Homework Assignment #13
Computational Geometry (Winter Term 2017/18)

Exercise 1
The dual of a line segment is a left-right double wedge, as was shown in the lecture.

a) What is the dual of the set of points inside a given triangle with vertices \(p, q\) and \(r\)? \([3 \text{ points}]\)

b) What type of object in the primal plane would dualize to a top-bottom double wedge? \([3 \text{ points}]\)

Exercise 2
Let \(P\) be a simple regular polygon with \(n\) vertices. Let \(\Delta_{\text{blue}}\) and \(\Delta_{\text{red}}\) be triangulations of \(P\), each given as a DCEL. We call the intersection points of edges from \(\Delta_{\text{blue}}\) and \(\Delta_{\text{red}}\) *Steiner vertices*. Let \(k\) be the number of Steiner vertices.

Give an \(O(n + k)\) algorithm to compute a DCEL describing the *overlay* of \(\Delta_{\text{blue}}\) and \(\Delta_{\text{red}}\), which consists of \(P, \Delta_{\text{blue}}, \Delta_{\text{red}}\) and the Steiner vertices. \([4 \text{ points}]\)

Exercise 3
Let \(R\) be a set of \(n\) red points in the plane, and let \(B\) be a set of \(n\) blue points in the plane. We call a line \(l\) a *separator* for \(R\) and \(B\) if \(l\) has all points of \(R\) to one side and all points of \(B\) to the other side.

a) Give a deterministic algorithm that can decide in \(O(n \log n)\) time whether \(R\) and \(B\) have a separator. \([5 \text{ points}]\)

b) Give a randomized algorithm that can decide in \(O(n)\) expected time whether \(R\) and \(B\) have a separator. \([5 \text{ points}]\)

This assignment is due at the beginning of the next lecture, that is, on February 7 at 10:15. Solutions will be discussed in the tutorial on Friday, February 9, 14:15–15:45 in room SE I.