Exercise 1

In this exercise, we look at the number of different triangulations that a set of \( n \) points in the plane admits.

a) Prove that no set of \( n \) points can be triangulated in more than \( 2^{\binom{n}{2}} \) ways. [2 points]

b) Can you give an (asymptotically) sharper upper bound on the number of possible triangulations? [2 extrapoints]

c) Prove that there are sets of \( n \) points that can be triangulated in at least \( 2^{n-2\sqrt{n}+1} \) different ways. [4 points]

d) Can you give an example that the points can be triangulated in more different ways (asymptotically)? [2 extrapoints]

Exercise 2

Prove that any two triangulations of a planar point set can be transformed into each other by edge flips.

Hint: Start by showing that any two triangulations of a convex polygon can be transformed into each other by edge flips. [5 extrapoints]

Exercise 3

Prove that the smallest angle of any triangulation of a convex polygon whose vertices lie on a circle is the same. This implies that any completion of the Delaunay graph of a set of points to a Delaunay triangulation maximizes the minimum angle.

Hint: You can use the result of Exercise 2. [4 points]

Please turn over.
Exercise 4

Let $P$ be a set of points in the plane and let $G_P = (P, E_P)$ be the graph with $(p, q) \in E_P$ if and only if $p$ and $q$ are the only points in $P$ that are contained inside the disk $D_{pq}$ with diameter $pq$, i.e., $D_{pq} \cap P = \{p, q\}$.

a) Prove that the Delaunay graph of $P$ contains the graph $G_P$.  
   \[3 \text{ points}\]

b) Prove that $(p, q)$ is an edge of $G_P$ if and only if the Delaunay edge between $p$ and $q$ intersects its dual Voronoi edge.  
   \[3 \text{ points}\]

c) Show how to compute $G_P$ for a set $P$ of $n$ points in $O(n \log n)$ time.  
   \[4 \text{ points}\]

This assignment is due at the beginning of the next lecture, that is, on January 10 at 10:15. Solutions will be discussed in the tutorial on Friday, January 12, 14:15–15:45 in room SE I.