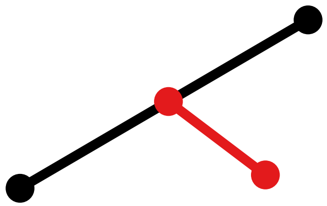


Line-Segment Intersection

Definition:

Line-Segment Intersection

Definition: Is  an intersection?

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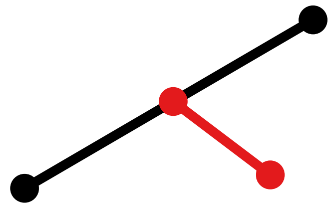
Definition: Is  an intersection?

Answer: Depends. . .

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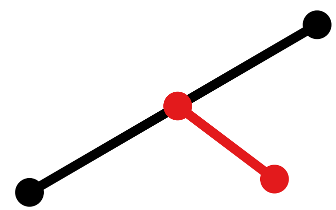
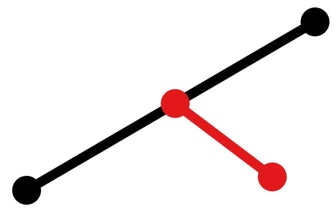
Definition: Is  an intersection?

Answer: Depends. . .

Problem: Given a set S of n *closed* non-overlapping line segments in the plane, compute. . .

Line-Segment Intersection

Definition: Is  an intersection?

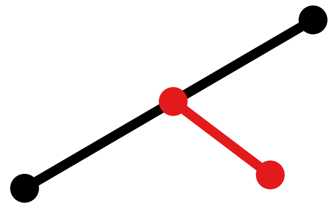
Answer: Depends…

Problem: Given a set S of n *closed* non-overlapping line segments in the plane, compute…

– all points where at least two segments intersect and

– for each such point report all segments that contain it.

Line-Segment Intersection

Definition: Is  an intersection?

Answer: Depends. . .

yes!

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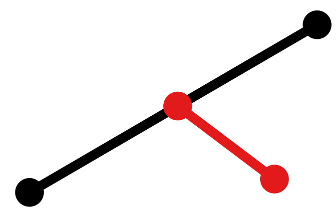
Line-Segment Intersection

Definition: Is  an intersection?

yes!

Answer: Depends…

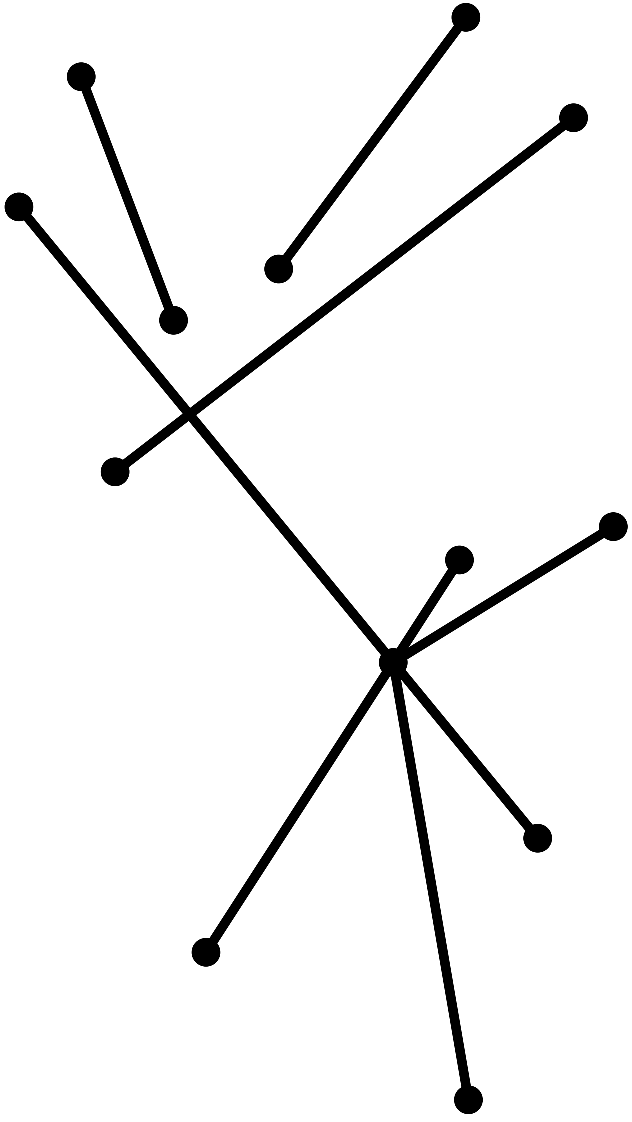
Problem: Given a set S of n *closed* non-overlapping line segments in the plane, compute…

– all points where at least two segments intersect and

– for each such point report all segments that contain it.

Task: Discuss with your neighbor:
how would you do it?

Example



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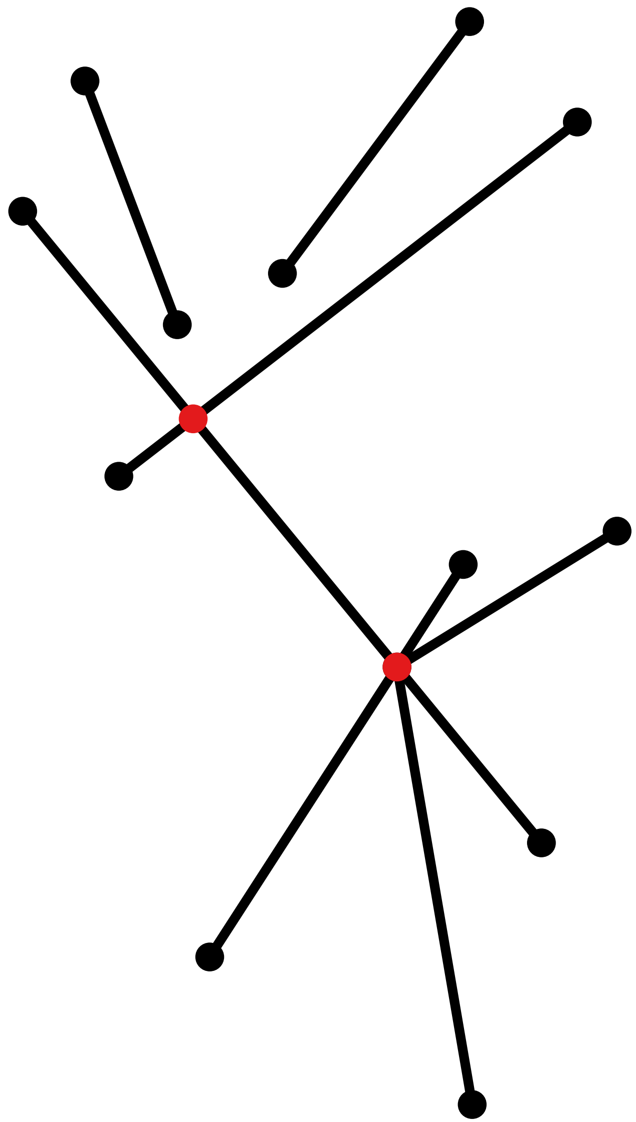

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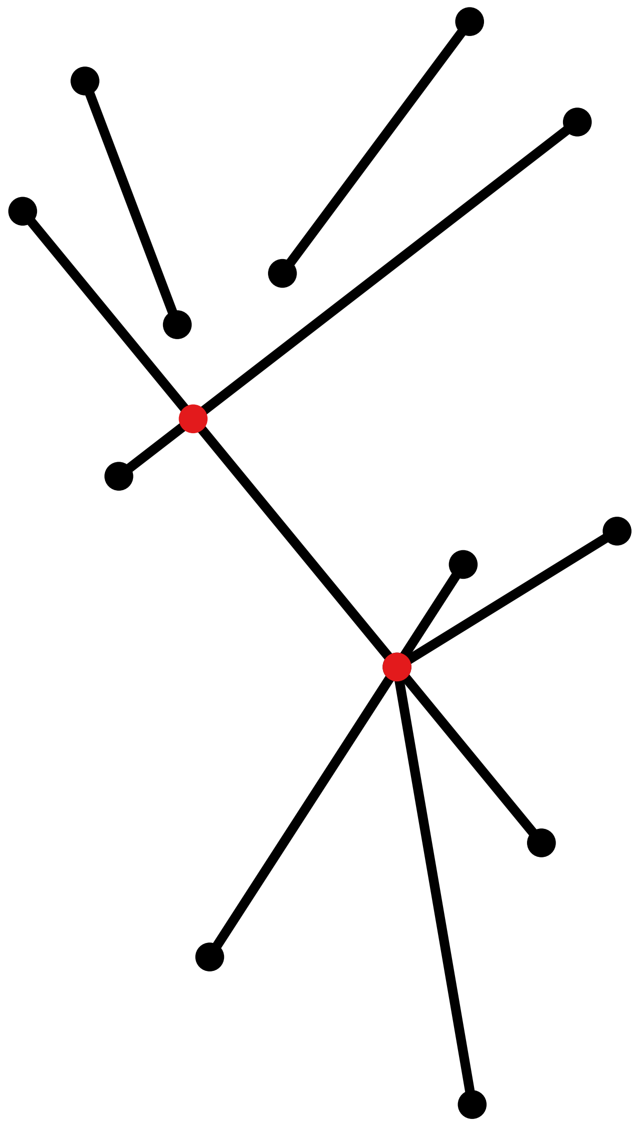
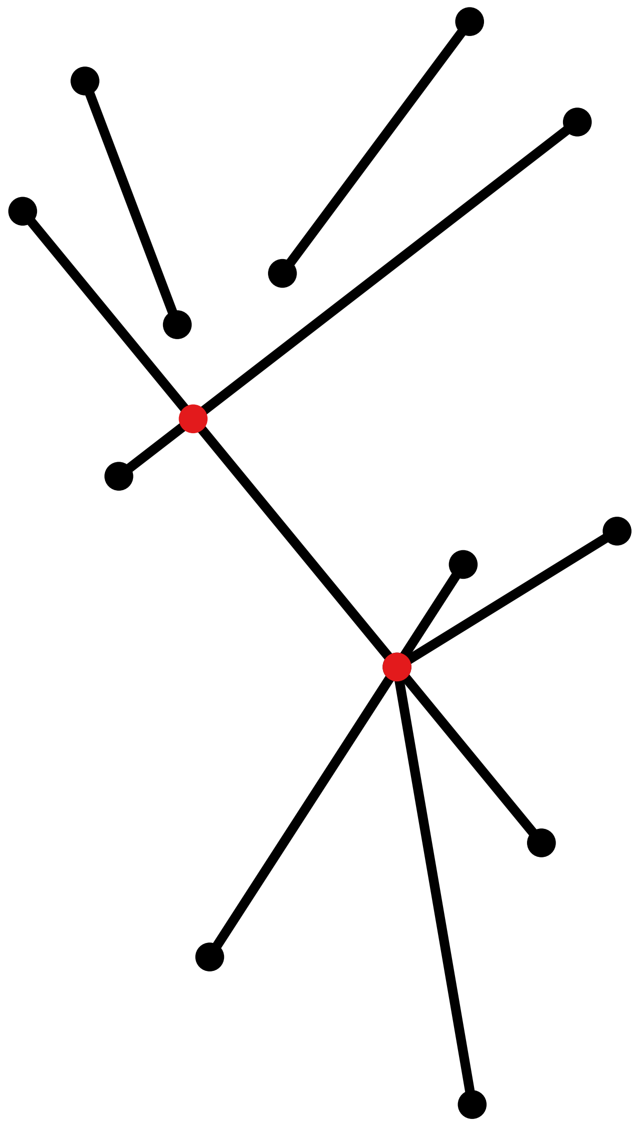


Brute Force?

$O(n^2)$ ... can we do better?

Example

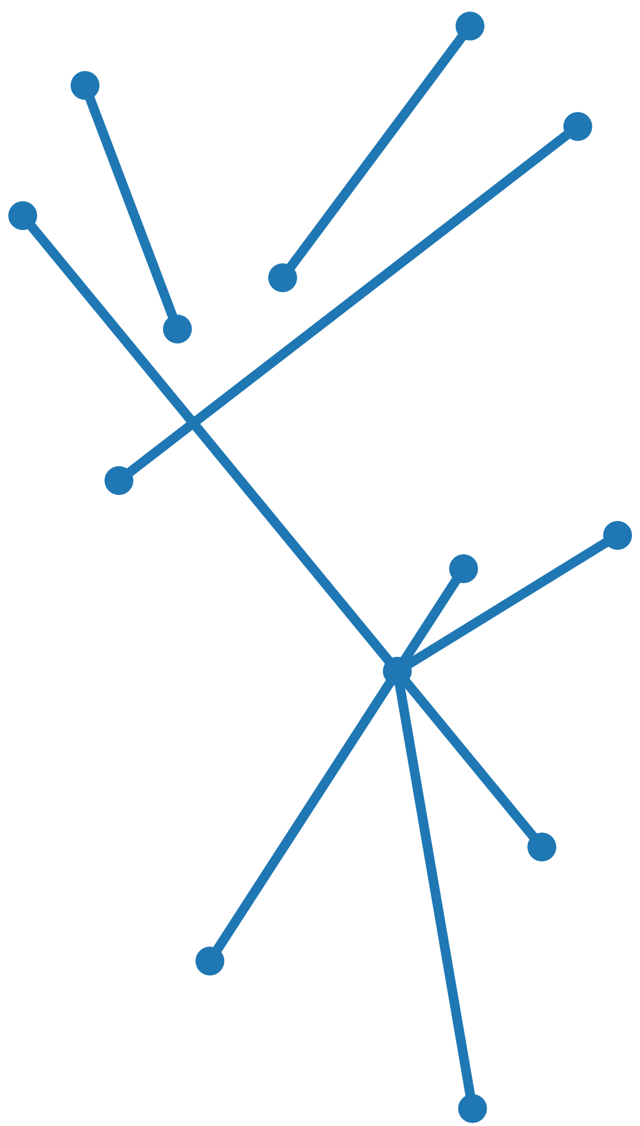


Brute Force?

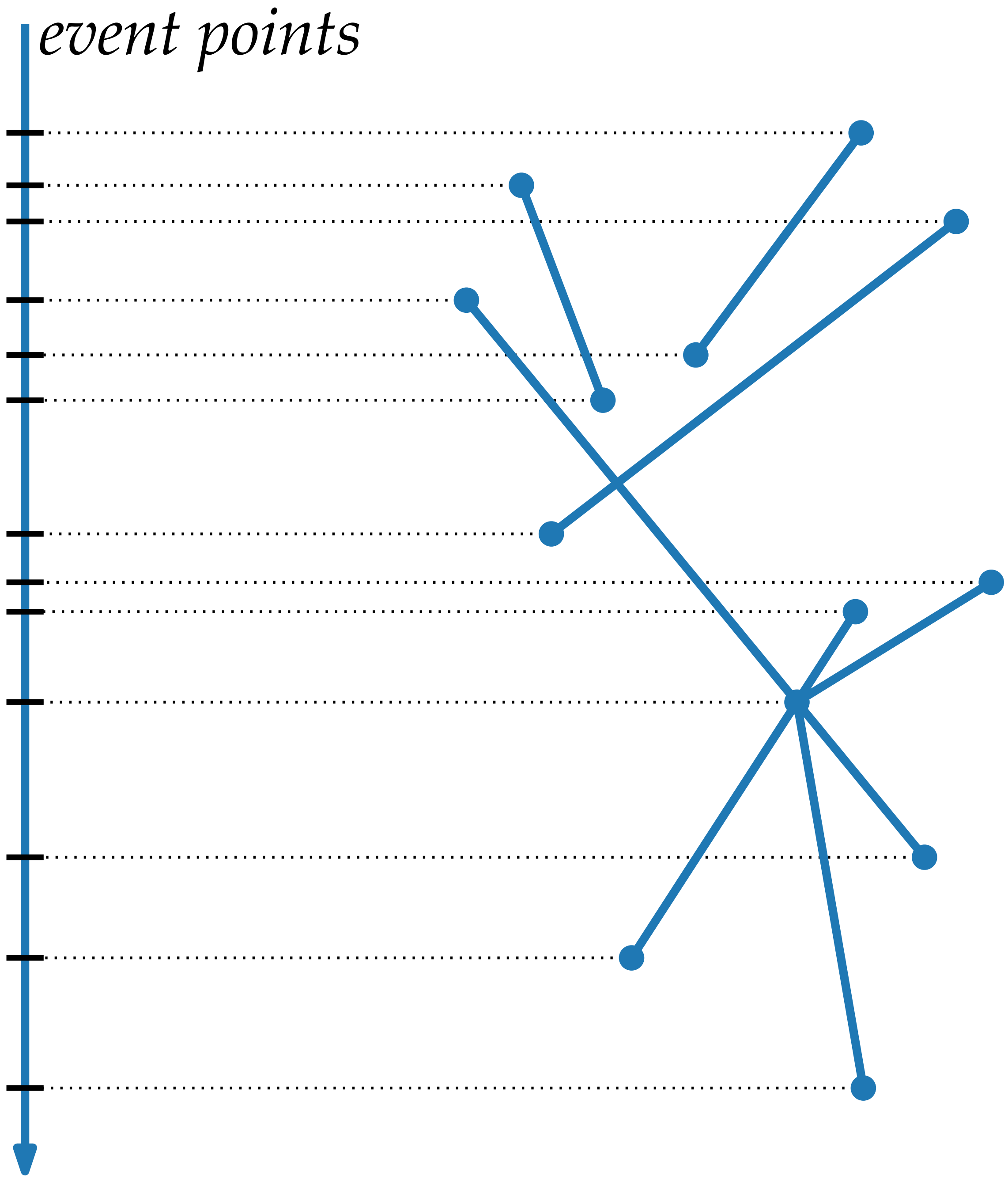
$O(n^2)$ ... can we do better?

Idea:
Process segments top-to-bottom using a "sweep line".

Sweep-Line Algorithm

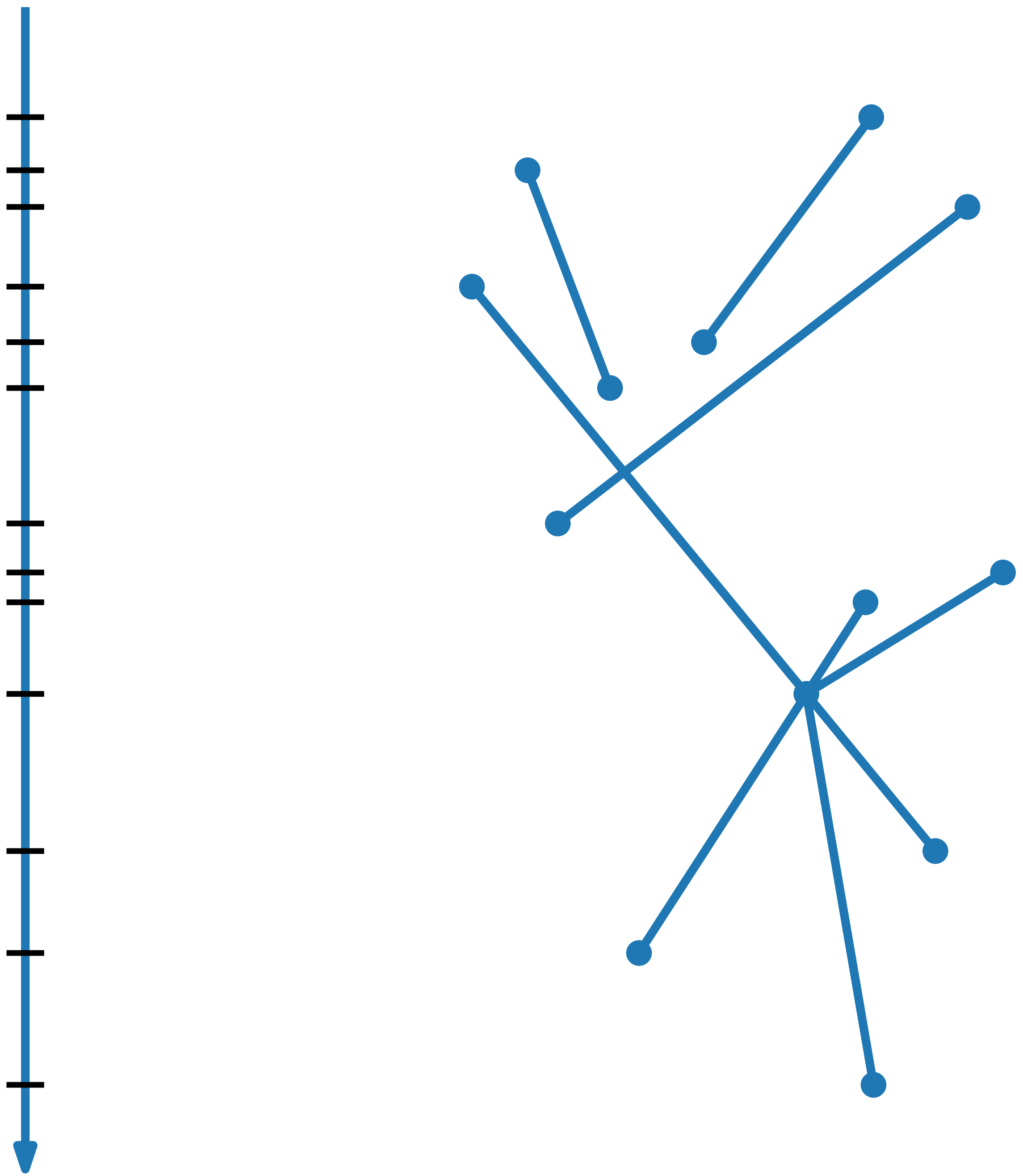


Sweep-Line Algorithm



Which active segments should be compared?

Sweep-Line Algorithm



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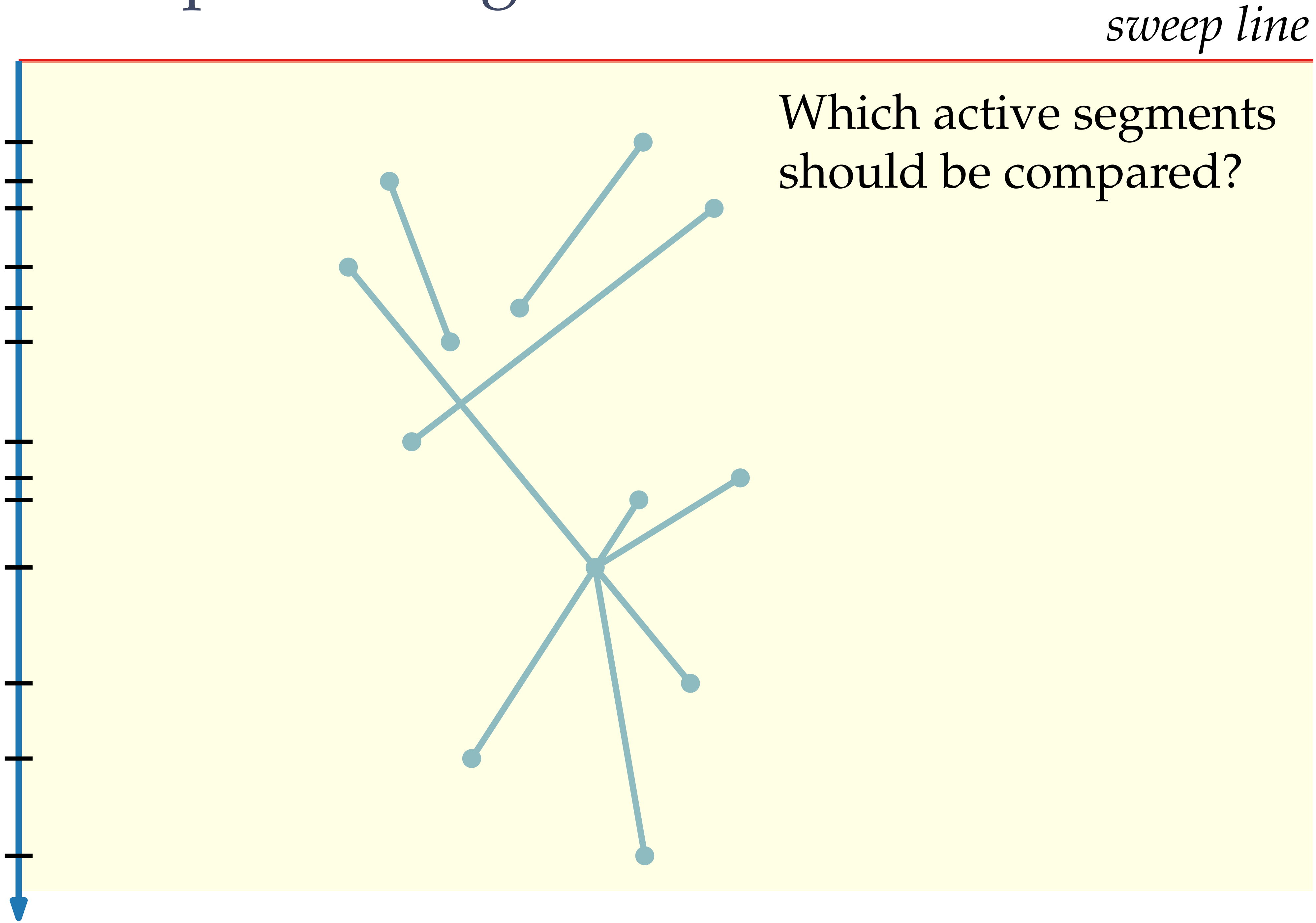

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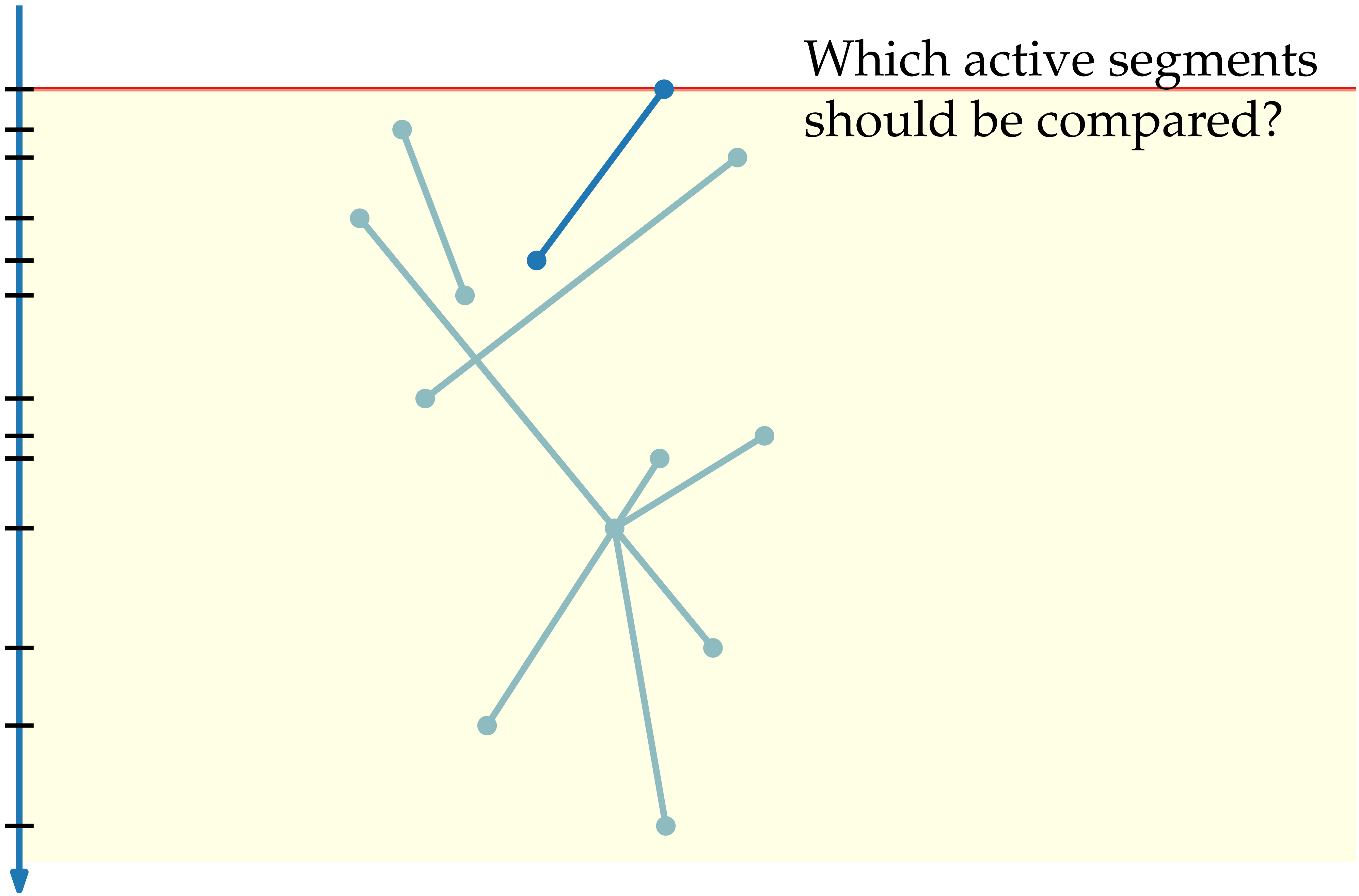

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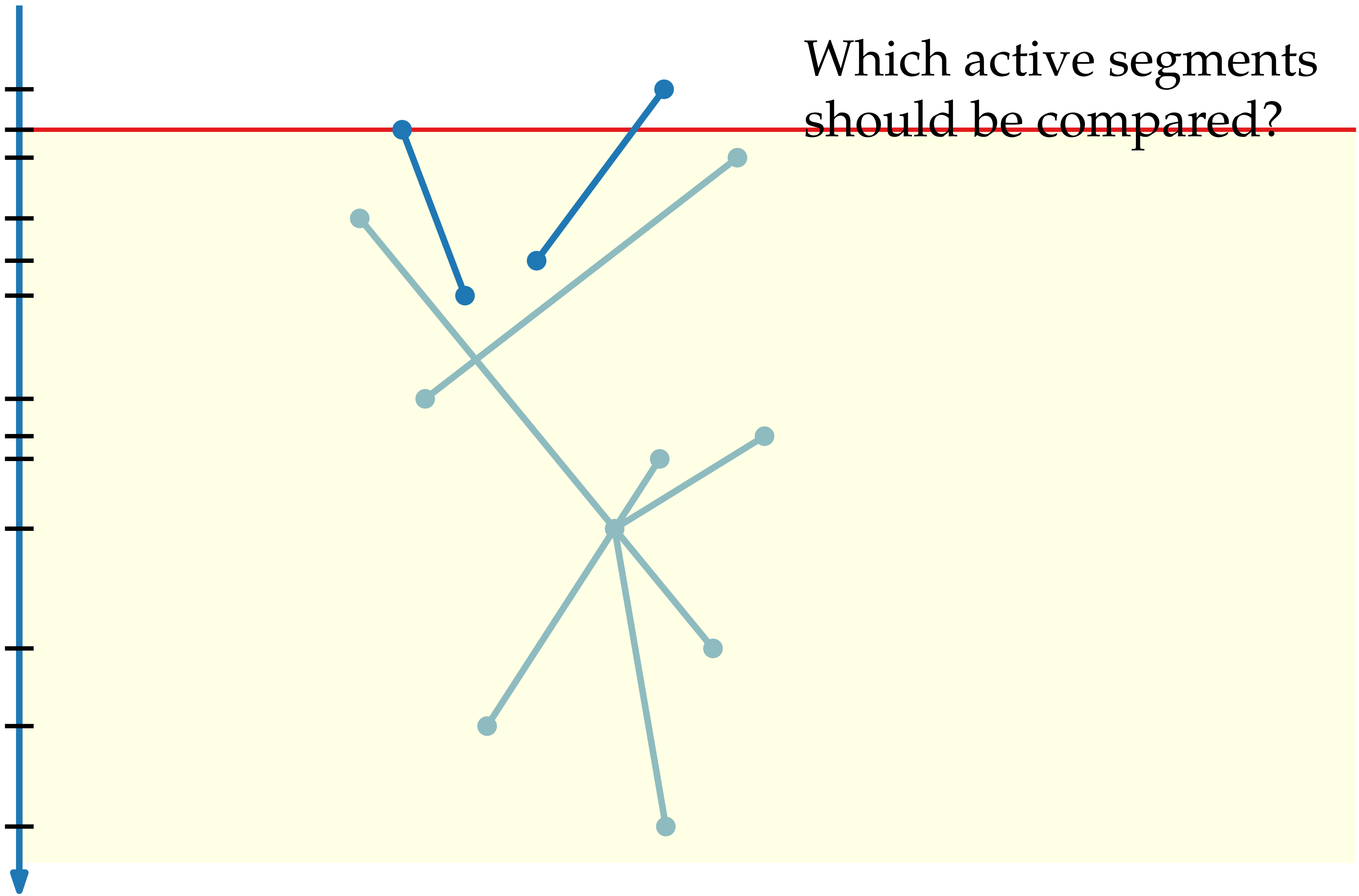

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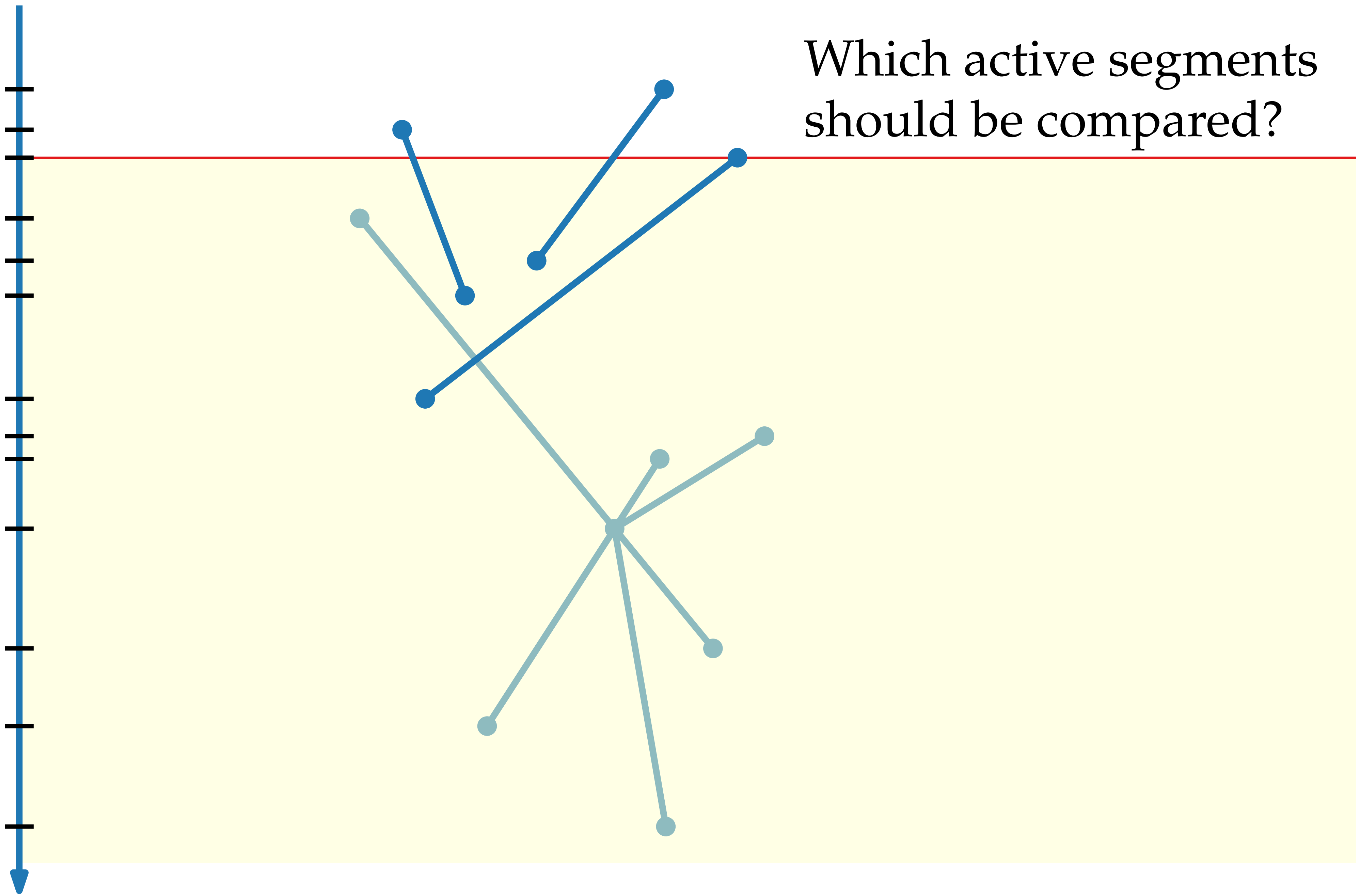

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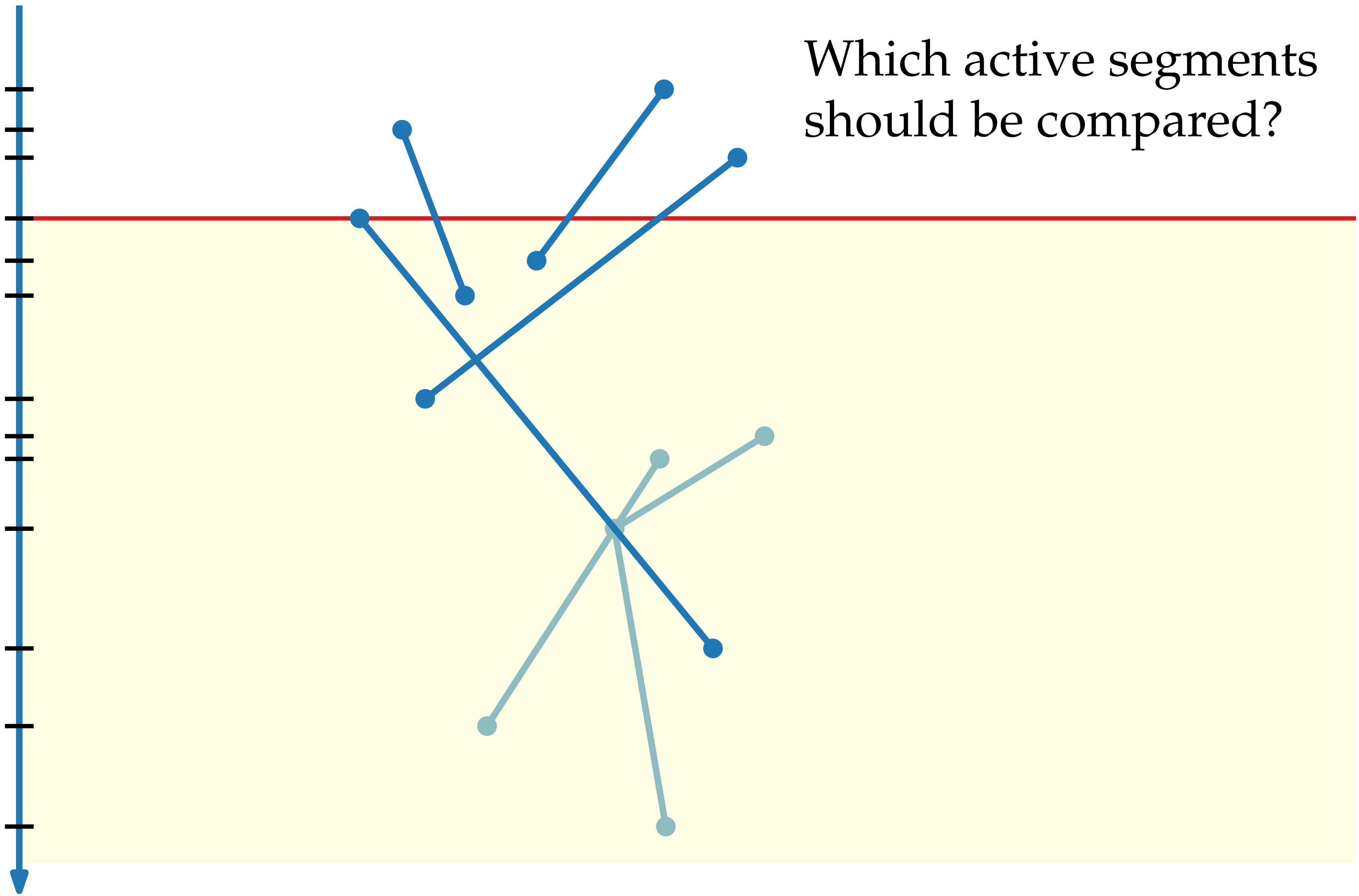

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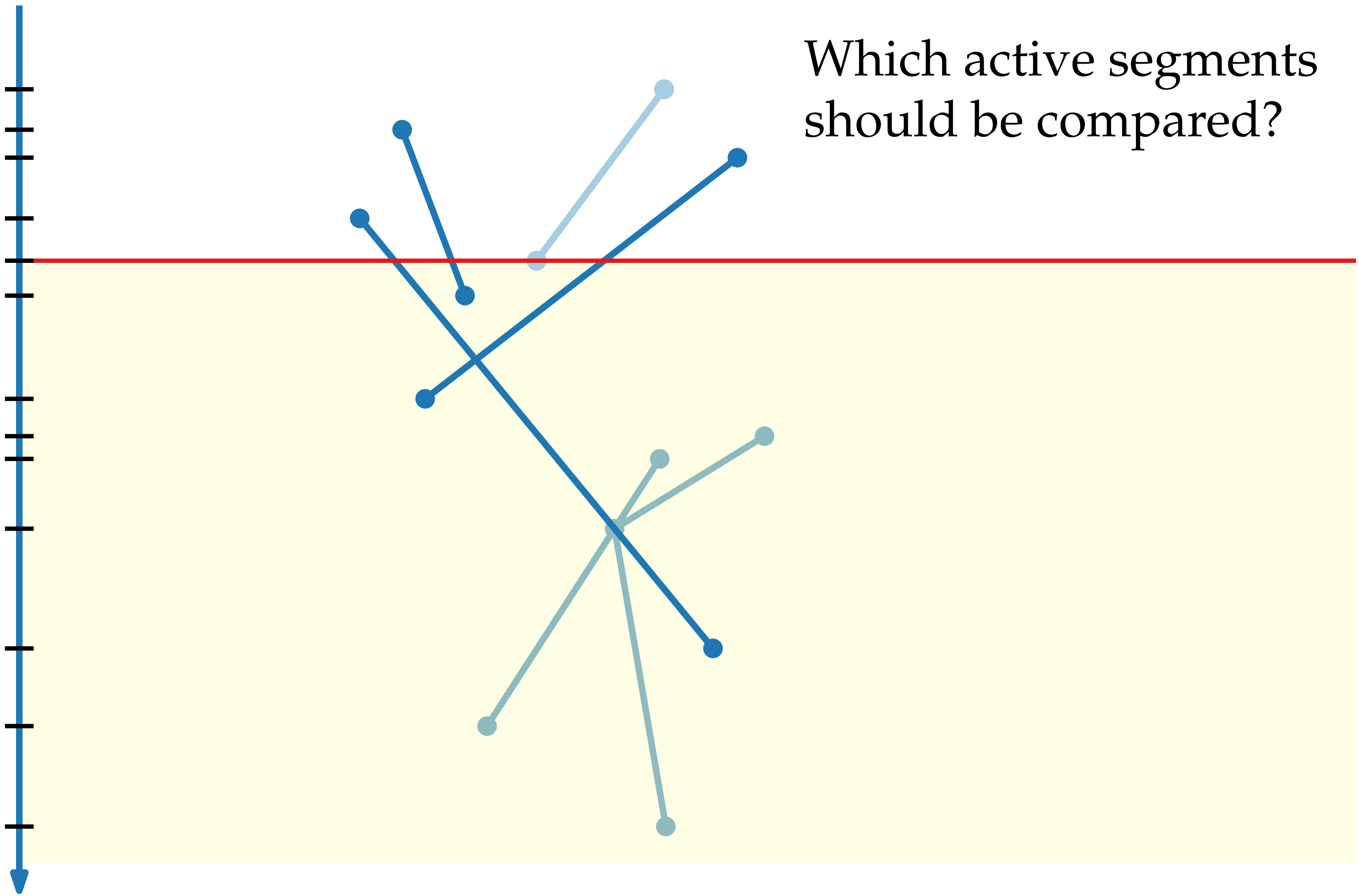

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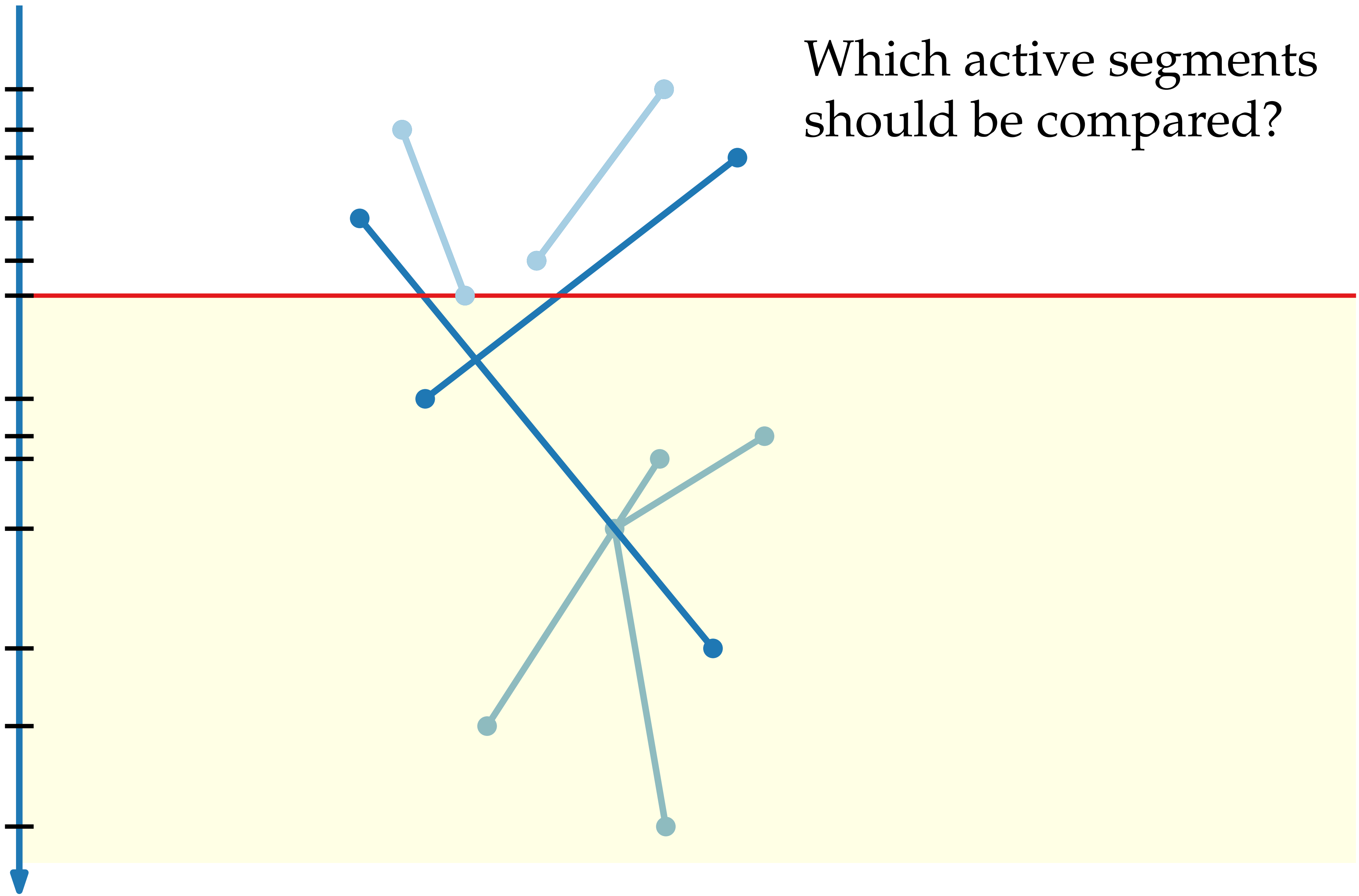

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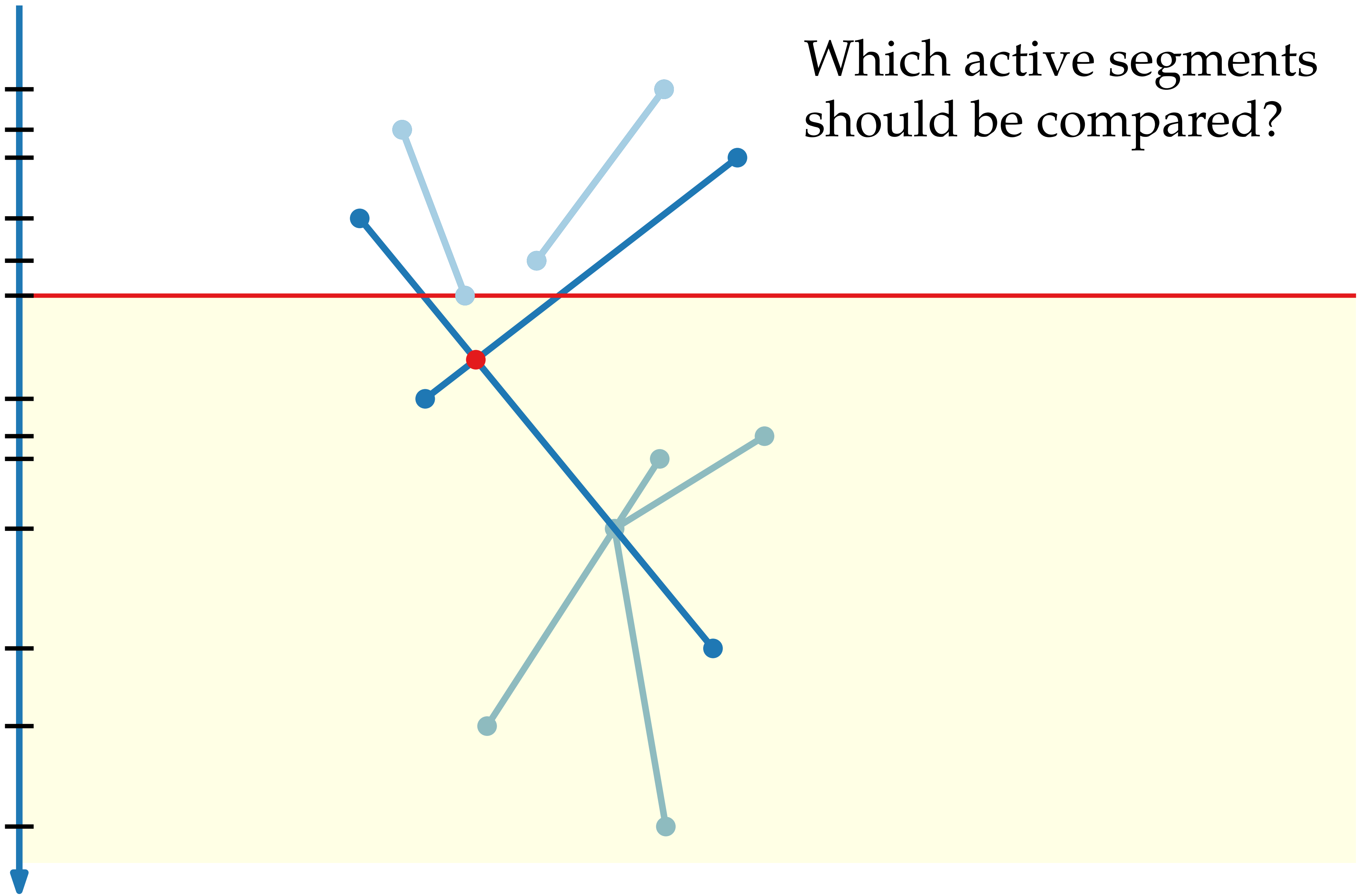

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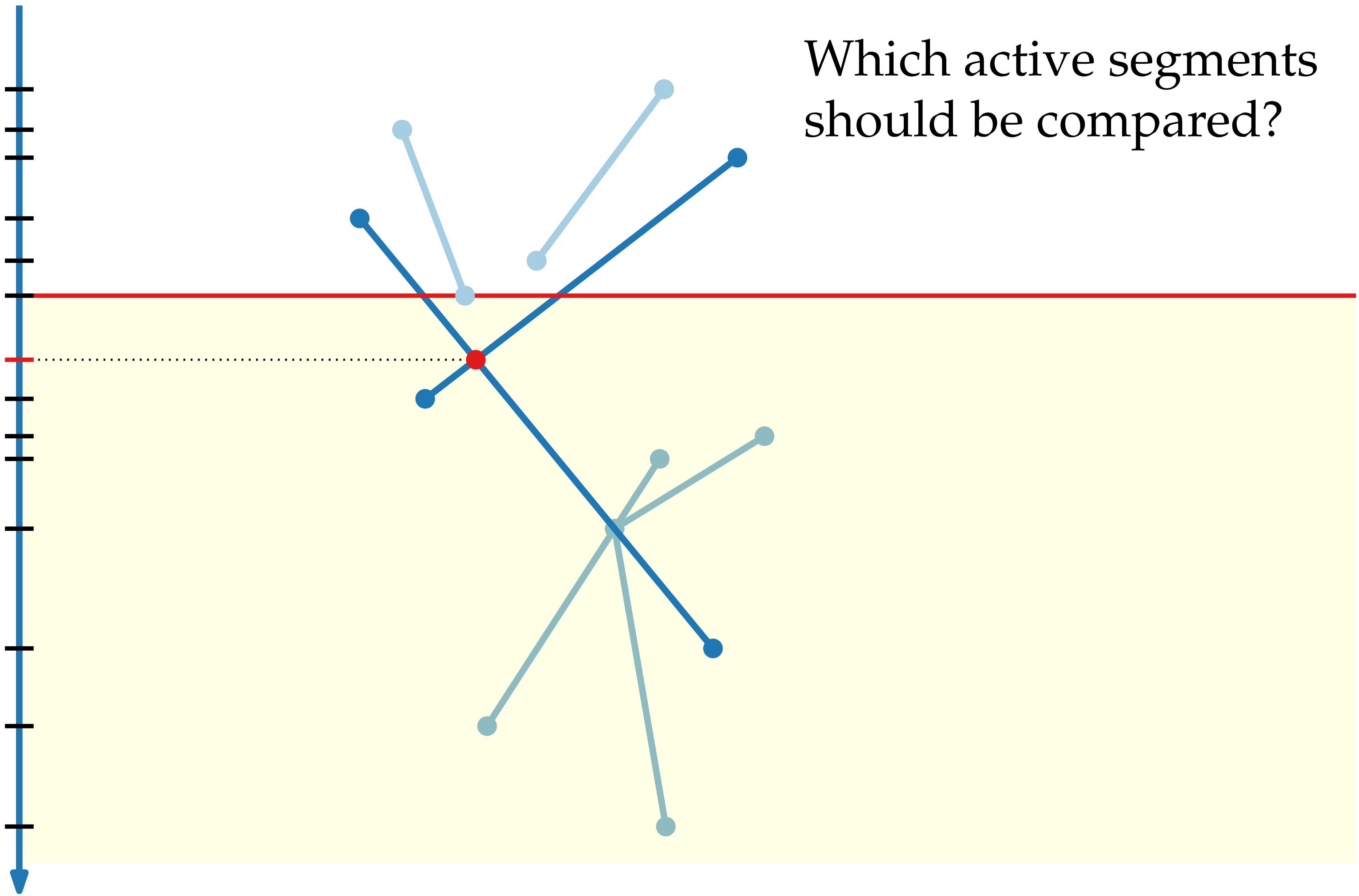

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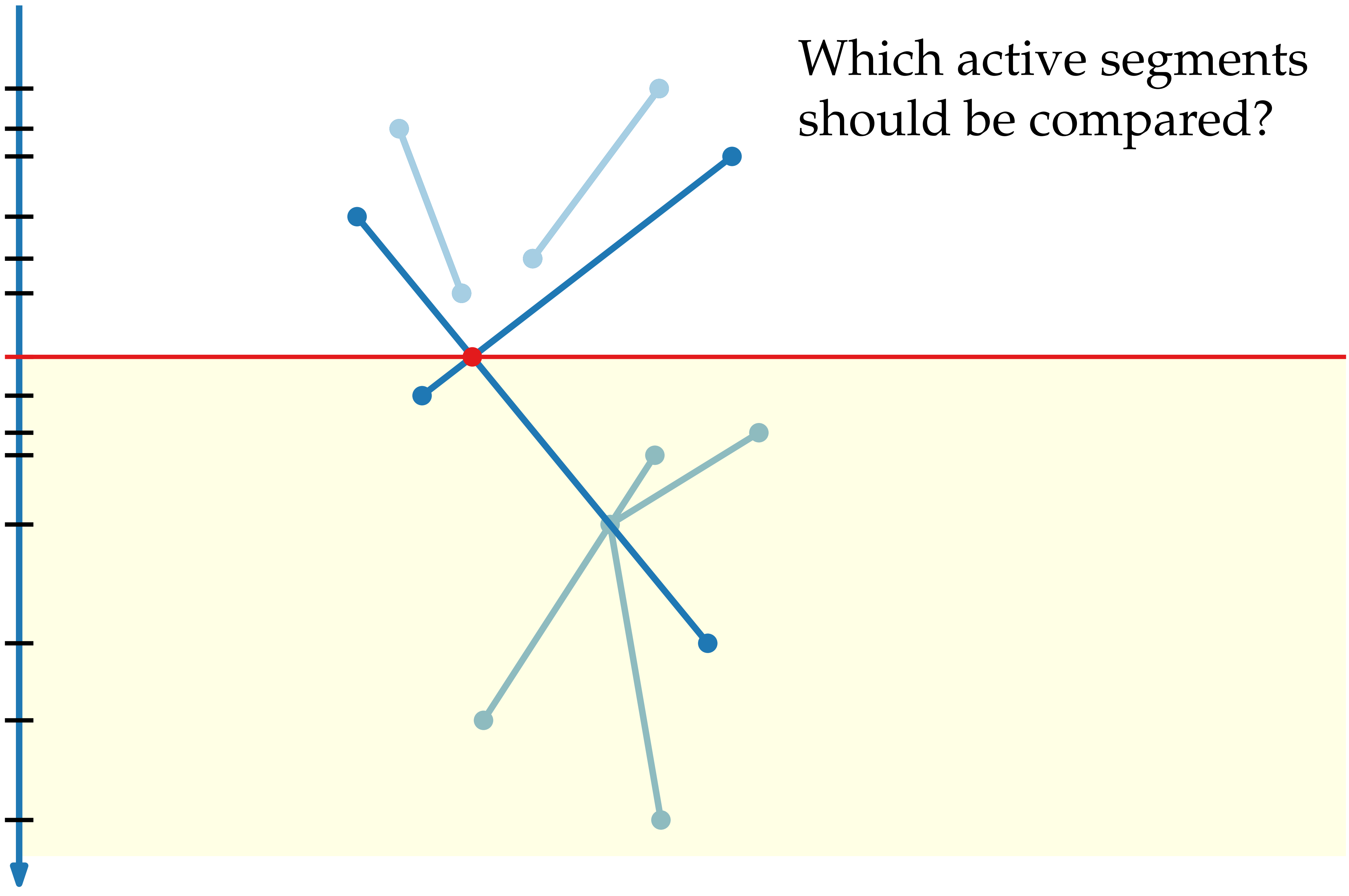

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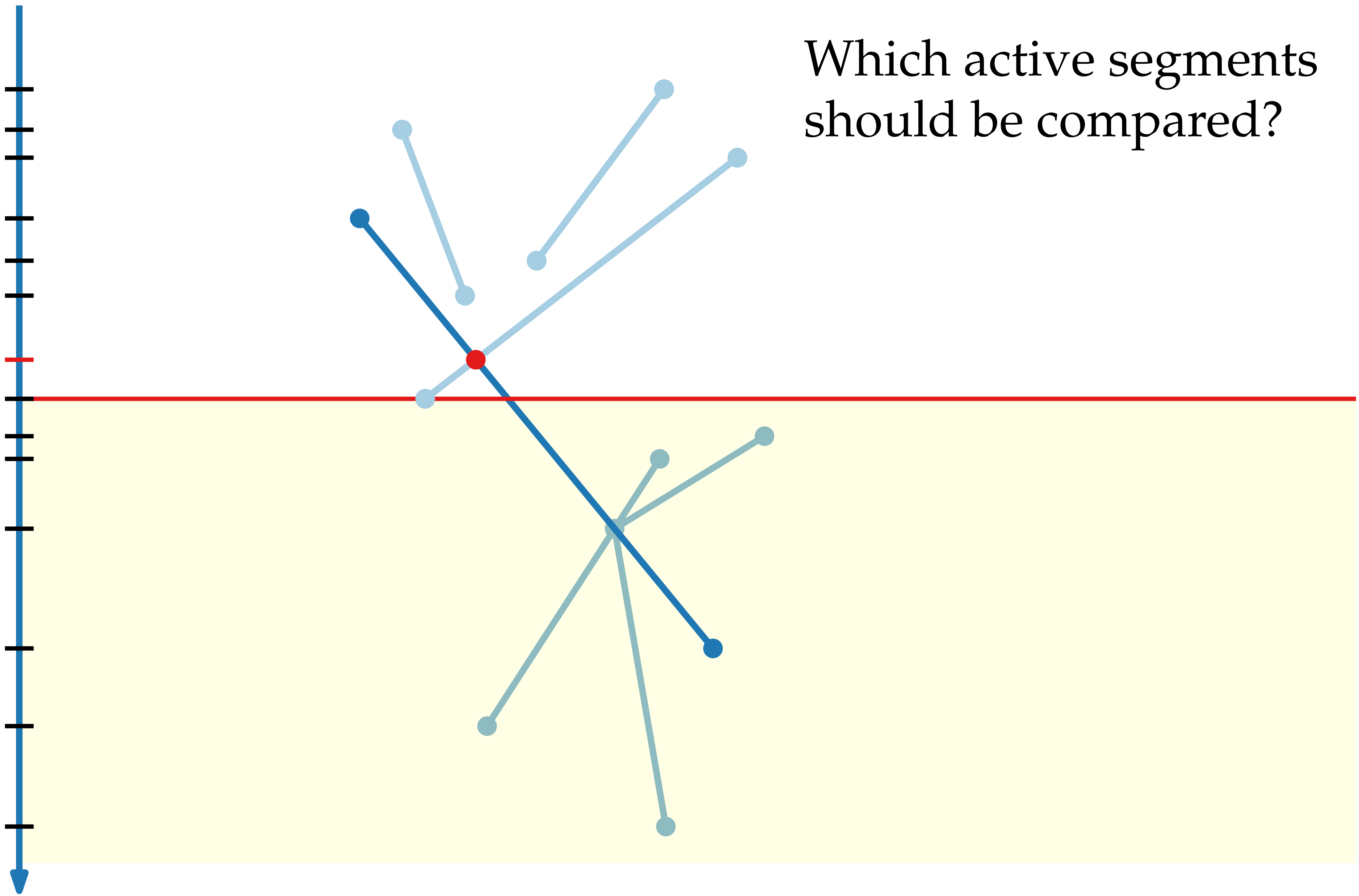

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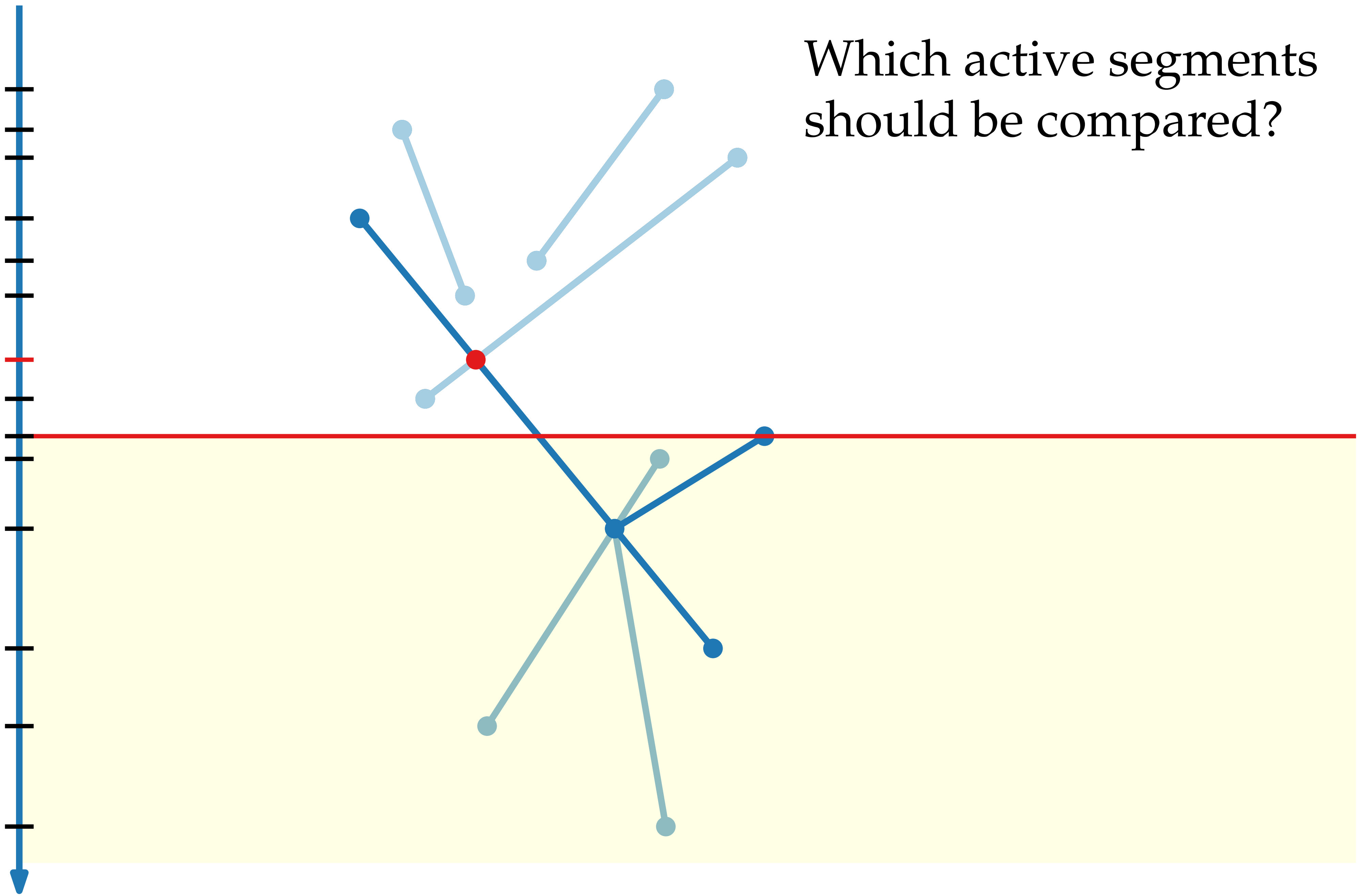

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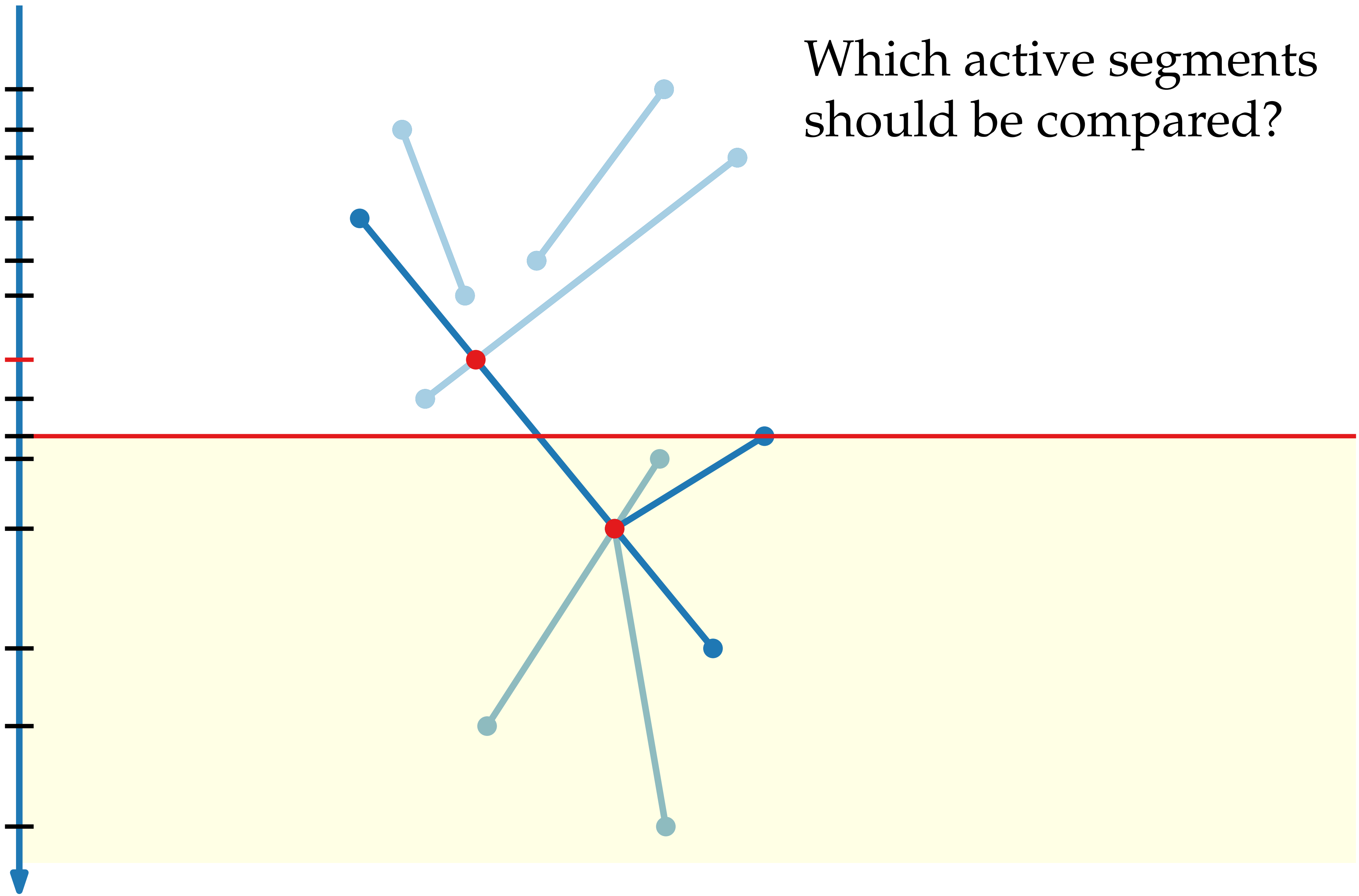

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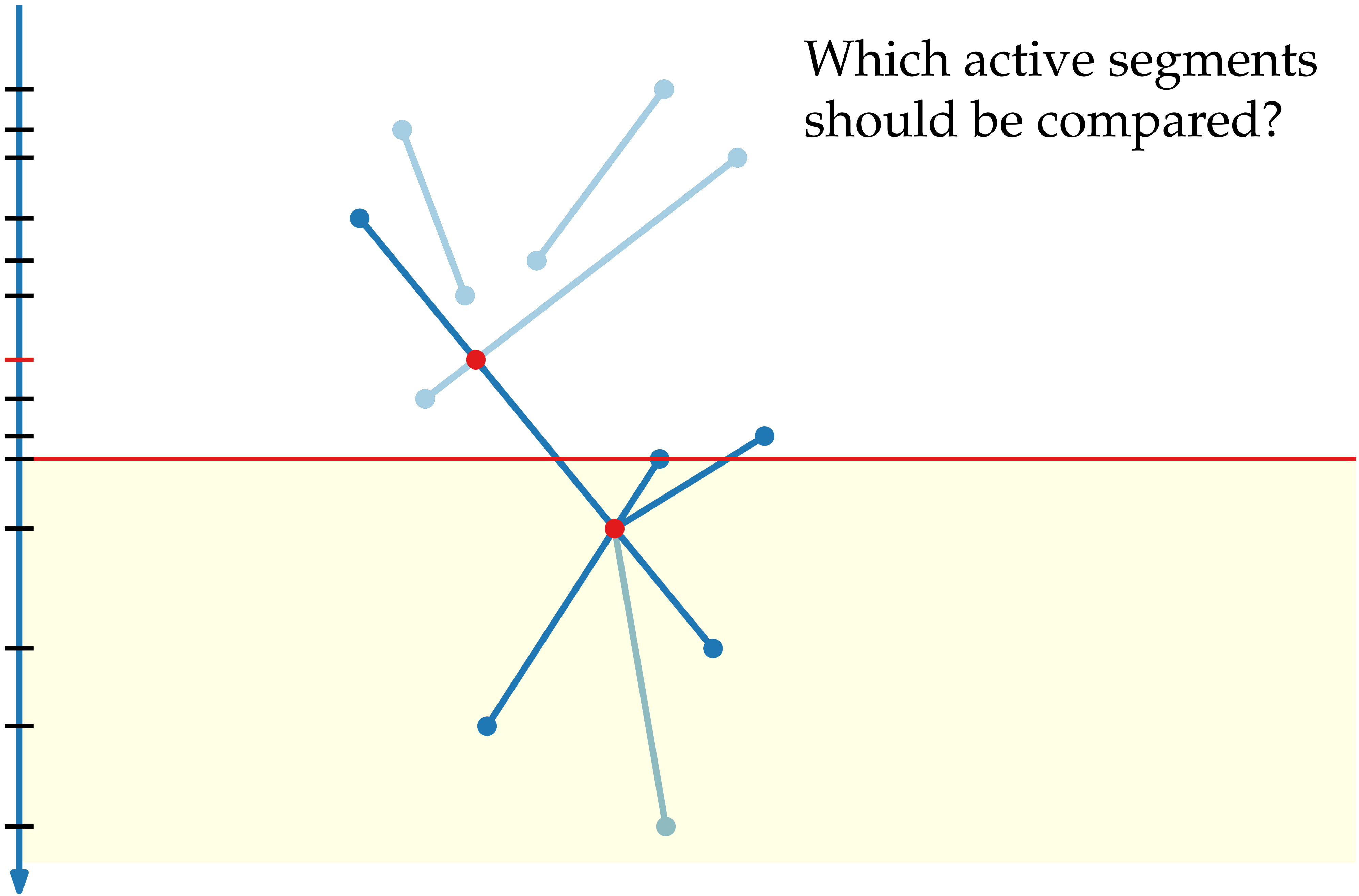

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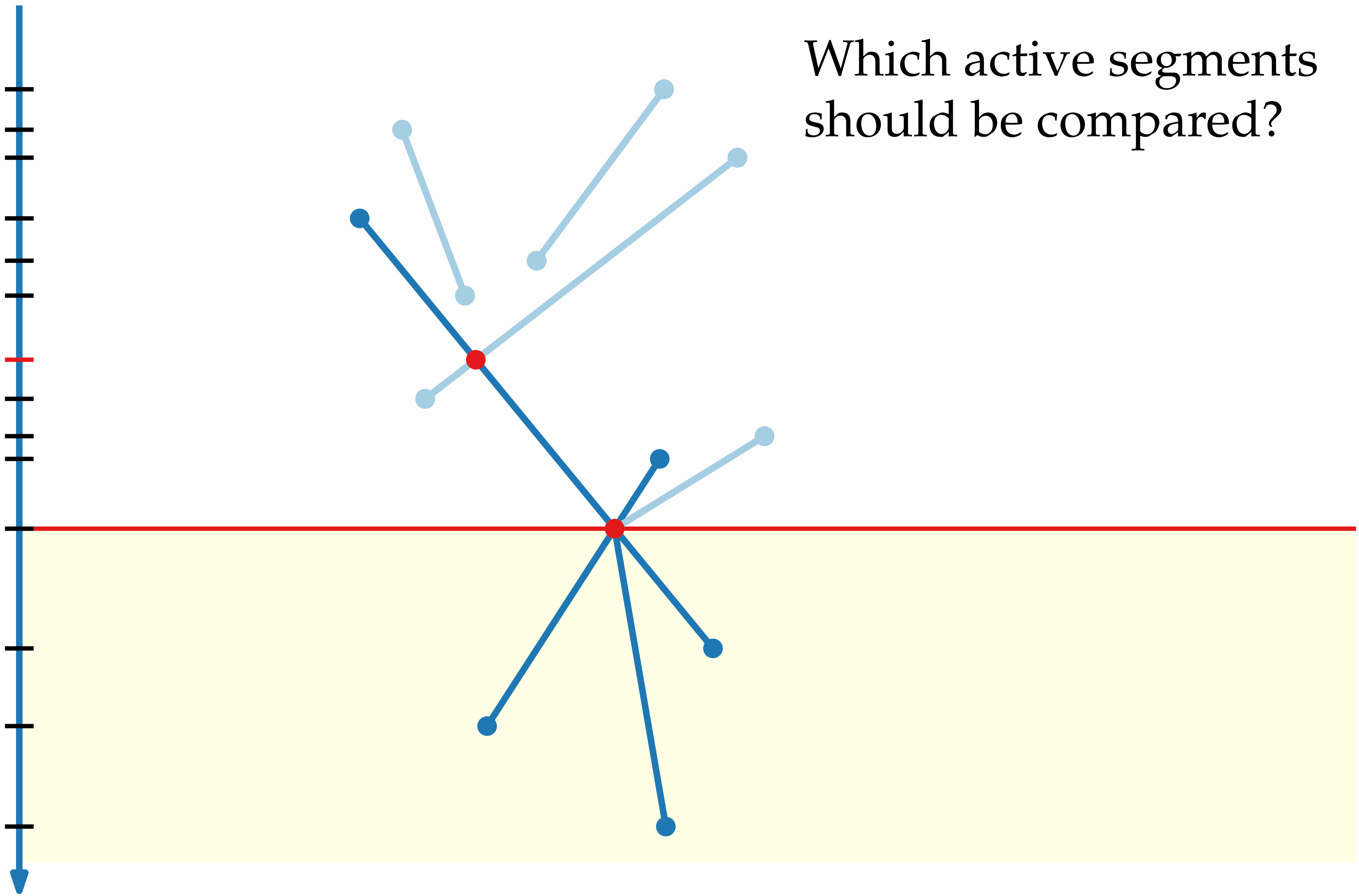

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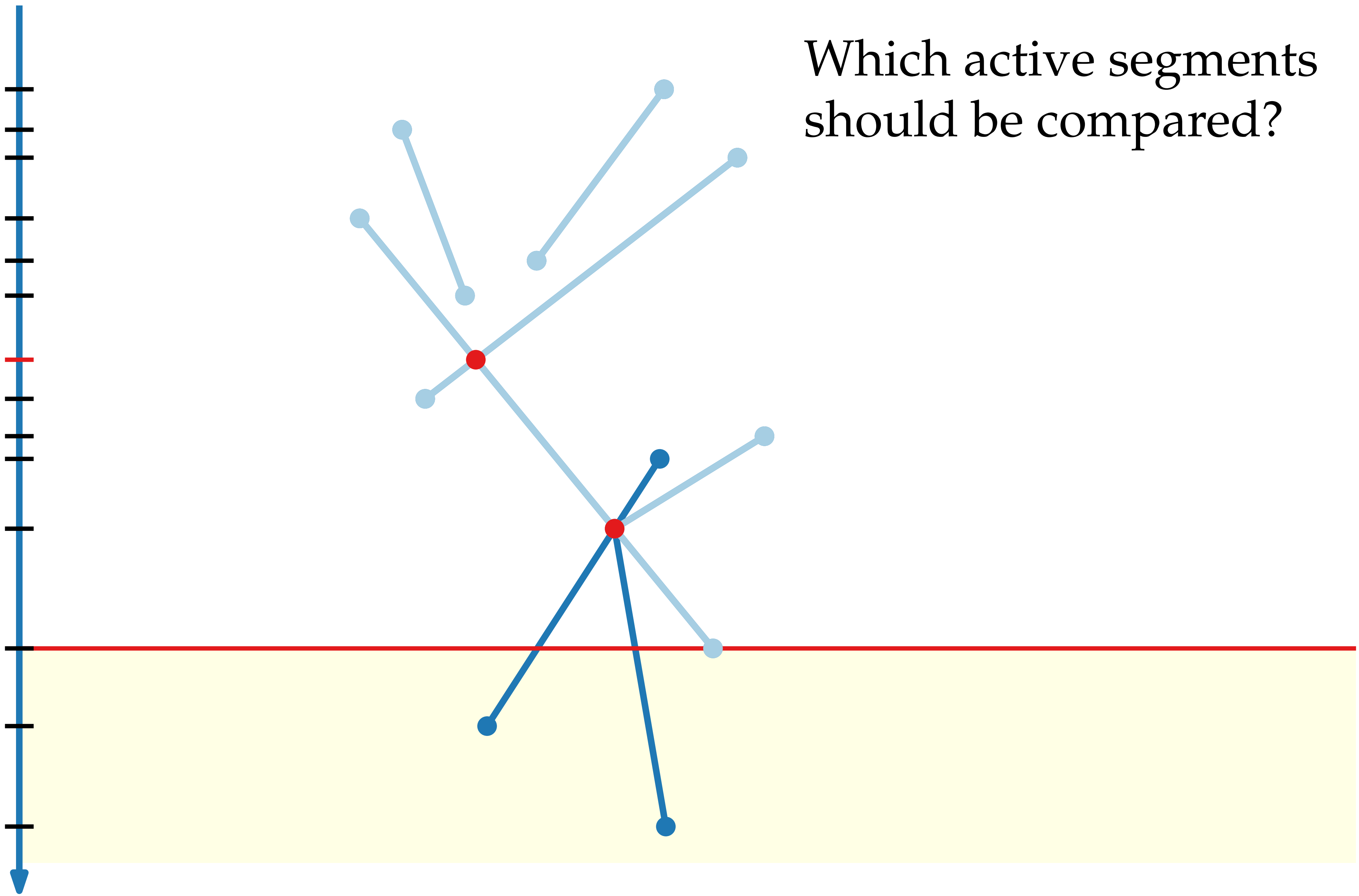

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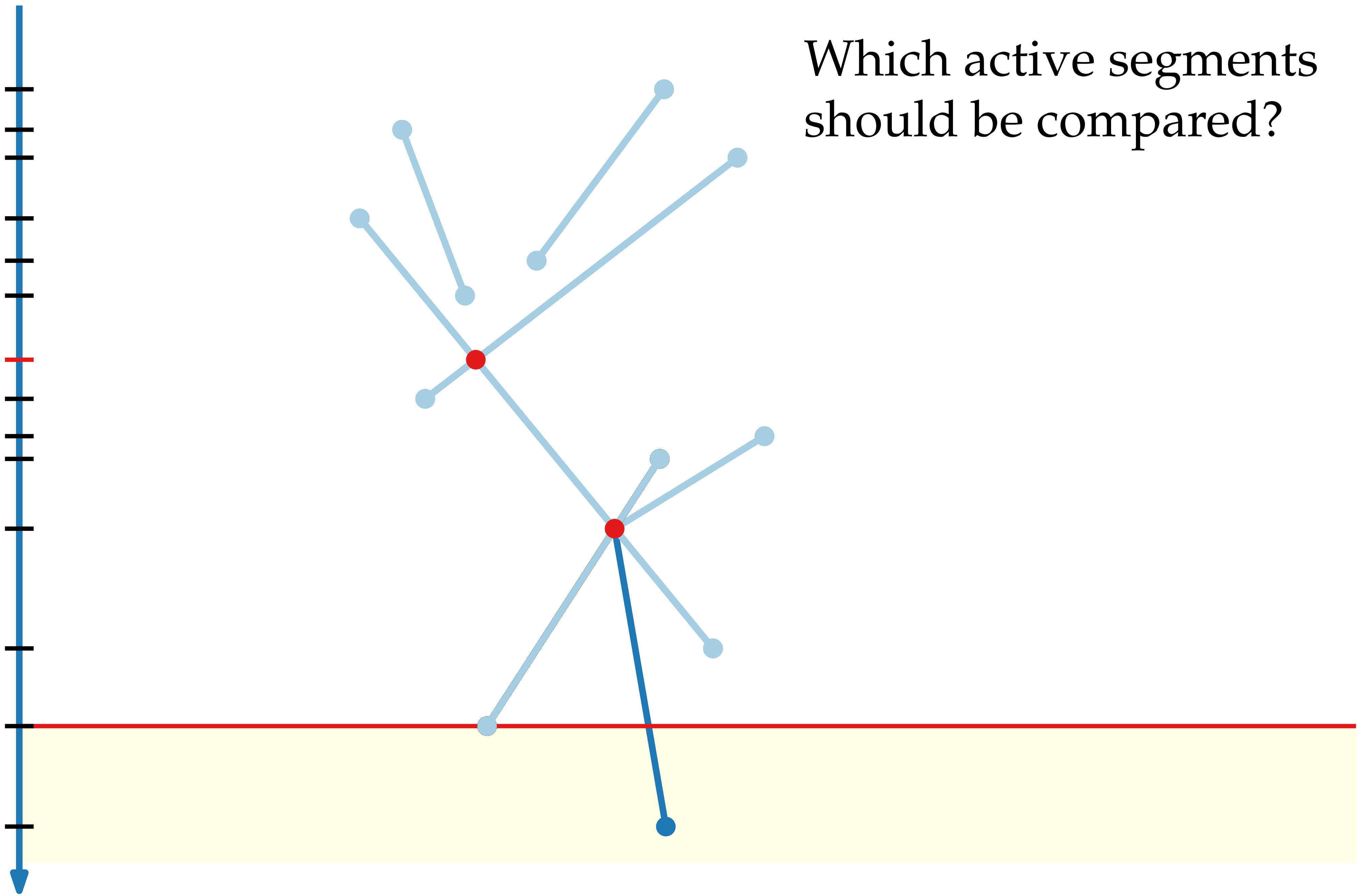

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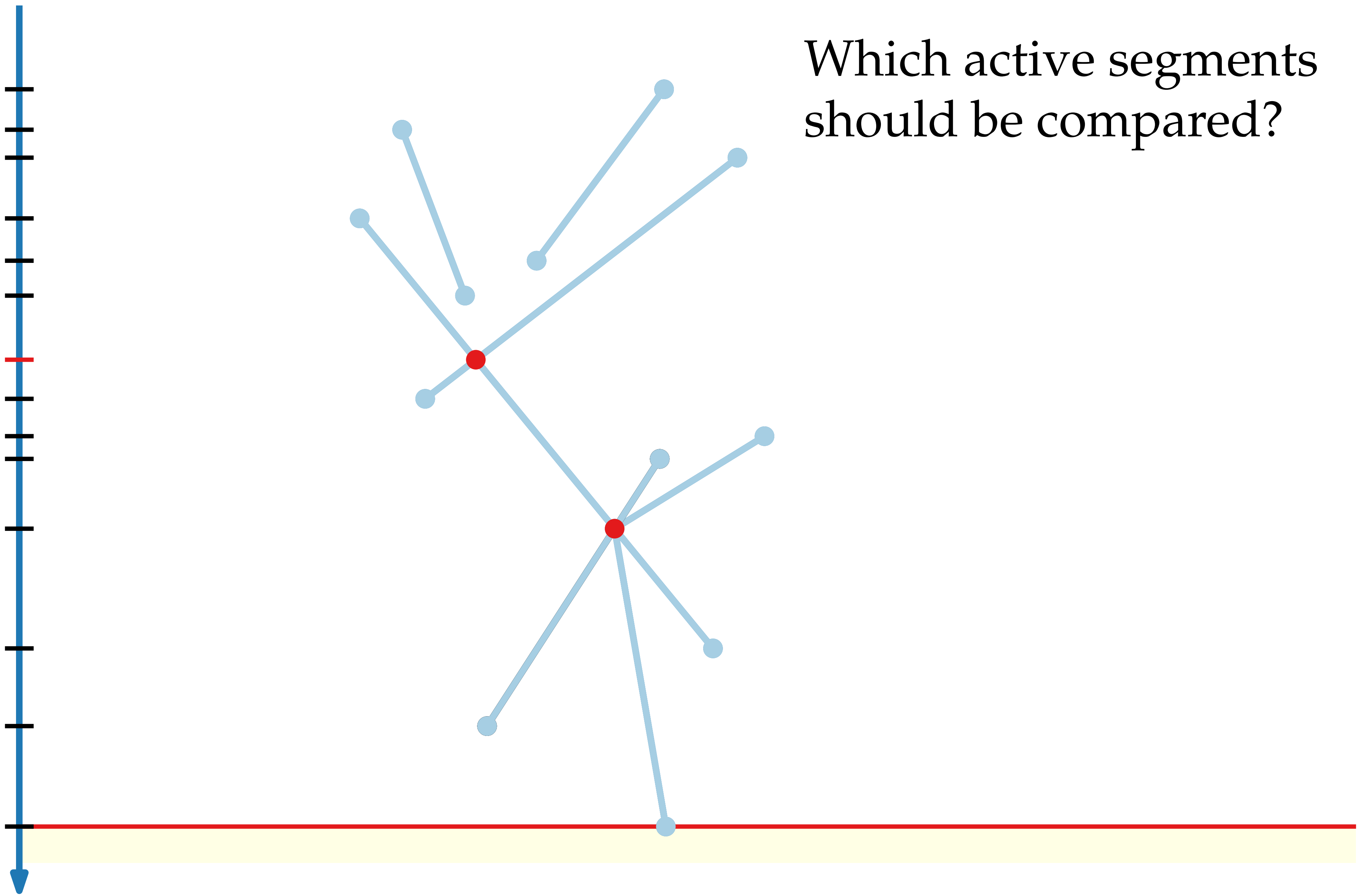

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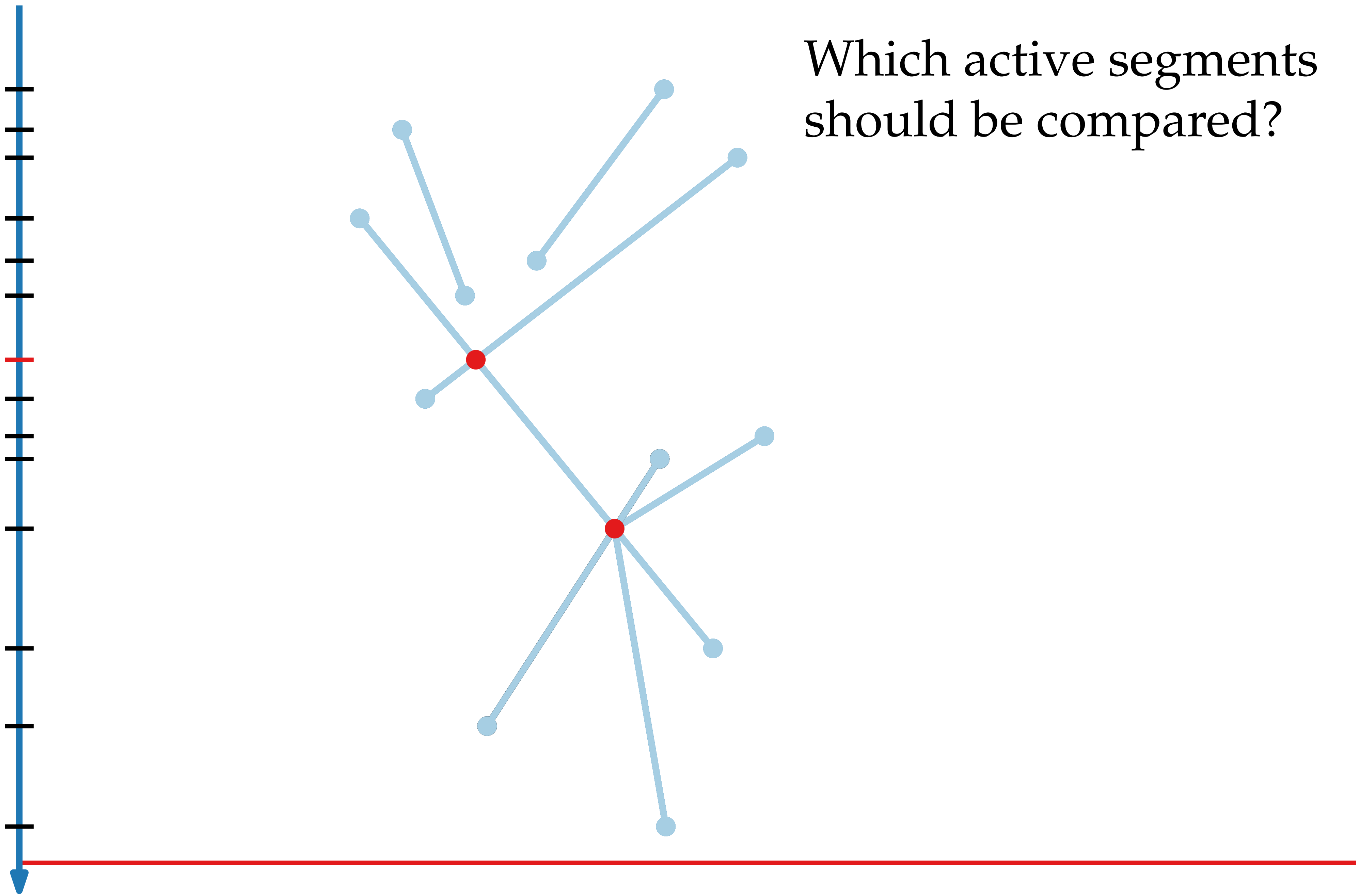

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Data Structures

1) event (-point) queue $\mathcal{Q}$

2) (sweep-line) status $\mathcal{T}$

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$$p \prec q \quad \Leftrightarrow_{\text{def.}}$$

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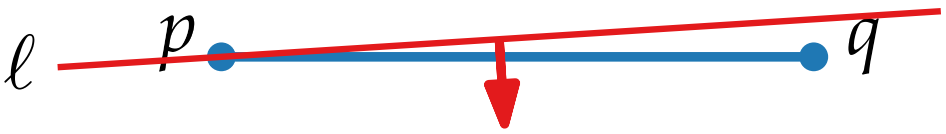

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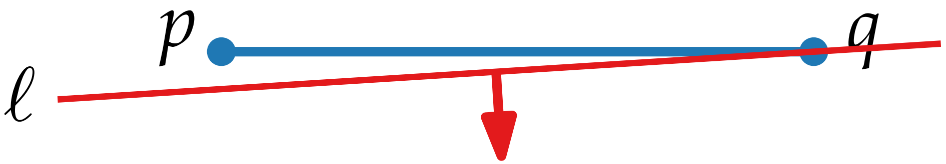

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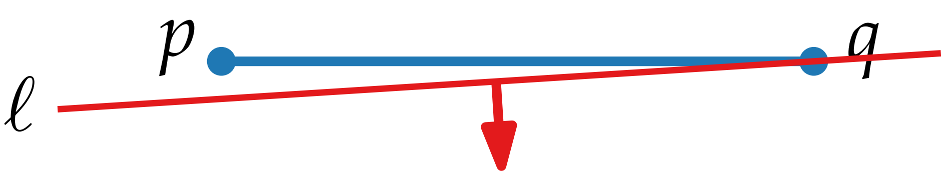

Store event pts in *balanced binary search tree* acc. to $\prec$

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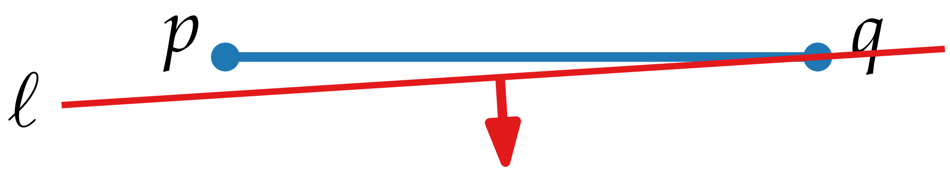

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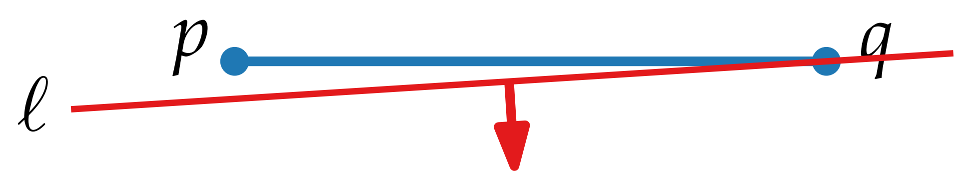

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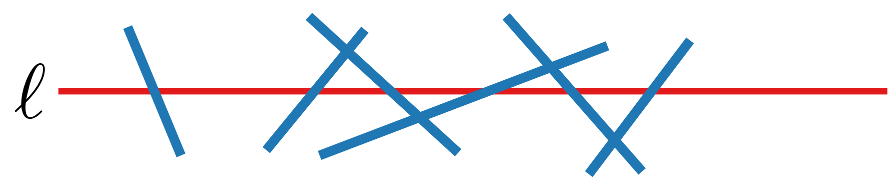

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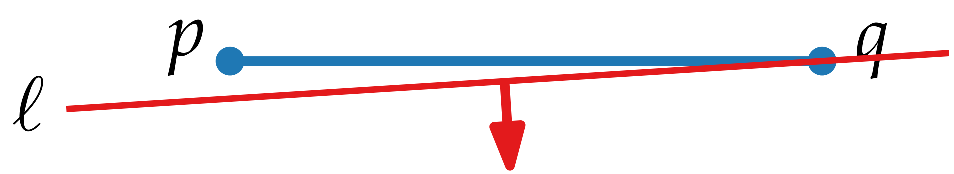

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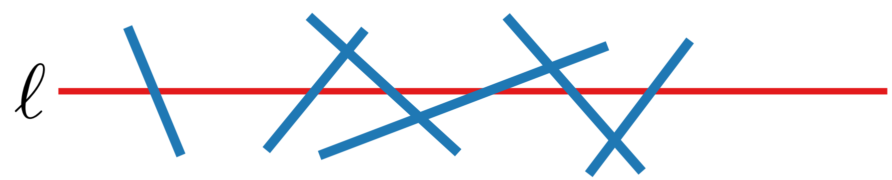

Store the segments intersected by ℓ in left-to-right order.

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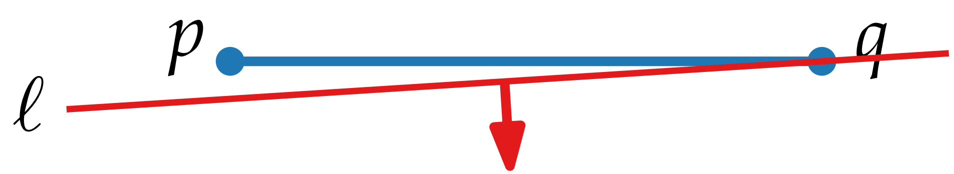

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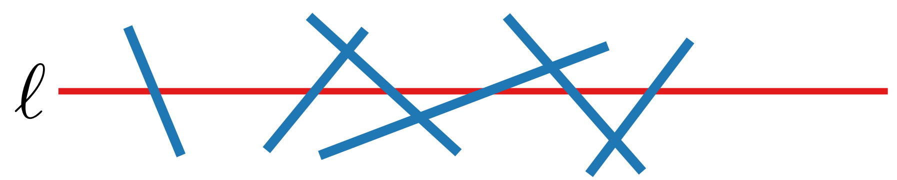

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How?

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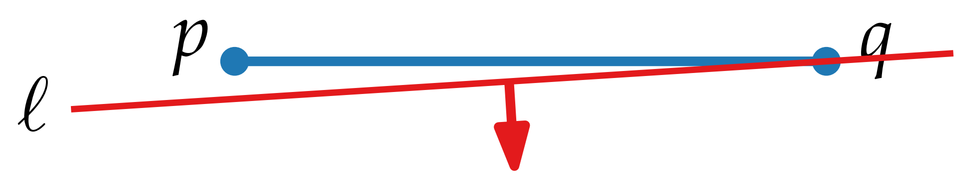

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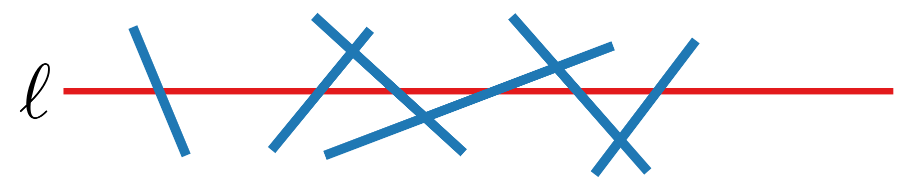
2) (sweep-line) status $\mathcal{T}$



Store the segments intersected by ℓ in left-to-right order.

How? In a balanced binary search tree!

Pseudo-code

findIntersections(S**)**

Input: set S of n non-overlapping closed line segments

Output:
– set I of intersection pts
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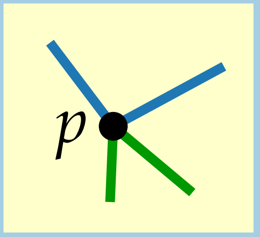

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```
Q ← ∅;  T ← ⟨ vertical lines at x = −∞ and x = +∞ ⟩   // sentinels
foreach s ∈ S do                                      // initialize event queue Q
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        if p ∉ Q then Q.insert(p); L(p) = U(p) = ∅
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while Q ≠ ∅ do
    p ← Q.nextEvent()
    Q.deleteEvent(p)
    handleEvent(p)
```

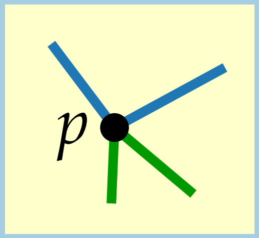

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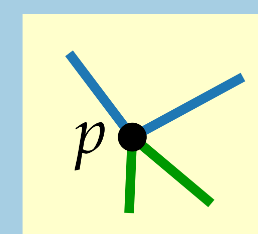

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How would you implement it?

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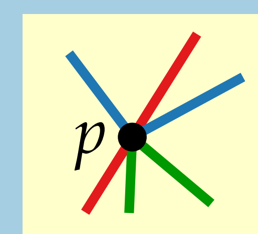

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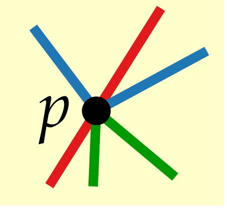
Handling an Event



$C(p), L(p), U(p)$

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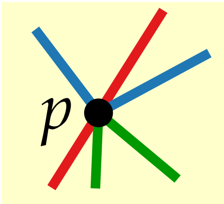

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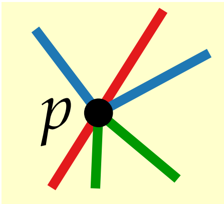

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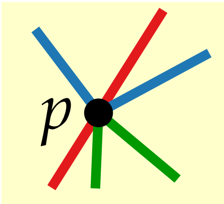

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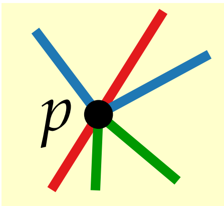

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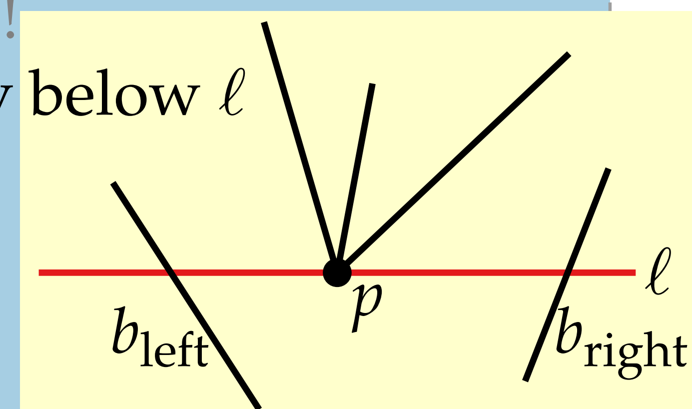

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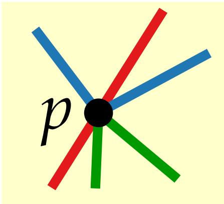

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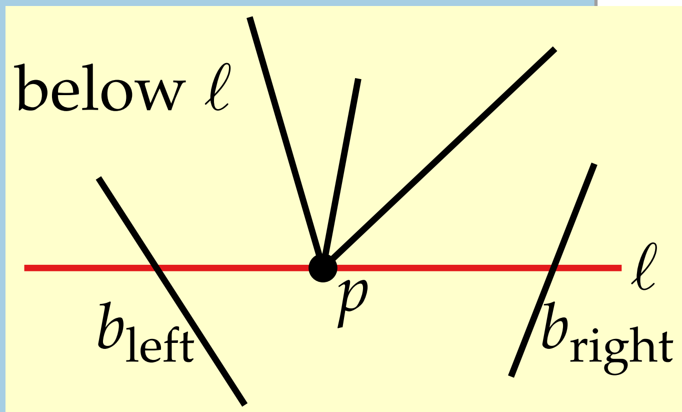

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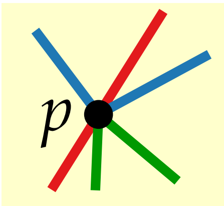

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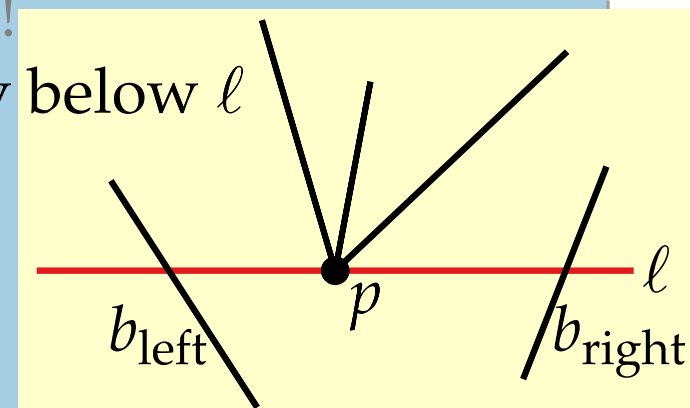

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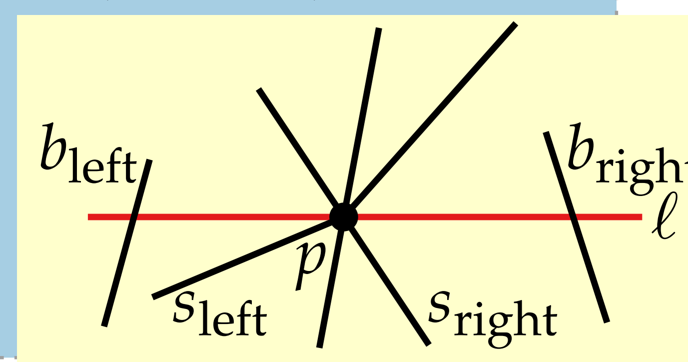

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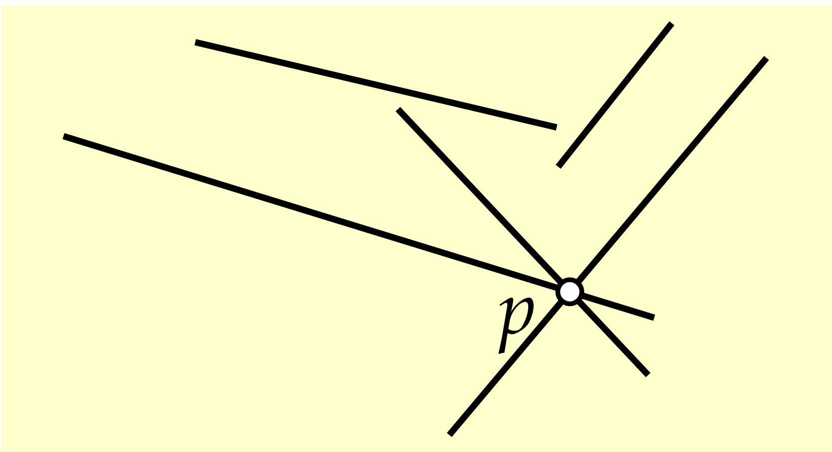

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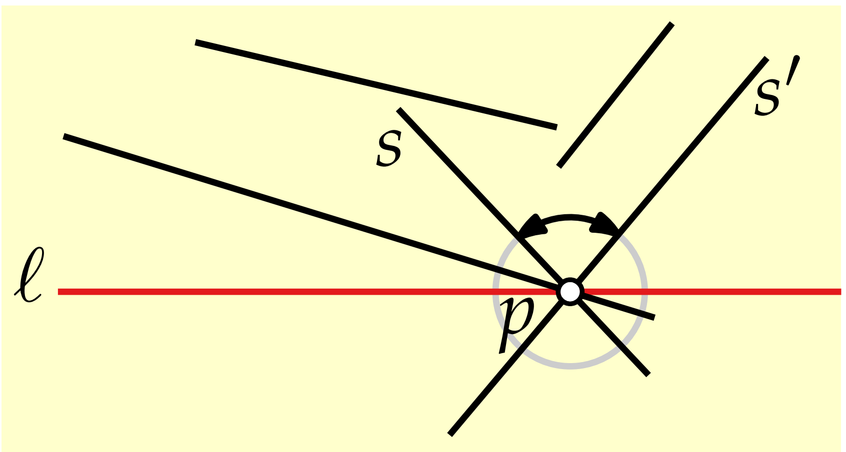

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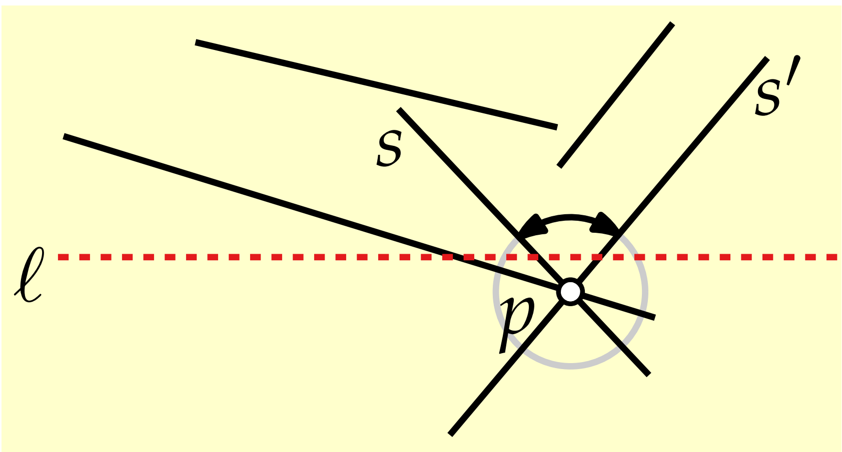

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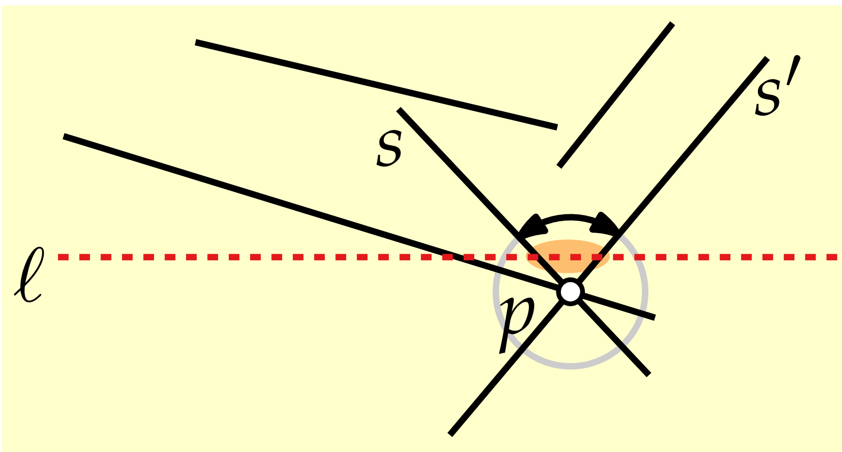

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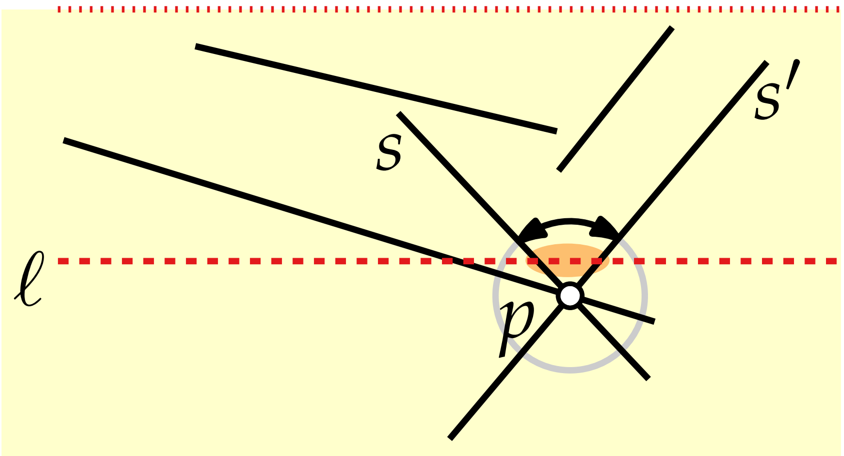

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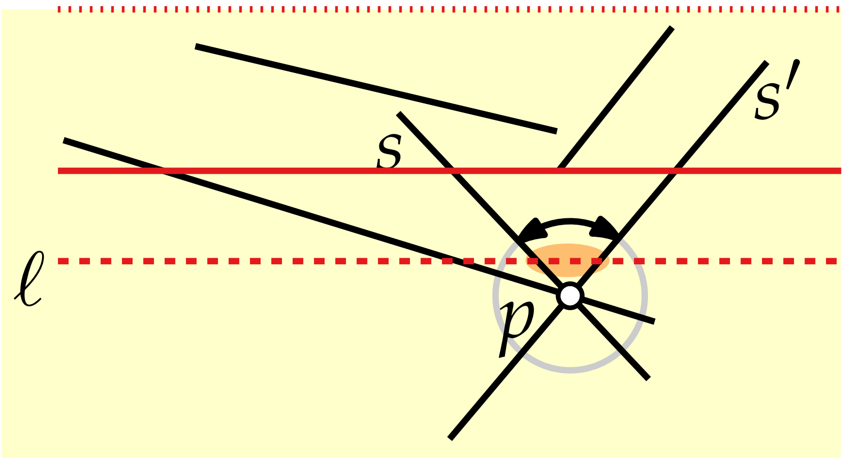

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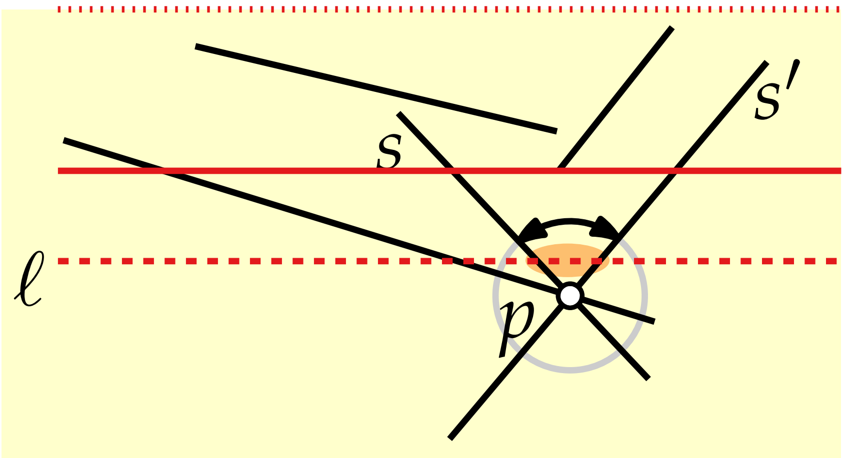

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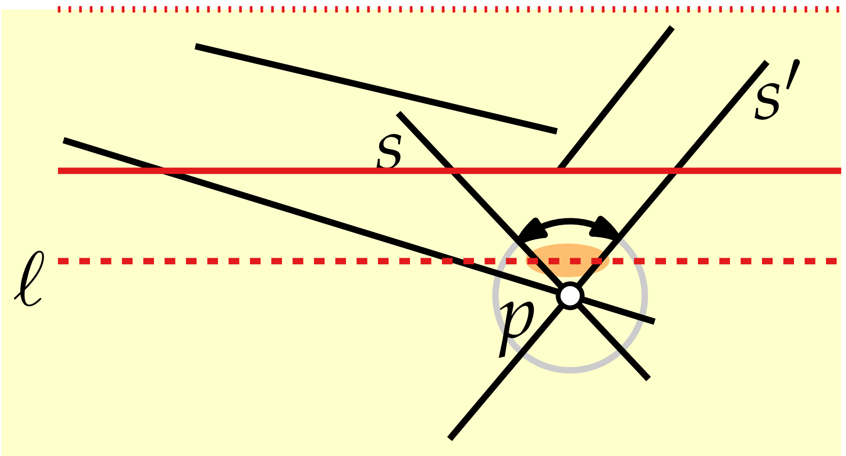

We also need that *every* segment with p as an interior point is added to $C(p)$.

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$\mathcal{Q} \leftarrow \emptyset$; $\mathcal{T} \leftarrow \langle$ vertical lines at $x = -\infty$ and $x = +\infty \rangle$ // sentinels

foreach $s \in S$ **do**

  foreach endpoint p of s **do**

    if $p \notin \mathcal{Q}$ **then** $\mathcal{Q}$.insert(p); $L(p) = U(p) = \emptyset$

    if p lower endpt of s **then** $L(p)$.append(s)

    if p upper endpt of s **then** $U(p)$.append(s)

while $\mathcal{Q} \neq \emptyset$ **do**

  $p \leftarrow \mathcal{Q}$.nextEvent()

  $\mathcal{Q}$.deleteEvent(p)

  handleEvent(p)

handleEvent(event p)

if $|U(p) \cup L(p) \cup C(p)| > 1$ **then**

  report intersection in p, report segments in $U(p) \cup L(p)$

delete $L(p) \cup C(p)$ from $\mathcal{T}$ // consecutive in $\mathcal{T}$!

insert $U(p) \cup C(p)$ into $\mathcal{T}$ in their order slightly below ℓ

if $U(p) \cup C(p) = \emptyset$ **then**

  $b_{\text{left}} / b_{\text{right}} =$ left/right neighbor of p in $\mathcal{T}$

  findNewEvent(b_{left}, b_{right}, p) $\rightarrow$ $\{x\} = s \cap s'$; **if** $x \notin \mathcal{Q}$ **then** $\mathcal{Q}$.insert(x)

else

  $s_{\text{left}} / s_{\text{right}} =$ leftmost/rightmost segment in $U(p) \cup C(p$

  $b_{\text{left}} =$ left neighbor of s_{left} in $\mathcal{T}$

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  findNewEvent(b_{left}, s_{left}, p)

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Running time?

```
Q ← ∅;  T ← ⟨ vertical lines at x = −∞ and x = +∞ ⟩ // sentinels
foreach s ∈ S do
    foreach endpoint p of s do
        if p ∉ Q then Q.insert(p); L(p) = U(p) = ∅
        if p lower  endpt of s then L(p).append(s)
        if p upper endpt of s then U(p).append(s)

while Q ≠ ∅ do
    p ← Q.nextEvent()
    Q.deleteEvent(p)
    handleEvent(p)
```

```
handleEvent(event p)
if |U(p) ∪ L(p) ∪ C(p)| > 1 then
    report intersection in p, report segments in U(p) ∪ L(p)
delete L(p) ∪ C(p) from T // consecutive in T!
insert U(p) ∪ C(p) into T in their order slightly below ℓ
if U(p) ∪ C(p) = ∅ then
    b_left / b_right = left/right neighbor of p in T
    findNewEvent(b_left, b_right, p)
else
    s_left / s_right = leftmost/rightmost segment in U(p) ∪ C(p
    b_left = left neighbor of s_left in T
    b_right = right neighbor of s_right in T
    findNewEvent(b_left, s_left, p)
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```
{x} = s ∩ s′
if x ∉ Q then Q.insert(x)
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Running Time

Check your knowledge about planar graphs!

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and $m = \sum_p m(p)$.

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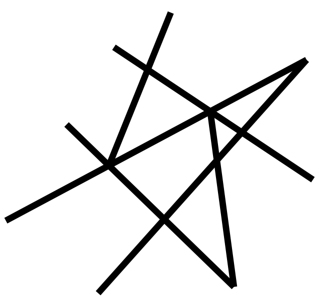

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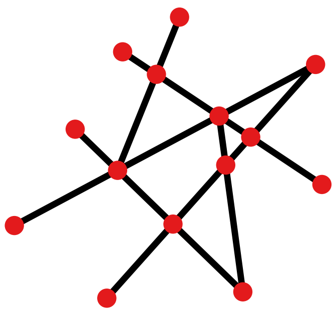

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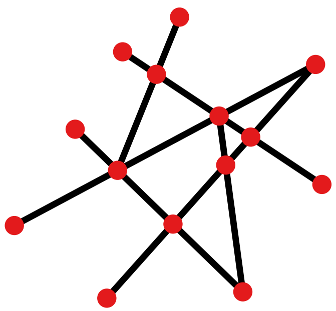

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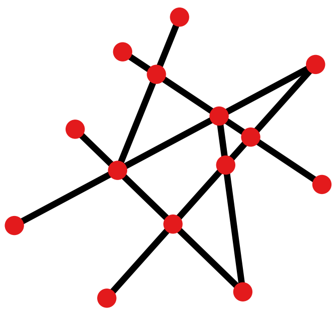

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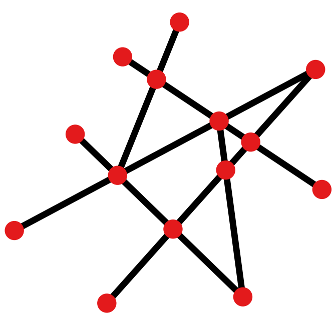

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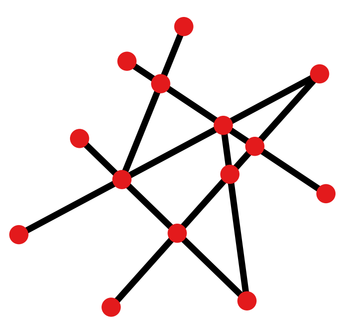

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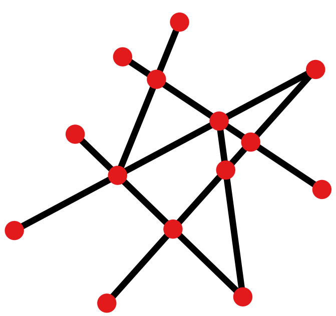

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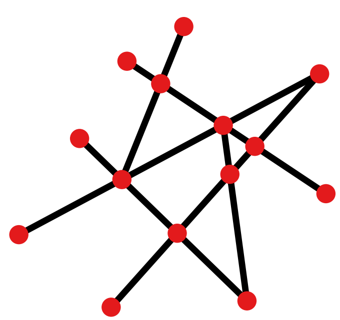

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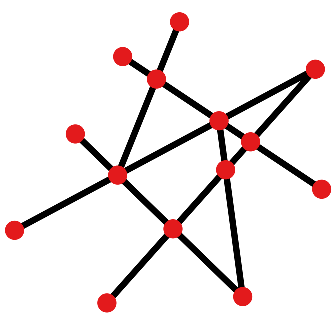

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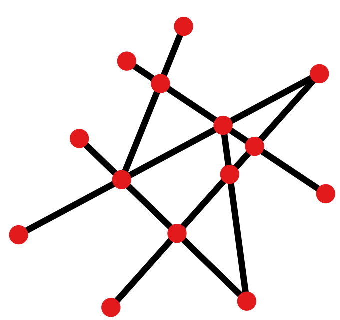

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Theorem. We can report all I intersection points among n non-overlapping line segments in the plane and report the segments involved in the intersections in $O((n+I)\log n)$ time and $O(n)$ space.

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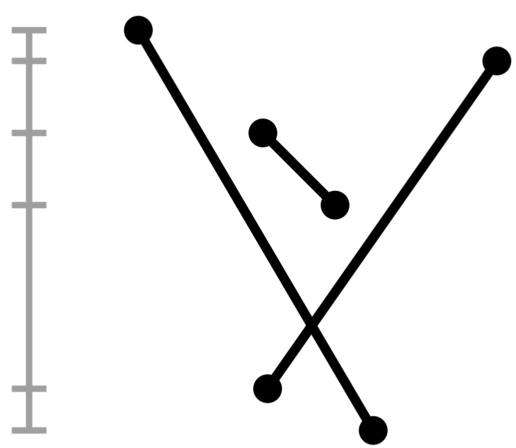

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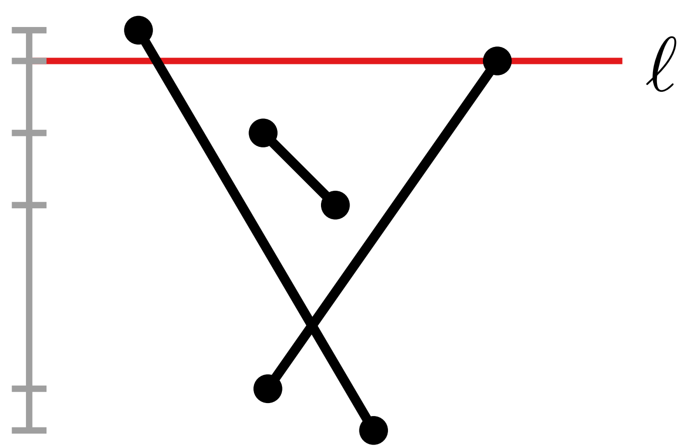

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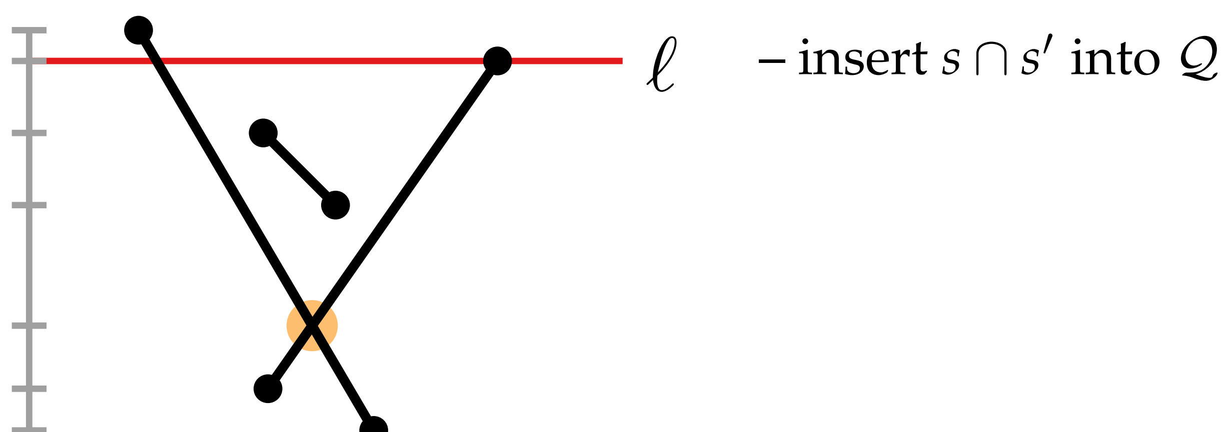

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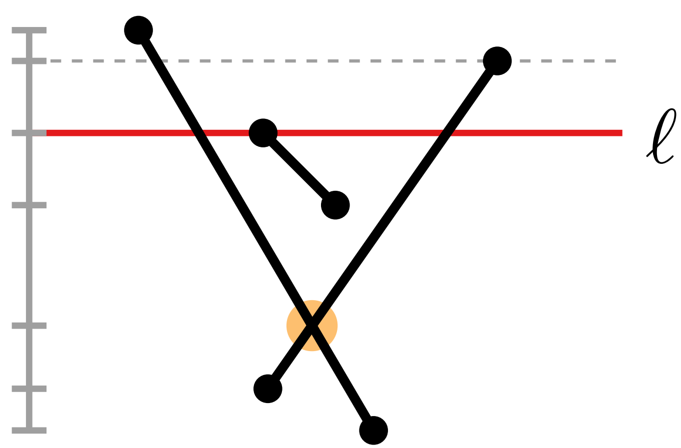

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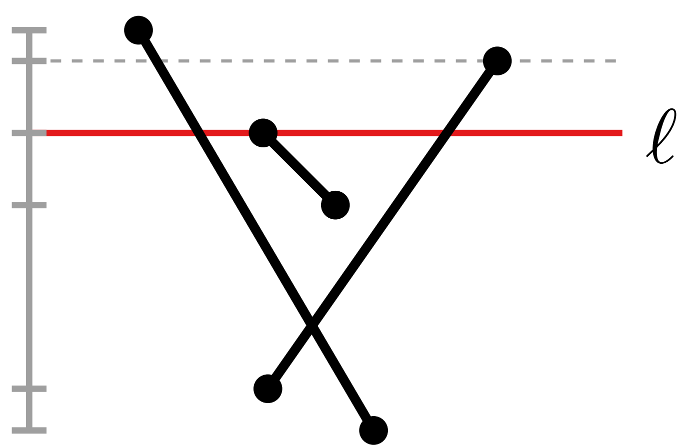

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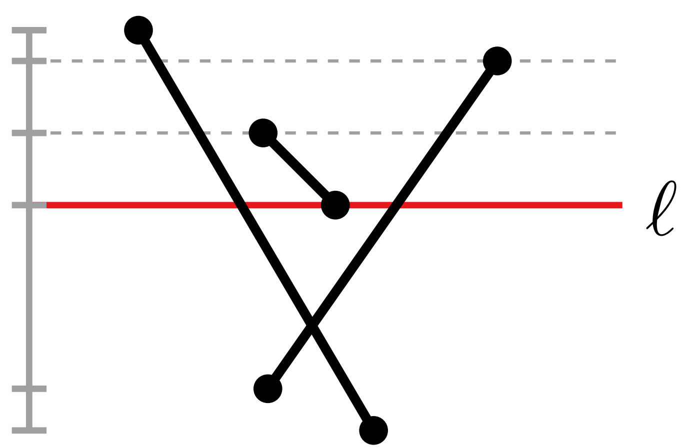

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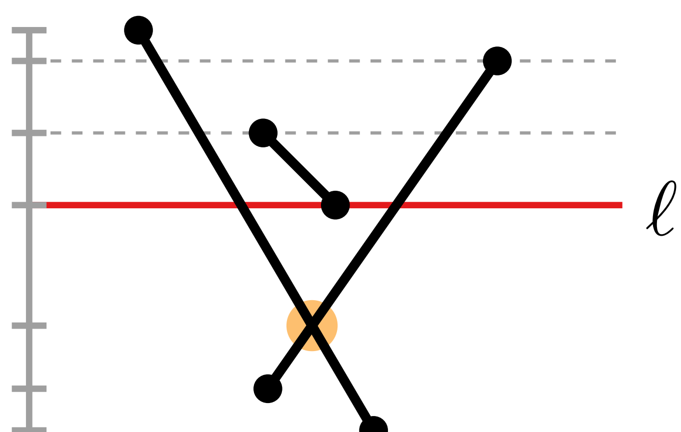

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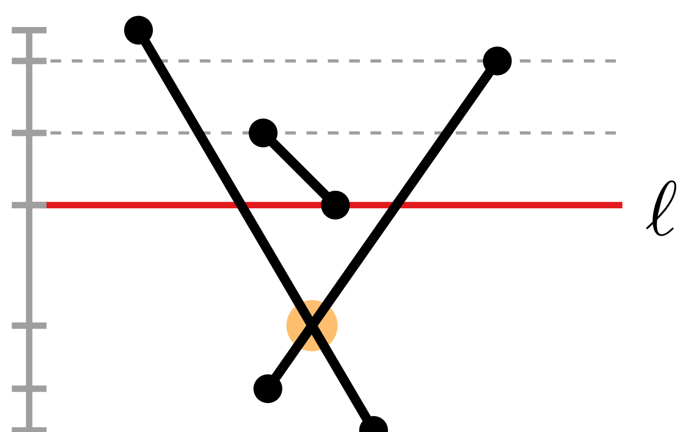

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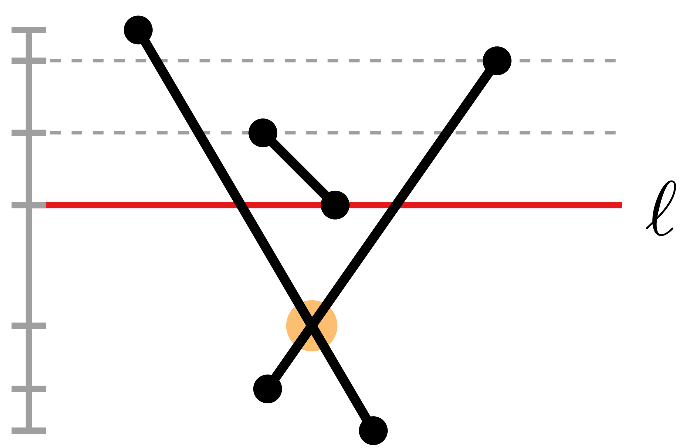

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$\Rightarrow$ need just $O(n)$ space; (asymptotic) running time doesn't change

$\square$